SIMPLE STRESSES AND STRAINS

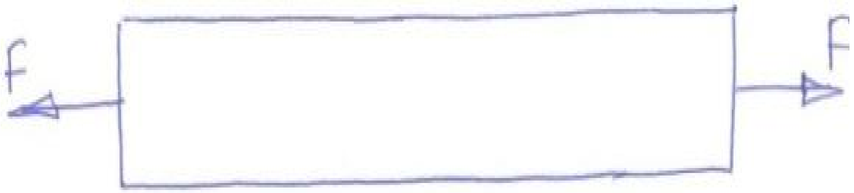
Prof.H.Sudarsana Rao

Civil Engineering Department

JNTUACEA, ANANTAPUR

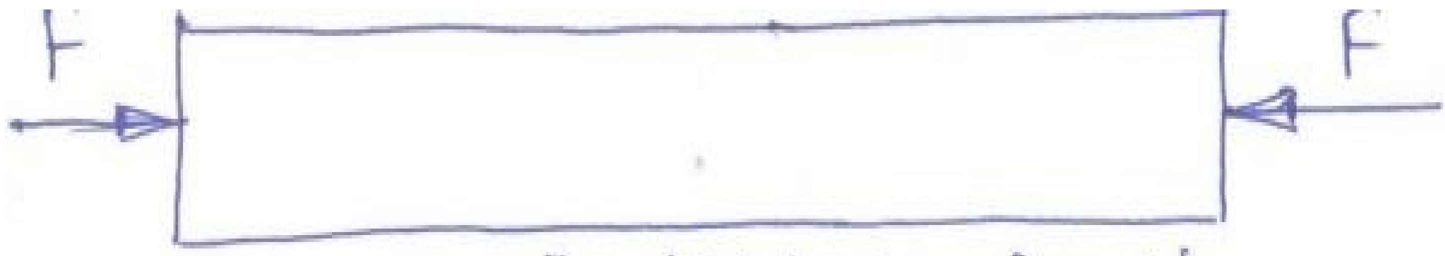
Classification of forces

- **a. Tensile force:** Tensile force is a type of loading in which the two sections of material on either side of a plane along its length tend to be pulled apart or elongated.



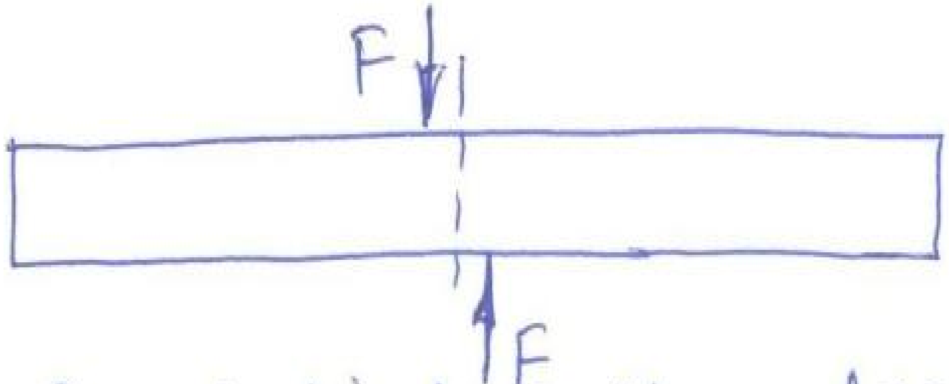
Bar subjected to tension

- **b. Compressive force:** Compressive force is a type of loading in which the two sections of material on either side of a plane along its length tend to be pushed or compressed.
- Tensile or compressive stress acts normal to the stress plane.



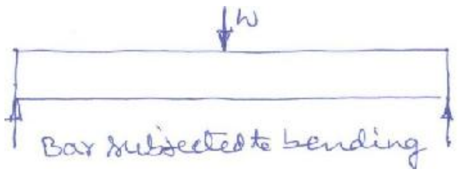
Bar subjected to compression

- **c. Shear force:** Shear involves applying a load parallel to a plane which caused the material on one side of the plane to want to slide across the material on the other side of the plane.
- A shearing stress acts parallel to the stress plane.
- Shear properties are primarily used in the design of mechanically fastened components, webs, and torsion members, and other components subject to parallel, opposing loads.



Bar subjected to shear force

- **d. Bending moment:** Bending involves applying a transverse load in a manner that
- causes a material to curve shape and results in compressing the material on one side and stretching it on the other.



- **e. Torsion or twicesting moment:** Torsion is the application of a force that causes twice sting in a material. Torsion induced in the material when the transverse load is not lying on the longitudinal axis (away from the longitudinal axis).

Bar subjected to twisting

Types of loads

- **Static loading** is a constant force acting on a material.
- **Dynamic or cyclic loading** is not constant force but fluctuates on the material.
- Load is the combined effect of external forces acting on a body.
  a) Point load or concentrated load: It is the load considered to act at a point.
  b) Distributed load : The load is distributed or spread in some manner over the length of the beam.
  c) Uniformly distributed load (u.d.l): The load is distributed or spread uniformly.
  d) Uniformly varying load (u.v.l): The load distributed or spread is not uniform i.e. varying along the length. eg.Triangular load, trapezoidal load, parabolic load, etc.
  e) Parabolic loading
  f) Concentrated moment
  g) Uniformly distributed moment
  h) Inclined load

Stress: Stress is defined as the internal resistance offered by the body against deformation.

$$\sigma = \frac{P}{A}$$

σ : Stress (tensile or compressive)

P : Axial load

A : Cross sectional area

Units of stress : N/mm^2, kN/m^2

Simple stress is also called direct stress.

Stress is the force required per unit cross sectional area.

- Crushing stress is a localized compressive stress at the area of contact between two members.
- Maximum stress of a material depends on the type of material.
- Stress at a point is tensor.

- **Compressive stress** (or compression) is the stress state caused by an applied load that acts to reduce the length of the material along the axis of the applied load. A simple case of compression is the uniaxial compression induced by the action of opposite, pushing forces.
- **Tensile stress** is the stress state caused by an applied load that tends to elongate the material along the axis of the applied load.
- **Shear stress** is the stress state caused by the combined energy of a pair of opposing forces acting along parallel lines of action through the material. It is the stress caused by faces of the material sliding relative to one another.

Strain: Strain is defined as the ratio of change in length to original length.

$$e = \frac{\delta l}{l}$$

e: Compressive or tensile strain

δl: Change in length

l: Original length

- Strain is the deformation produced by stress.
- Strain is the deformation per unit length.
- Strain rosettes are used to measure the linear strain.

Linear strain is the deformation of the bar per unit length in the direction of the applied force.

Lateral strain is the deformation of the bar per unit length in a direction right angle to the direction of the applied force.

Shear strain: Shear strain is measured by the angle through which the body distorts.

$$\text{Shear strain, } e_s = \tan\phi \qquad\qquad e_s = \phi$$

When a square or rectangular block subjected to shear force is in equilibrium, the shear stress in one plane is always associated with a complementary shear stress of equal magnitude in the other plane at right angles to it.

Volumetric strain: It is defined as the ratio of change in volume to original volume of the body.

$$e_v = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta V}{V}$$

Hook's law: Hook's law is defined as the stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{a constant}$$

- proposed by Robert Hook.

Modulus of Elasticity: Modulus of Elasticity is defined as the linear stress to linear strain. It is denoted by E.

$$E = \frac{\text{Linear stress}}{\text{Linear strain}} = \frac{\sigma}{e}$$

- E is the slope of a linear part of stress-strain curve.

Material	Modulus of elasticity, GPa
Aluminium	70
Bronze	80
Brass	100
Copper	120
Steel	200
Diamond	1200

Modulus of Rigidity: Modulus of Rigidity is defined as the shear stress to shear strain. It is denoted by G, C or N. It is also called shear modulus of elasticity.

$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

Bulk modulus: Bulk modulus is defined as the normal stress to volumetric strain. It is denoted by K.

$$K = \frac{\text{Volumetric stress}}{\text{volumetric strain}} = \frac{\sigma_v}{e_v}$$

Elongation of a bar:

Elongation of a prismatic bar, $\delta l = \dfrac{Pl}{AE}$

Elongation of a circular tapered bar, $\delta l = \dfrac{4Pl}{\pi d_1 d_2 E}$

P : Applied Load

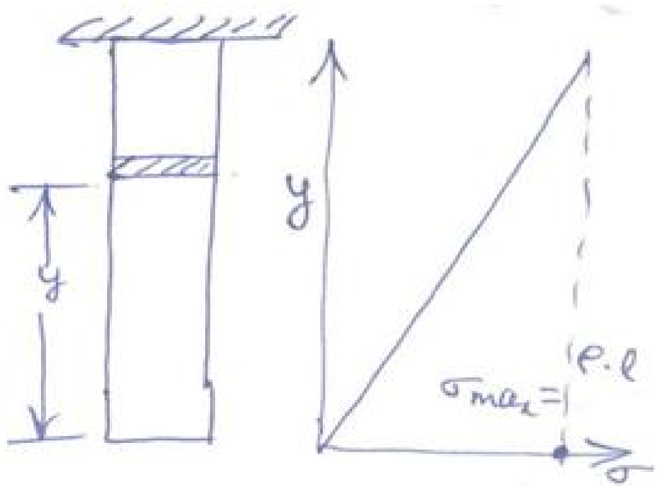
l : Length of the bar

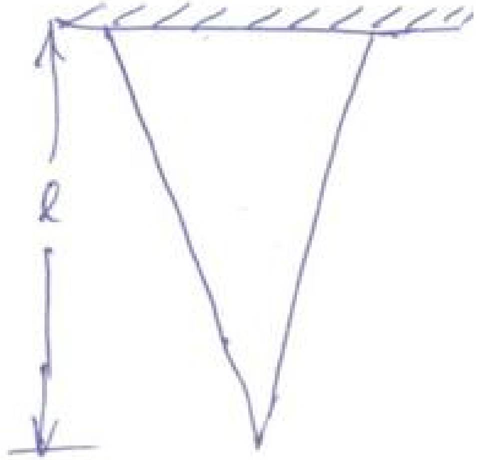
A : Cross sectional area

E : Modulus of elasticity of the material of the bar

d_1, d_2 : Diameters at the ends of the bar

Elongation of a bar due to self weight





a. Prismatic bar

Stress at lower end = 0

Stress at the support, $\sigma_{\max} = \rho.l$

Elongation of bar, $\delta l = \dfrac{\rho l^2}{2E}$

ρ : Density of the material of the bar

l : Length of the bar

E : Modulus of elasticity

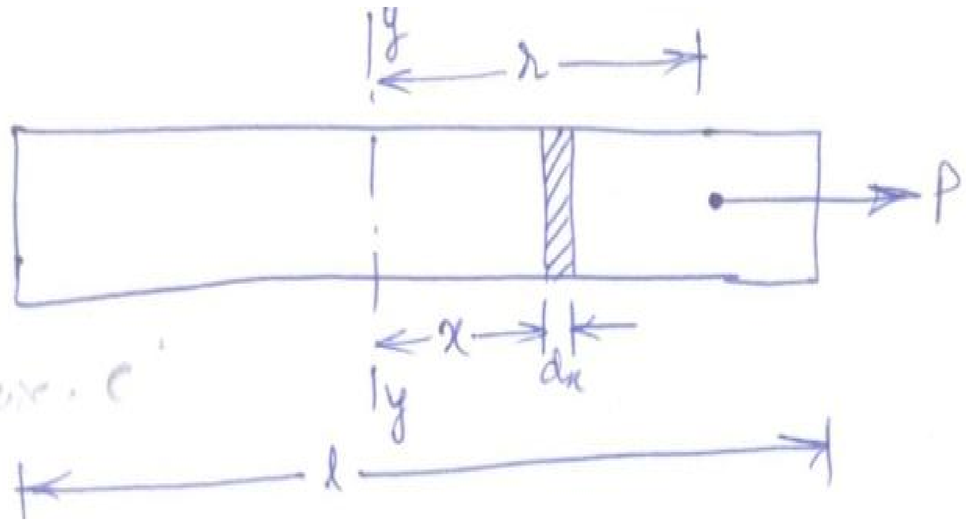
b. Conical bar:

Elongation of bar, $\delta l = \dfrac{\rho l^2}{6E}$

Stress at lower end = 0

Stress at the support, $\sigma_{\max} = \dfrac{\rho.l}{3}$

Elongation of a bar due to Rotation

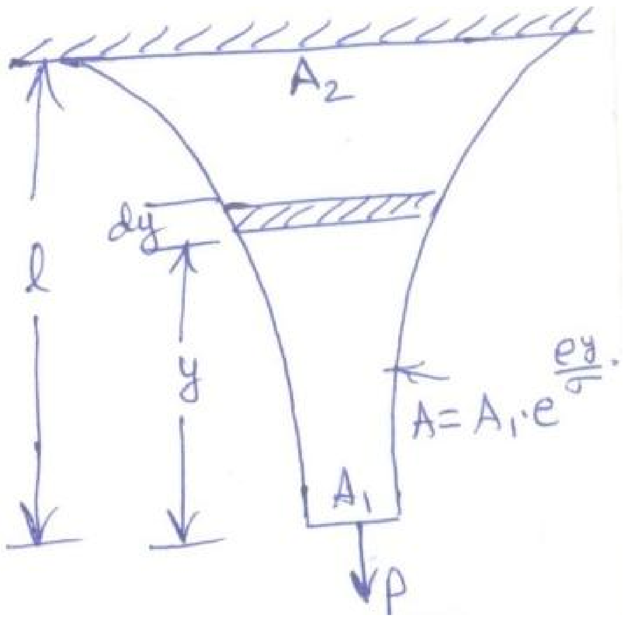


The maximum stress in the bar due to rotation occurs at the point of rotating axis.

$$\sigma_{\max} = \frac{1}{8}\rho.\omega^2.l^2$$

Elongation of bar, $\delta l = \dfrac{\rho\omega^2 l^3}{12E}$

Bar of uniform strength



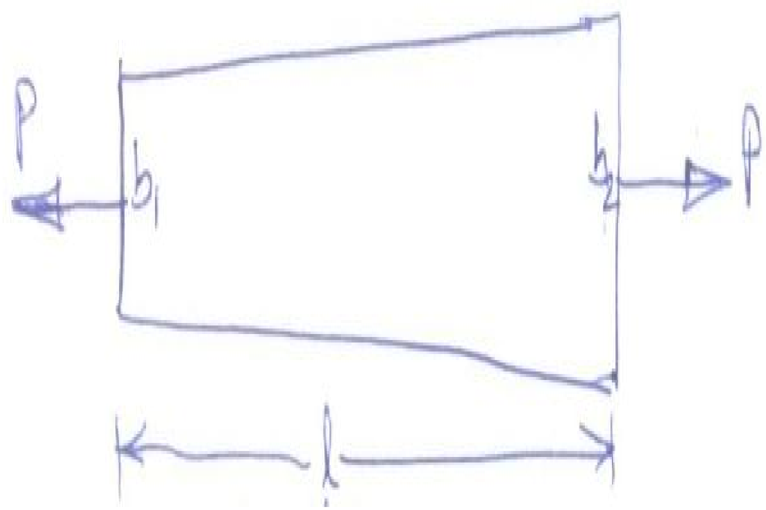
$$A_2 = A_1 . e^{\frac{\rho l}{\sigma}}$$

Elongation of bar, $\delta l = \dfrac{4Pl}{\pi E D d}$

σ : Uniform stress in the bar

D, d : Diameters at the ends of the bar

Tapered flat



Elongation of a tapered flat, $\delta l = \dfrac{Pl}{(b_2 - b_1)tE} \log_e \left(\dfrac{b_2}{b_1} \right)$

t: Thickness of the flat

b_1, b_2: Width of the flat at the ends

Poisson's ratio

Poisson's ratio is defined as the ratio of lateral strain to longitudinal strain. It is denoted by μ or $1/m$ or ν.

$$\mu \text{ or } \frac{1}{m} = \frac{\text{lateral (or transverse) strain}}{\text{longitudional (or primary) strain}}$$

The value of m varies between 3 and 4 for differentt materials.

- For most metals, μ varies between 0.25 to 0.35

μ is a constant for a given material.

Material	Poisson's ratio
Cork	0.0
Glass	0.02 to 0.03
Beryllium	0.08
Concrete	0.1 to 0.2
Cost Iron	0.270
Wrought Iron	0.278
Steel	0.288
Wrought Iron	0.300
Stainless steel	0.305
Ice	0.330
Aluminum	0.330
Brass	0.340
Bronze	0.340
Copper	0.355
Nylon	0.400
Gold	0.440
Lead	0.440
Re-entrant foam	-0.7

- Poisson's ratio is a very important diagnostic property of an isotropic elastic material.
- Range of Poisson's ratio values for isotropic elastic solid : $-1 \leq \mu \leq 0.5$
- For incompressible materials like clay, paraffin and rubber, $\mu = 0.5$
- The maximum Poisson's ratio of 0.5 is for an ideal elastic incompressible material whose volumetric strain is zero.
- For a perfectly rigid material, $\mu = 0.5$, then stretching a specimen causes no lateral contraction
- For a perfectly isotropic elastic material, Poisson's Ratio is 0.25.
- Polymer foams have negative values of Poisson's ratio.
- Some anisotropic materials have are or more Poisson's ratios above 0.5 in some directions.
- Poisson's ratio is an isotropic and not anisotropic concept.
- Some bizarre materials have $\mu < 0$, on stretching a round bar of such a material, the diameter of bar increases.

Strain along length of member, $e = \frac{\delta l}{l} = \frac{\text{Change in length}}{\text{Original length}}$

Strain along width of member, $e_b = -\mu.e \Rightarrow \frac{\delta b}{b} = -\mu.e$

Strain along thickness of member, $e_t = -\mu.e \Rightarrow \frac{\delta t}{t} = -\mu.e$

b : Width of the member

t : Width of the member

δb : Change in width of the member

δt : Change in width of the member

Relationship between the Elastic Modulii:

$$E = 2G(1+\mu)$$

$$E = 3K(1-2\mu)$$

$$E = \frac{9KG}{3K+G} \qquad \mu = \frac{3K-2G}{6K-2G}$$

E : Modulus of elasticity

G : Modulus of rigidity

K : Bulk modulus

For an isotropic material, $E > K > G$.

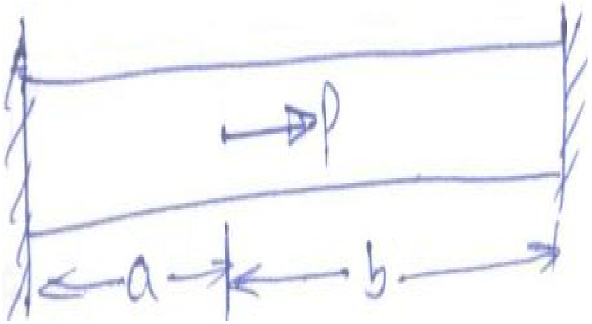
If $\mu < \frac{1}{3}$, $E > K$

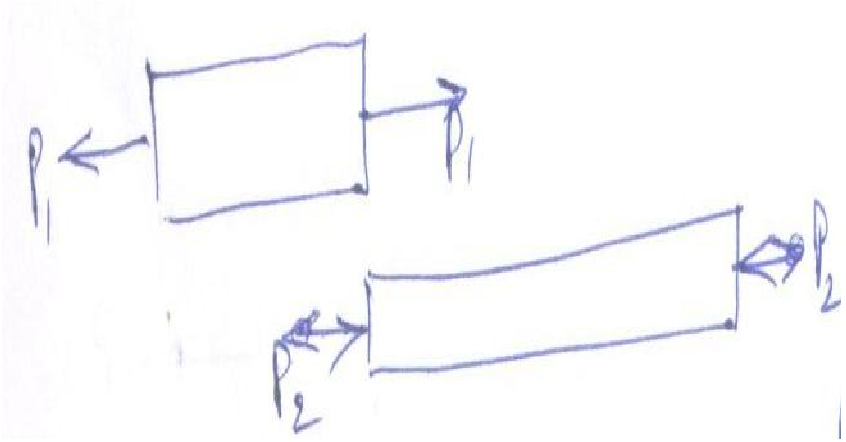
If $\mu = \frac{1}{3}$, $E = K$

If $\mu > \frac{1}{3}$, $E < K$

Lami's constant, $\lambda = \dfrac{E.\mu}{(1+\mu)(1-2\mu)} = \dfrac{2G\mu}{(1-2\mu)}$

Bar fixed at both ends subjected to axial force:





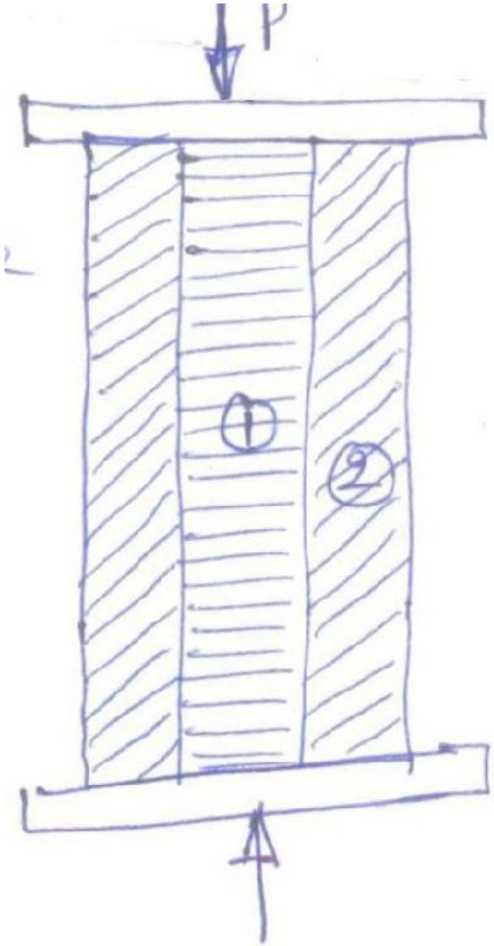
$$\frac{P_1.a}{AE}=\frac{P_2.b}{AE} \Rightarrow P_1 = P_2.\frac{b}{a}$$

$$\mathrm{P}_1 + P_2 = \mathrm{P}$$

$$P_2\left(1+\frac{b}{a}\right)=P \Rightarrow \; P_2 = \frac{Pa}{l}, \quad P_1 = \frac{Pb}{l}$$

Compound Bars

- Compound bar is statically indeterminate to degree 1.
- One compatibility equation and one equilibrium equation is required for analyzing the problem.
- The external applied axial load is shared by the two bars.



$$P = P_1 + P_2 \qquad \text{Equilibrium equation}$$

$$P = \sigma_1 . A_1 + \sigma_2 . A_2$$

P_1, P_2 : Load shared by the bars 1 and 2

σ_1, σ_2 : Stresses in the bars 1 and 2

A_1, A_2 : Cross sectional areas of the bars 1 and 2

e_1, e_2 : ΣΣΣtrain produced in the bars 1 and 2

$\delta l_1, \delta l_2$: Σtrain produced in the bars 1 and 2

The strain produced in the two bars is sameme.

$$e_1 = e_2 \Rightarrow \delta l_1 = \delta l_2 \qquad \text{Compatibility equation}$$

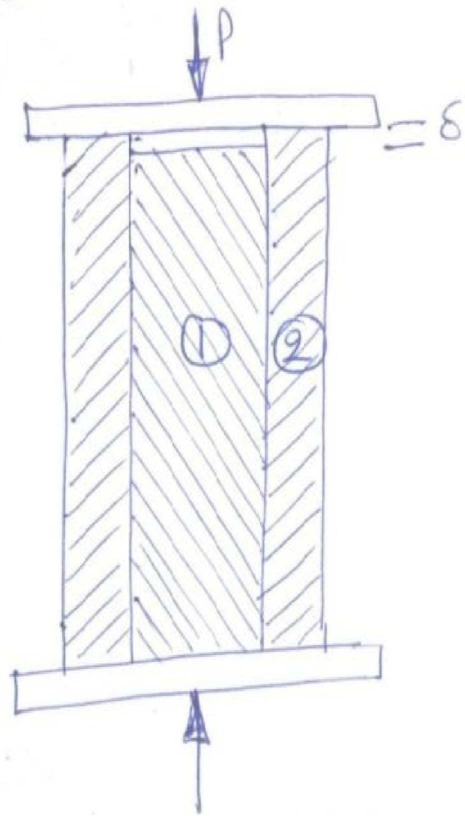
$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2}$$

When the differentce in length of bars is δ :

Equilibrium equation, $P = P_1 + P_2$

Compatibility equation, $\delta l_2 = \delta l_1 + \delta$

$$e_2 = e_1 + \frac{\delta}{L}$$



Change in length of compound bar, $\delta l = \dfrac{PL}{A_1E_1 + A_2E_2}$

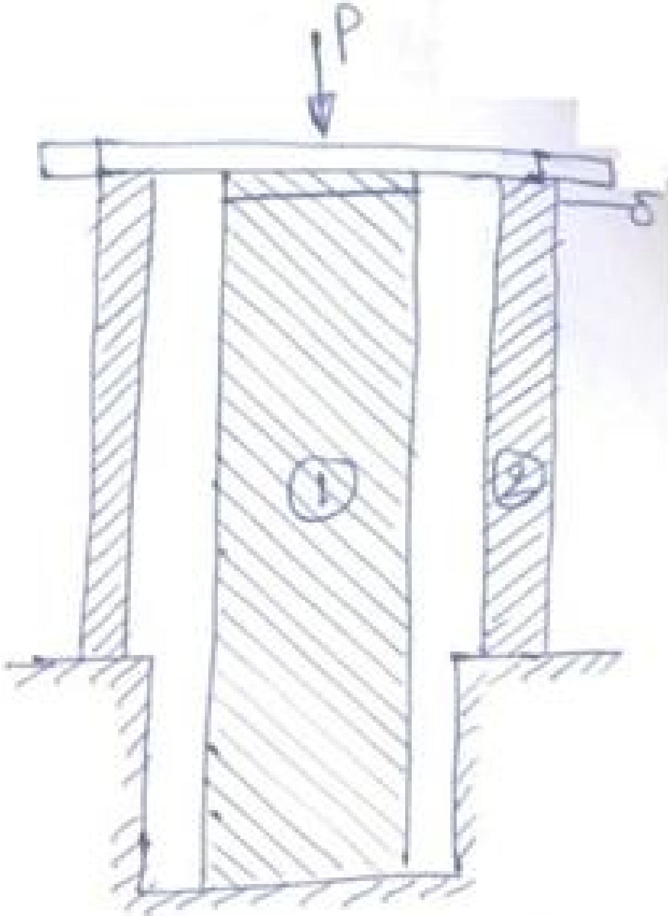
Load shared by the bars 1, $P_1 = \dfrac{PA_1E_1}{A_1E_1 + A_2E_2}$

Load shared by the bars 2, $P_2 = \dfrac{PA_2E_2}{A_1E_1 + A_2E_2}$

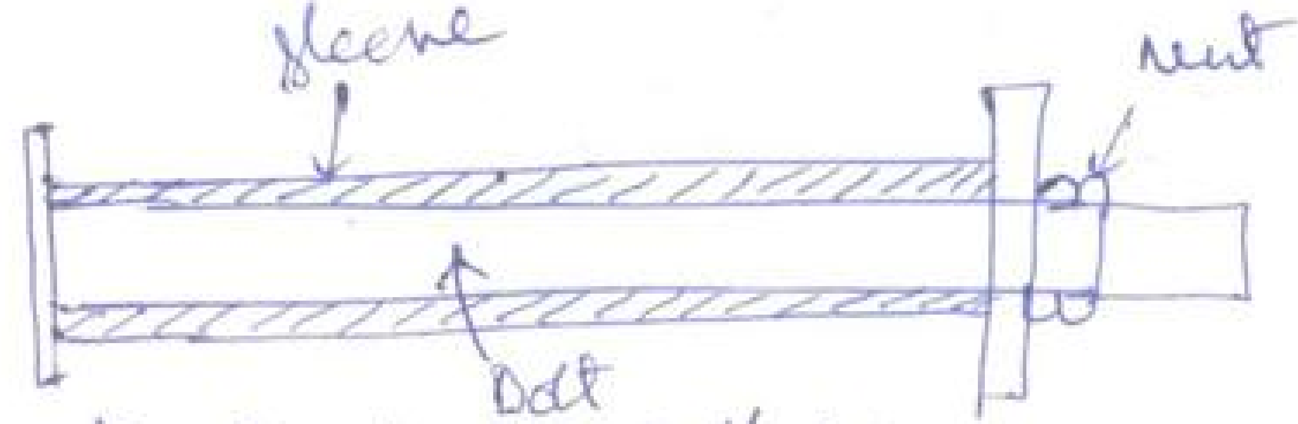
Compound bars of differentt length:

Equilibrium equation, $P = P_1 + P_2$

Compatibility equation, $\delta l_2 = \delta l_1 + \delta$



Compound bar between two rigid supports:



note - shank - comp stress
bolt → Tensile stress

Thermal stresses and strains:

If the temperature of the bar is raised or lowered, then its length increases or decreases correspondingly.

δl : Change in length due rise / fall in temperature

$$\delta l = l\alpha T$$

l : Length of the bar

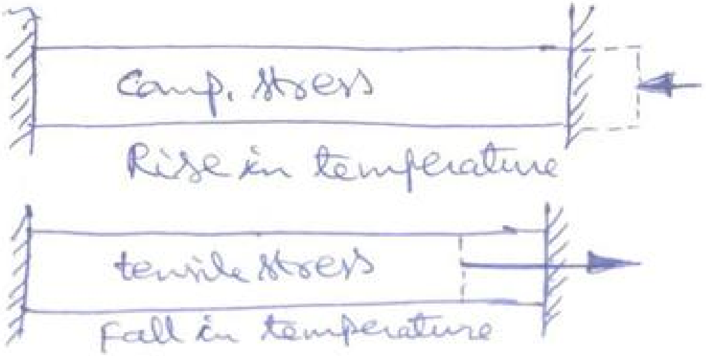
α : Coefficient of linear expansion

T : Rise/fall in temperature, $T = t_2 - t_1$

t_1 : Initial temperature of the bar

t_2 : Final temperature of the bar

If the change in length of bar is prevented by some means, temperature strain produced and hence induces temperature stresses.



e_t : Temperature strain

$e_t = \alpha T$

Temperature strain is directly proportional to change in temperature.

σ_t : Temperature stress

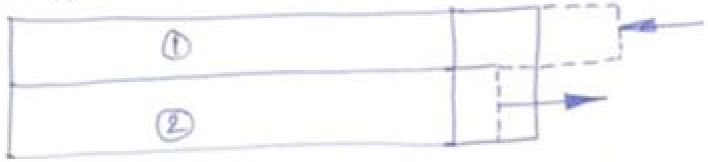
$\sigma_t = e_t.E = \alpha.T.E$

Temperature stress induced in a bar depends on α, T and E.

If the temperature of the bar is raised, the temperature strain and stress will be compressive in nature.
If the temperature of the bar is lowered, the temperature strain and stress will be tensile in nature.
If the bar is allowed to expand / contract freely due to rise / fall of temperature, no strain and stress will be induced in the bar.

Compound bar subjected to thermal stresses and strain:

Rise in temperature:



If $\alpha_1 > \alpha_2$, the extension of bar 1 is more than that of bar 2 when subjected to rise in temperature.

Since the bars 1 and 2 are brazed together, the bar 1 will try to pull the bar 2 and the bar 2 push the bar 1. Therefore, compressive stress induced in bar 1 and tensile stress induced in bar 2.

$$e_1 = \alpha_1 t - e$$

$$e_2 = e - \alpha_2 . t$$

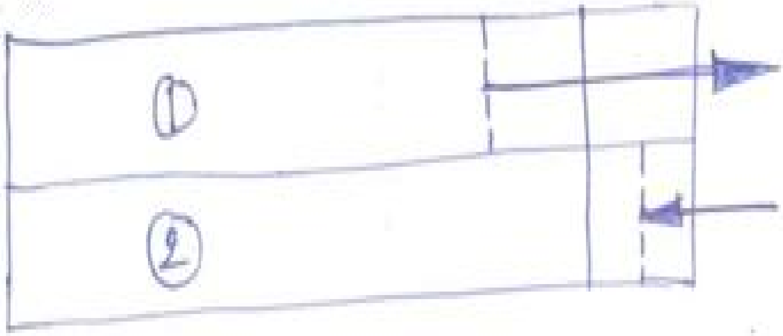
$$e_1 + e_2 = t(\alpha_1 - \alpha_2) \quad ....(i)$$

$$e_1 = \frac{\sigma_1}{E_1}, \quad e_2 = \frac{\sigma_2}{E_2}$$

$$P_1 = P_2$$

$$\sigma_1 A_1 = \sigma_2 A_2 \quad ....(ii)$$

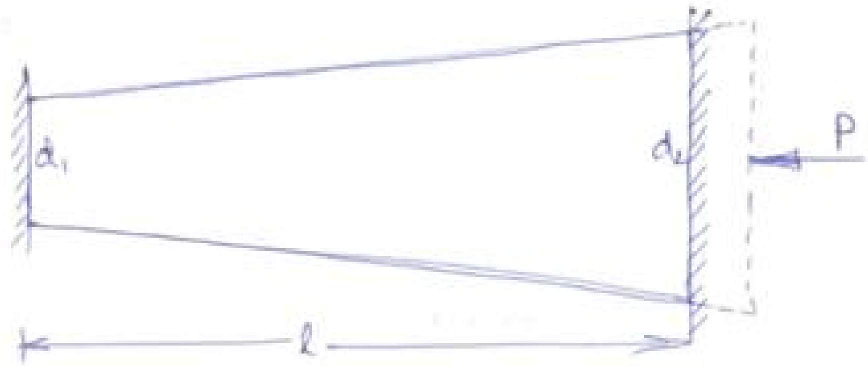
Fall in temperature:



If $\alpha_1 > \alpha_2$, the contraction of bar 1 is more than that of bar 2 when subjected to fall in temperature.

Since the bars 1 and 2 are brazed together, the bar 1 will try to push the bar 2 and the bar 2 will try to pull the bar 1. Therefore, tensile stress induced in bar 1 and compressive stress induced in bar 2.

Circular taper bar subjected to rise/fall temperature:



Free expansion due to rise in temperature, $\delta l = l\alpha T$

Elongation of a circular tapered bar, $\delta l = \dfrac{4Pl}{\pi d_1 d_2 E}$

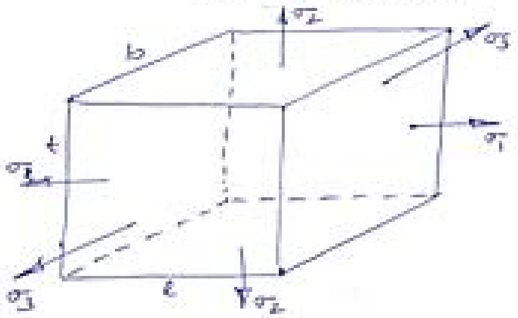
Load required to prevent the free expansion/contraction, $P = \dfrac{\alpha.T.E.\pi d_1 d_2}{4}$

Maximum stress induced at the section having diameter d_1, $\sigma_{\max} = \alpha TE.\dfrac{d_2}{d_1}$

Minimum stress induced at the section having diameter d_2, $\sigma_{\min} = \alpha TE.\dfrac{d_1}{d_2}$

Volumetric strain for a rectangular section:

Volumetric strain, $e_v = \dfrac{\text{Change in volume}}{\text{Original volume}}$, $\quad e_v = \dfrac{\delta V}{V}$



Volumetric strain, $e_v = e_x + e_y + e_z$

Volumetric strain is equal to the algebraic sum of the strains along the three principal axes.

Strain in x direction, $e_x = \dfrac{\sigma_1}{E} - \dfrac{1}{m}.\dfrac{\sigma_2}{E} - \dfrac{1}{m}.\dfrac{\sigma_3}{E}$

Strain in y direction, $e_y = -\dfrac{1}{m}.\dfrac{\sigma_1}{E} + \dfrac{\sigma_2}{E} - \dfrac{1}{m}.\dfrac{\sigma_3}{E}$

Strain in Z direction, $e_z = -\dfrac{1}{m}.\dfrac{\sigma_1}{E} - \dfrac{1}{m}.\dfrac{\sigma_2}{E} + \dfrac{\sigma_3}{E}$

Volumetric strain, $e_v = e_x + e_y + e_z = \dfrac{1}{E}(\sigma_1 + \sigma_2 + \sigma_3)\left(1 - \dfrac{2}{m}\right)$

If $\sigma_2 = \sigma_3 = 0$, $e_v = \dfrac{\sigma_1}{E}\left(1 - \dfrac{2}{m}\right)$

If $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$, $e_v = \dfrac{3\sigma}{E}\left(1 - \dfrac{2}{m}\right)$

Volumetric strain of a circular rod:

Let L : Length of the circular rod

d : Diameter of the circular rod

Volumetric strain, $e_v = 2.e_d + e_l$

Volumetric strain of a circular rod is the sum of strain in length and twicece the strain in diameter.

Volumetric strain of a sphere:

Let d : Diameter of the sphere

Volumetric strain, $e_v = 3.e_d$

Volumetric strain of a sphere is equal to three times the strain in diameter.

$$\text{Modulus of elasticity, } E = \frac{\text{Linear stress}}{\text{Linear strain}}$$

$$\text{Modulus of rigidity, } G = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\text{Bulk modulus, } K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}}$$

Hoop stress

If any thin tyre of steel or any other metal is to be shrunk on to a wheel, the diameter of the tyre is to be slightly smaller than that of the wheel so that it does not come out easily.

D : Diameter of the wheel

d : Diameter of the tyre

Temperature strain, $e = \dfrac{\text{Contraction prevented}}{\text{Original length}} = \dfrac{\pi D - \pi d}{\pi d} = \dfrac{D-d}{d}$

Hoop or circumferential stress due to fall of temperature, $\sigma = e.E$

$$\sigma = \frac{D-d}{d}.E$$

Minimum temperature to which tyre is to be raised so that it can be fitted over the wheel is given by

$$\pi D = \pi d(1 + \alpha T)$$

Objective questions and answers

01. The maximum value of Poisson's ratio for an elastic material is CE 1991

a. 0.25 b. 0.5 c. 0.75 d. 0.1

01. b

The maximum Poisson's ratio is 0.5 for an ideal elastic incompressible material whose volumetric strain is zero.

02. A cantilever beam of tubular section consists of two materials, copper as outer cylinder and steel as inner cylinder. It is subjected to a temperature rise of 20°C and $\alpha_{copper} > \alpha_{steel}$. The stresses developed in the tubes will be CE 1991

a. compression in steel and tension in copper

b. tension in steel and compression in copper

c. no stress in both

d. tension in both the materials

02. b

Rise of temperature, $T = 20^0C$

$$\alpha_{copper} > \alpha_{steel}$$

Extension of beam due to rise of temperature, $\delta l = L.\alpha T$

L : Length of the beam

α : Coefficient of linear expansion

T : Change of temperature

Since $\alpha_{copper} > \alpha_{steel}$, the free expansion of copper is more than the steel.

$$(\delta l)_{copper} > (\delta l)_{steel}$$

Since the two materials are brazed together, copper tube try to pull the steel tube and steel tube push the copper tube. Therefore, tensile stress induced in steel tube and compressive stress induced in copper tube.

03. The shear modulus (G), modulus of elasticity(E) and the Poisson's ratio(μ) of a material are related as, CE 2002

a. $G = \dfrac{E}{2(1+\mu)}$ b. $E = \dfrac{G}{2(1+\mu)}$ c. $G = \dfrac{E}{2(1-\mu)}$ d. $G = \dfrac{E}{2(\mu-1)}$

03. a

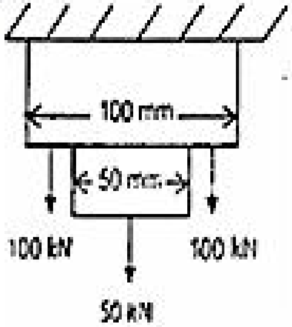
G : Shear modulus

E : Modulus of elasticity

μ : Poisson's ratio

Relationship between the above parameters is $E = 2G(1+\mu)$

04. A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self weight, the maximum tensile stress in N/mm^2 anywhere is



CE 2003

a. 16.0 b. 20.0 c. 25.0 d. 30.0

04. c

Tensile stress, $\sigma = \frac{P}{A}$

Load in lower bar, $P_1 = 50$ kN

Load on upper bar, $P_2 = 100+100+50 = 250$ kN

Cross sectional area of lower bar, $A_1 = 50\times 50 = 2500\ mm^2$

Cross sectional area of upper bar, $A_2 = 100\times 100 = 1\times 10^4 mm^2$

Tensile stress in lower bar, $\sigma_1 = \frac{50\times 10^3}{250} = 20\ N/mm^2$

Tensile stress in upper bar, $\sigma_2 = \frac{250\times 10^3}{1\times 10^4} = 25\ N/mm^2$

Maximum tensile stress = 25 N/mm^2

05. For an isotropic material, the relationship between the Young's modulus (E), shear modulus (G) and Poisson's ratio (μ) is given by CE 2007

a. $G = \frac{E}{2(1+\mu)}$ b. $E = \frac{G}{2(1+\mu)}$ c. $G = \frac{E}{(1+2\mu)}$ d. $G = \frac{E}{2(1-\mu)}$

05. a

E : Young's modulus

G : Shear modulus

μ : Poission's ratio

$$E = 2G(1+\mu) \Rightarrow G = \frac{E}{2(1+\mu)}$$

06. A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10^0C. If the coefficient of thermal expansion is 12×10^{-6} per °C and the Young's modulus is 2×10^5 MPa, the stress in the bar is

a. zero b. 12 MPa c. 24 MPa d. 2400 MPa CE 2007

06. c

Length of the bar, $L = 100$ mm

Temperature increase $= \Delta T$

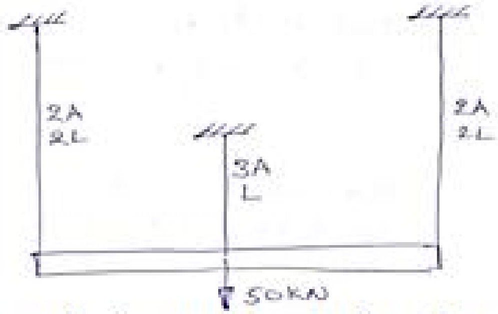
Coefficient of thermal expansion, $\alpha = 12 \times 10^{-6}$ per °C

Young's modulus, $E = 2 \times 10^{-5}$ MPa

Stress in the bar due to change of temperature, $\sigma = \alpha.T.E$

$$= 12 \times 10^{-6} \times 10 \times 2 \times 10^5 = 24 \text{ MPa}$$

07. A rigid bar is suspended by three rods made of the same material as shown in the figure. The area and length of the central rod are $3A$ and L, respectively while that of the two outer rods are $2A$ and $2L$, respectively. If a downward force of 50 kN is applied to the rigid bar, the forces in the central and each of the outer rods will be CE 2007



a. 16.67 kN each b. 30 kN and 15 kN **c.** 30 kN and 10 kN d. 21.4 kN and 14.3 kN

07. c

Let P_1 : Force in the central rod P_2 : Force in each outer rods

$$P_1 + 2P_2 = 50 \quad \ldots\ldots\ldots\ldots(i)$$

Since the rigid bar is symmetric, the elongation of central rod and outer rod is same.

$$\frac{P_1 L_1}{A_1 E} = \frac{P_2 L_2}{A_2 E}\,; \quad \frac{P_1 L}{3AE} = \frac{P_2 2L}{2AE}\,; \quad P_1 = 3P_2 \quad \ldots\ldots\ldots\ldots(ii)$$

$$3P_2 + 2P_2 = 50 \Rightarrow P_2 = 10 \text{ kN},\ P_1 = 30 \text{ kN}$$

Force in central rod, $P_1 = 30$ kN Force in each of outer rod, $P_2 = 10$ kN

08. The number of independent elastic constants for a linear elastic isotropic and homogeneous material is CE 2010

a. 4 b. 3 c. 2 d. 1

08. c

For a linear elastic isotropic and homogeneous material,

$$E = 2G\left(1+\frac{1}{m}\right) = 3K\left(1-\frac{2}{m}\right)$$

All the three elastic constants can be found if any two of them are known. Hence, the number of independent elastic constant are 2.

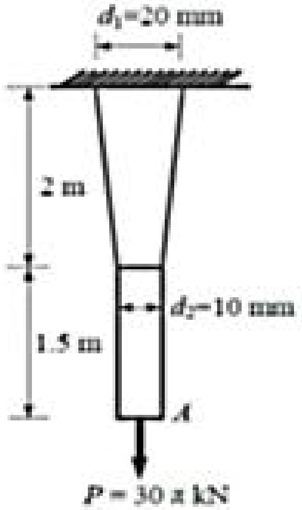
09. The Poisson's ratio is defined as CE 2012

a. $\left|\frac{\text{axial stress}}{\text{lateral stress}}\right|$ b. $\left|\frac{\text{lateral strian}}{\text{axial strian}}\right|$ c. $\left|\frac{\text{lateral stress}}{\text{axial stress}}\right|$ d. $\left|\frac{\text{axial strian}}{\text{lateral strian}}\right|$

09. b

Poisson's ratio is defined as the ratio of lateral strain to axial strain.

10. A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity, $E = 2 \times 10^5$ MPa. When subjected to a load $P = 30\pi$ kN, the deflection at point A is ....mm CE1 2015



10. 15

Diameters at the ends of tapered bar: d_1=20 mm, d_2=10 mm, L_1=1.5 m,

Diameter of the uniform bar, d_2=10 mm, $L_2 = 2$ m

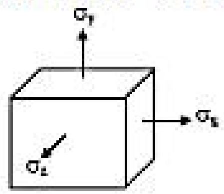
Axial load, $P = 30\pi$kN

Deflection of point A, $\delta = \delta_1 + \delta_2$

$$\delta = \frac{4PL}{\pi E d_1 d_2} + \frac{PL}{AE} = \frac{4PL}{\pi E d_1 d_2} + \frac{4PL}{\pi E d_2^2}$$

$$= \frac{4 \times 30\pi \times 2000}{\pi \times 2 \times 10^5 \times 20 \times 10} + \frac{4 \times 30\pi \times 1500}{\pi (10)^2 \times 2 \times 10^5} = 6 + 9 = 15 \text{ mm}$$

11. An elastic isotropic body is in a hydrostatic state of stress as shown in the figure. For no change in the volume to occur, what should be is Poisson's ratio?



CE2 2016

a. 0.00 b. 0.25 c. 0.50 d. 1.00

11. c

Volumetric strain, $e_v = e_x + e_y + e_z$

For a hydrostatic state of stress, $\sigma_x = \sigma_y = \sigma_z = \sigma$

Strain in x-direction, $e_x = \frac{\sigma}{E}(1-2\mu)$

$$e_v = \frac{3\sigma}{E}(1-2\mu)$$

For no change in volume, $e_v = 0$

$$\frac{3\sigma}{E}(1-2\mu) = 0 \Rightarrow 1-2\mu = 0 \Rightarrow \mu = 0.5$$

12. An elastic bar of length L, uniform cross sectional area A, coefficient of thermal expansion α, and Young's modulus E is fixed at the two ends. The temperature of the bar is increased by T, resulting in an axial stress σ. Keeping all other parameters unchanged,if the length of the bar is doubled, the axial stress would be

a. σ b. 2σ c. 0.5σ d. $0.25\alpha\sigma$ CE1 2017

12. a

L : Length of the bar

A : Uniform cross sectional area

α : Coefficient of thermal expansion

E : Young's modulus

T : Rise of temperature

σ : Axial stress induced in the bar

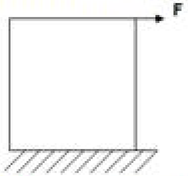
Thermal stress, $\sigma = \alpha . T . E$

σ does not depends on the length of the bar.

If the length of the bar is doubled, there is no change in axial stress.

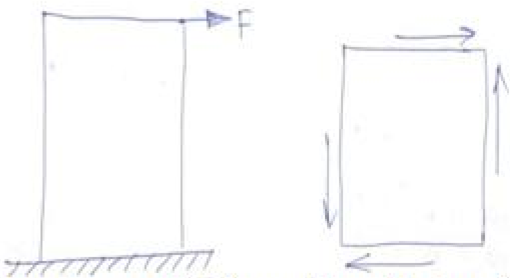
2.1 SIMPLE STRESSES AND STRAINS (GATE ME)

01. A block of steel is loaded by a tangential force on its top surface while the bottom surface is held rigidly. The deformation of the block is due to GATE ME 1992



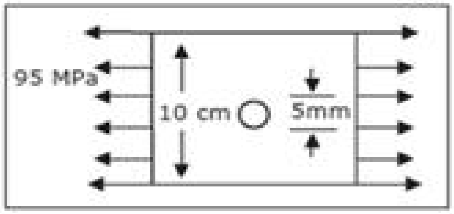
a. shear only b. bending only **c.** shear and bending d. torsion

01. c.



If a block is subjected to a tangential force F on its top face, an equal and opposite force of magnitude F is exerted at the rigid surface has a tendency to rotate in clockwise direction. It is also subjected to a bending moment of $F.h$, where h is the height of steel block. Shear force and bending moment causes relative displacement of the material in the direction of the face.

02. A large uniform plate containing a rivet-hole is subjected to uniform uni-axial tension of 95 MPa. The maximum stress in the plate is : GATE ME 1992



a. 100 MPa b. 285 MPa c. 190 MPa d. Indeterminate

02. a.

σ_1 : Axial stress in the plate across the rivet

A_1 : Cross sectional area of plate hole across the rivet hole.

σ : Axial stress in the plate $= 95$ MPa

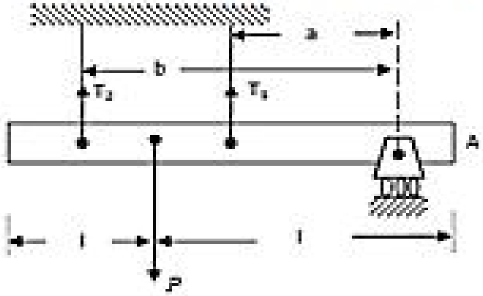
A : cross sectional area of plate

$P =$ maximum stress induced in the plate across the rivet hole.

P : Load carrying capacity of the plate

$$P = \sigma.A = \sigma_1.A_1 \Rightarrow 95 \times 100t = \sigma_1 \times 95 \times t \Rightarrow \sigma_1 = 100 \text{ MPa}$$

03. Figure below shows a rigid bar hinged at A and supported in a horizontal by two vertical identical steel wires. Neglect the weight of the beam. The tension T_1 and T_2 inducted in these wires by a vertical load P applied as shown are



GATE ME 1994

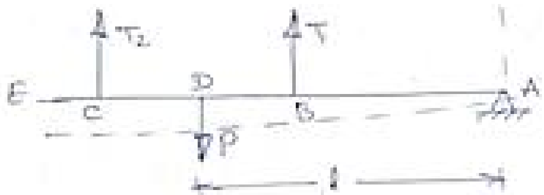
a. $T_1 = T_2 = \frac{P}{2}$

b. $T_1 = \frac{Pal}{a^2+b^2}$, $T_2 = \frac{Pbl}{a^2+b^2}$

c. $T_1 = \frac{Pbl}{(a^2+b^2)}$; $T_2 = \frac{Pal}{(a^2+b^2)}$

d. $T_1 = \frac{Pbl}{2(a^2+b^2)}$;

03. b



The defamation of the rigid bar is shown in fig.

$$\frac{\delta_1}{a} = \frac{\delta_2}{b} \Rightarrow \frac{\frac{T_1 L}{AE}}{a} = \frac{\frac{T_2 L}{AE}}{b}; \qquad T_1 = T_2 \cdot \frac{a}{b}$$

Taking moments of all forces about the hinge A,

$$\Sigma M_A = 0 \Rightarrow T_1 . a + T_2 . b - Pl = 0 \Rightarrow T_1 . a + T_2 . b = Pl; \qquad T_2 . \frac{a^2}{b} + T_2 . b = P.l$$

$$T_2\left(a^2+b^2\right) = Pbl; \qquad T_2 = \frac{Pbl}{a^2+b^2}; \qquad T_1 = \frac{Pbl}{a^2+b^2} \cdot \frac{a}{b} = \frac{Pal}{a^2+b^2}$$

04. A free bar of length L uniformly heated from 0^0C to a temperature t^0C. α is the coefficient of linear expansion and E is the modulus of elasticity. The stress in the bar is GATE ME 1995

a. $\alpha t E$ b. $\frac{\alpha t E}{2}$ c. zero d. None of the above

04. c

No stress induced in the bar as the bar is free to allow to expand

05. The relationship Young's modulus (E), Bulk modulus (K) and Poisson's ratio (μ) is given by GATE ME 2002

a. $E = 3K(1-2\mu)$ b. $K = 3E(1-2\mu)$ c. $E = K(1-\mu)$ d. $K = 3E(1-\mu)$

05. a

$$E = 3K(1-2\mu)$$

E : Young's modulus,

K : Bulk modulus

μ : Poisons ratio

06. Two identical circular rods of same diameter and same length are subjected to same magnitude of axial tensile force. One of the rods is made out of mild steel having the modulus of elasticity of 206 GPa. The other rod is made out of cast iron having the modulus of elasticity of 100 GPa. Assume both the materials to be homogeneous and isotropic and the axial force causes the same amount of uniform stress in both the rods. The stresses developed are within the proportional limit of the respective materials. Which of the following observations is correct?

a. Both rods elongate by the same amount GATE ME 2003

b. Mild steel rod elongates more than the cast iron

c. Cast iron rod elongates more than the mild steel rod

d. As the stresses are equal strains are also equal in both the rods

06. c.

For mild steel,

Modulus of elasticity, $E_1 = 206GPa$

For cast Iron,

Modulus of elasticity, $E_2 = 100\,\text{GPa}$

Elongation, $\delta L = \frac{PL}{AE} \Rightarrow \delta L \propto \frac{1}{E}$

$P_1 = P_2 = P \qquad L_1 = L_2 = L \qquad A_1 = A_2 = A$

$$\frac{\delta L_1}{\delta L_2} = \frac{E_2}{E_1} = \frac{100}{206} = 0.485$$

Therefore, elongation of cast Iron is more than the elongation of mild steel rod.

Strain, $e = \frac{\sigma}{E} \qquad e \propto \frac{1}{E}$

$$\frac{E_1}{e_2} = \frac{E_2}{E_1} = \frac{100}{206} = 0.485$$

Strain in cast iron is more than the strain in mild steel.

07. In terms of Poisson's ratio (μ) the ratio of Young's Modulus (E) to Shear Modulus (G) of elastic materials is GATE ME 2004

a. $2(1+\mu)$ b. $2(1-\mu)$ c. $\frac{1}{2}(1+\mu)$ d. $\frac{1}{2}(1-\mu)$

07. a

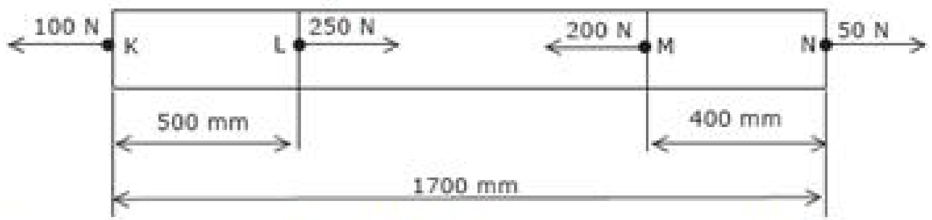
Poisson's ratio $=\mu$

Young's modulus $=E$

Shear modulus $=G$

$$E=2G(1+\mu)\Rightarrow\frac{E}{G}=2(1+\mu)$$

08. The figure below shows a steel rod of 25 mm^2 cross sectional area. It is loaded at four points K, L, M and N. Assume E_{steel} = 200 GPa. The total change in length of the rod due to loading is GATE ME 2004



a. 1 μm **b.** -10 μm c. 16 μm d. -20 μm

08. b

The free body diagram for the bar is shown in fig.

δ: Change in length of bar

$$= \delta_1 + \delta_2 + \delta_3$$

$$= \frac{100 \times 500}{AE} - \frac{150 \times 800}{AE} + \frac{50 \times 400}{AE} = \frac{10^3}{25 \times 200 \times 10^3}(50 - 120 + 20)$$

$$= \frac{1}{25 \times 200} \times (-50) = -\frac{1}{100}\,\text{m} = -0.01\,\text{m} = -10 \times 10^{-3}\,\text{mm} = -10\,\mu\text{m}$$

09. A steel bar of 40 mm $\times$ 40 mm square cross-section is subjected to an axial compressive load of 200 kN. If the length of the bar is 2 m and $E = 200$ GPa, the elongation of the bar will be: GATE ME 2006

a. 1.25 mm b. 2.70 mm c. 4.05 mm d. 5.40 mm

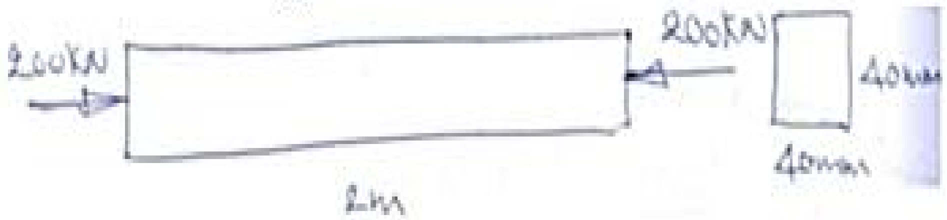
09. a

Axial compressive load, $P = 200$ kN

Cross sectional area, $A = 40 \times 40$ mm^2

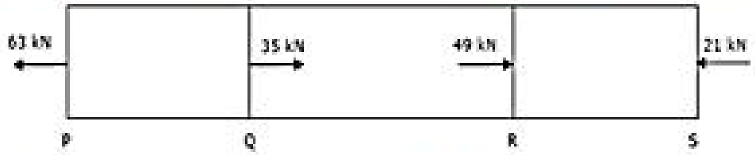
Length of the bar, $l = 2$ m

Modulus of elasticity $E = 200$ GPa



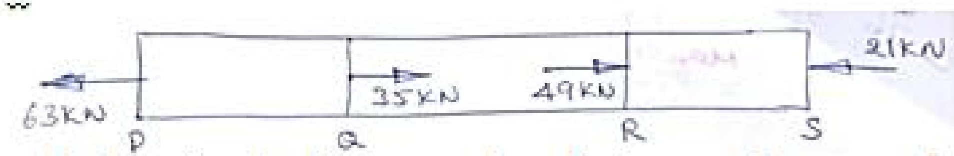
Elongation of the bar, $\delta = \dfrac{Pl}{AE} = \dfrac{200 \times 10^3 \times 2 \times 10^3}{40 \times 40 \times 200 \times 10^3} \Rightarrow \delta = 1.25$ mm

10. A bar having a cross sectional area of 700 mm^2 is subjected to axial loads at the positions indicated. The value of stress in the segment QR is : GATE ME 2006

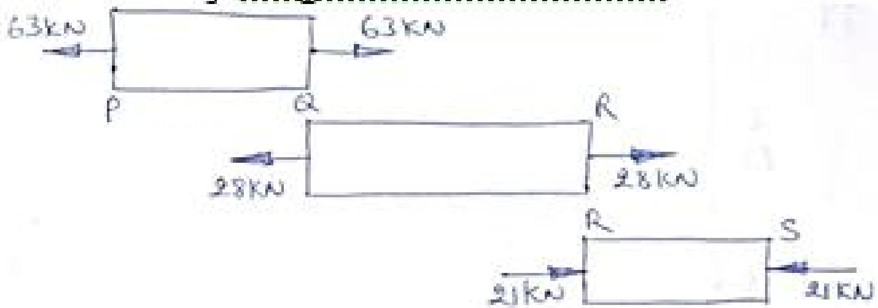


a. 40 MPa b. 50 MPa c. 70 MPa d. 120 MPa

10. a



The free body diagrams for the bar is shown in fig.



Cross sectional area, $A = 700\,\text{mm}^2$

Force in member QR, $P = 28\text{kN}\,(\text{Tensile})$

Stress in bar QR, $\sigma_{QR} = \dfrac{P}{A} = \dfrac{28\times10^3}{700} = 40\,\text{N/mm}^2$

11. A steel rod of length L and diameter D, fixed at both ends, is uniformly heated to a temperature rise of ΔT. The Young's modulus is E and the coefficient of linear expansion is α. The thermal stress in the rod is

a. 0 b. $\alpha\Delta T$ **c.** $E\alpha\Delta T$ d. $E\alpha\Delta TL$ GATE ME 2007

11. c

L: length of the bar

D: Diameter of the bar

ΔT: Rise of temperature

E: Modulus of elasticity of the material

α: coefficient of linear expansion change in length of bar, $\Delta L = L.\alpha.\Delta T$ strain, $e = \alpha.\Delta T$

Stress in the bar, $\sigma = e.E = \alpha.\Delta T.E$

12. A $200\times100\times50\,\text{mm}$ steel block is subjected to a hydrostatic pressure of $15\,\text{MPa}$. The Young's modulus and Poisson's ratio of the material are 200 GPa and 0.3 respectively. The change in the volume of the block in mm^3 is GATE ME 2007

a. 85 **b.** 90 c. 100 d. 110

12. b.

Size of block: $200\times100\times50$ mm

Hydrostatic pressure on block, $\sigma = 15\,\text{MPa}$

Modulus of elasticity, $E = 200$ GPa

Poisson's ratio, $\mu = 0.3$

Change in volume of the block, $\delta V = ?$

Volumetric strain, $e_v = \dfrac{3\sigma}{E}(1-2\mu)$

$$\frac{\Delta V}{V} = \frac{3\times15}{200\times10^3}(1-2\times0.3) = 9\times10^{-5}$$

$$\Delta V = 9\times10^{-5}\times V = 9\times10^{-5}\times200\times100\times50 = 90\,\text{mm}^3$$

13. A rod of length L and diamter D is subjected to a tensile load P. Which of the following is suffiecient to calculate the resulting change in diameter?

a. Young' modulus   b. Shear modulus   GATE ME 2008

c. Possion's ratio   **d.** Both Young' modulus and Shear modulus

13. d

$$e_l = \frac{\delta L}{L}; \quad \delta L = \frac{PL}{AE}$$

$$e_d = -\mu . e_l; \quad \delta D = -\mu . e_l . D$$

To calculate change in diameter, Modulus of elasticity and poisons ratio are required. Poisson's ratio be found if shear modulus is known. Hence both young's modulus and shear modulus are required to calculate the change in diameter of the bar.

14. A free bar of length L uniformly heated from 0^0C to a temperature t^0 C . α is the coefficient of linear expansion and E is the modulus of elasticity . The stress in the bar is GATE ME 2010

a. αTE b. $\frac{\alpha TE}{2}$ **c.** zero d. None of the above

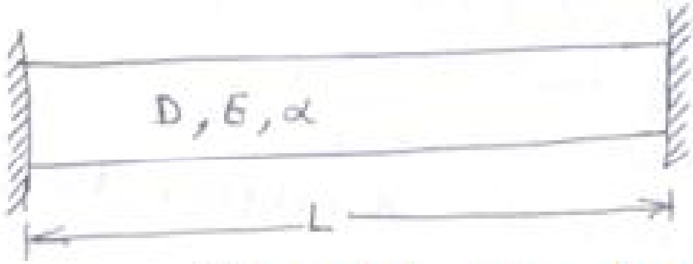
14. c

If a body is allowed to expand or contract freely due to rise or fall of temperature, no thermal stresses are induced in the body.

15. A steel rod of length L and diameter D, fixed at both ends, is uniformly heated to a temperature rise of ΔT. The young's modulus is E and the co-efficient of linear expansion is α. The thermal stress in the rod is GATE ME 2010

a. 0 b. $\alpha\Delta T$ **c.** $E\alpha\Delta T$ d. $E\alpha\Delta TL$

15. c



Thermal strain, $e = \dfrac{\text{Change in length}}{\text{Origial length}} = \dfrac{L\alpha.\Delta T}{L} = \alpha.\Delta T$

Thermal stress, $\sigma = e.E = \alpha.\Delta T.E$

16. A solid steel cube constrained on all six faces is heated so that the temperature rises uniformly by ΔT. If the thermal coefficient of the material is α, Young's modulus is E and the Poisson's ratio is ν, the thermal stress developed in the cube due to heating is GATE ME 2012

a. $-\frac{\alpha(\Delta T)E}{(1-2\nu)}$ b. $-\frac{2\alpha(\Delta T)E}{(1-2\nu)}$ c. $-\frac{3\alpha(\Delta T)E}{(1-2\nu)}$ d. $-\frac{\alpha(\Delta T)E}{3(1-2\nu)}$

16. a

Temperature rise $=\Delta T$

Coefficient of thermal expansion, $=\alpha$

Young's modulus $=E$

Poisson's ratio $=\mu$

Thermal stress developed in the cube, $\sigma=?$

Let a be the side of the cube. Since the cube is constrained to expand on all six faces, the stress induced in all the three directions will be the same.

$\sigma_x=\sigma_y=\sigma_z=\sigma$

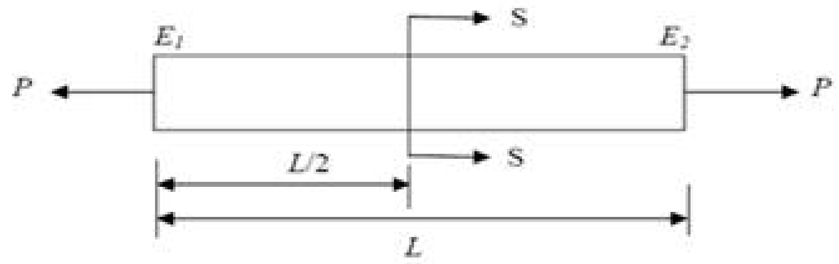
Strain in x direction, $e_x=\frac{\sigma_x}{E}-\frac{\mu}{E}.\sigma_y-\frac{\mu}{E}\sigma_z$

$e_x=\frac{\sigma}{E}(1-2\mu)$

Thermal strain in x direction, $e_x=-\alpha\Delta T$

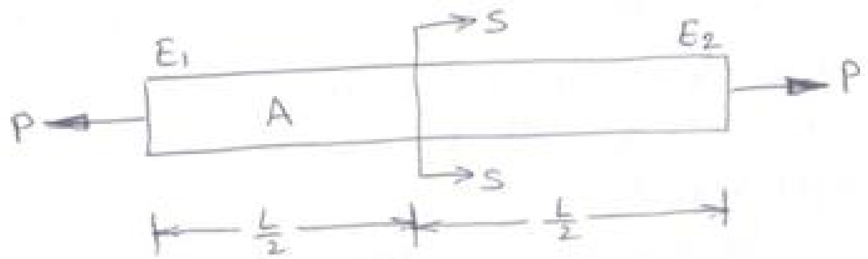
$-\alpha.\Delta T=\frac{\sigma}{E}(1-2\mu)$ $\sigma=-\frac{\alpha.\Delta T.E}{(1-2\mu)}$

17. A rod of length L having uniform cross-sectional area A is subjected to a tensile force P as shown in the figure below. If the Young's moudlus of the material varies lineraly from E_1 and E_2 along the length of the rod, the normal stress developed at the section–SS is GATE ME 2013



a. $\frac{P}{A}$ b. $\frac{P(E_1 - E_2)}{A(E_1 + E_2)}$ c. $\frac{PE_2}{AE_1}$ d. $\frac{PE_1}{AE_2}$

17. a



Normal stress at SS, $\sigma = \frac{P}{A}$

The stress induced in the bar does not depends on the variation of modulus of Elasticity.

18. A metallic rod of 500 mm length and 50 mm diameter, when subjected to a tensile force of 100 kN at the ends, experiences a *n* increase in its length by 0.5 mm and a reduction in its diameter by 0.015 mm. The Poisson's ratio of the rod material is _______ GATE ME1 2014

18. 0.3

Length of the rod, $L = 500\,\text{mm}$

Diameter of the rod, $D = 50\,\text{mm}$

Tensile force, $P = 100\,\text{kN}$

Increase in length, $\delta L = 0.5\,\text{mm}$

Reduction in diameter, $\delta D = 0.015\,\text{mm}$

Poisson ratio, $\mu = ?$

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{\delta D / D}{\delta L / L} = \frac{0.015}{50} \times \frac{500}{0.5} = 0.3$$

19. A 200 mm long, stress free rod at room temperature is held between two immovable rigid walls. The temperature of the rod is uniformly raised by 250^0 C. If the Young's modulus and coefficient of thermal expansion are 200 GPa and $1\times10^{-5}/^0$ C, respectively, the magnitude of the longitudinal stress (in MPa) developed in the rod is _______ GATE ME1 2014

19. 500

Length of the rod, $L = 200$ mm

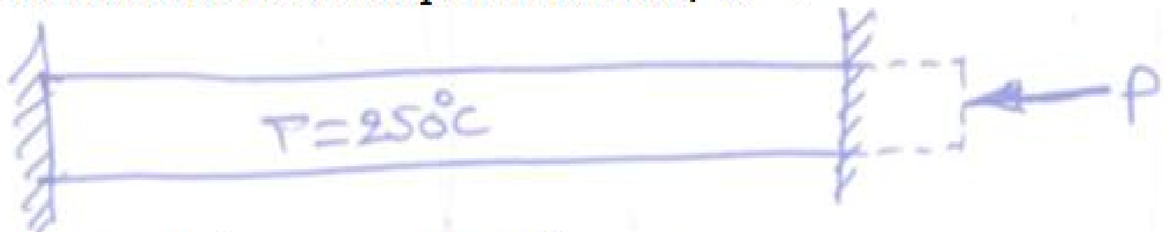
Rod is held between two immovable rigid walls.

Rise of temperature, $T = 250^0$ C

Modulus of elasticity, $E = 200$ GPa

Coefficient of thermal expansion, $\alpha = 1\times10^{-5}/^0$ C

Longitudional stress developed in the rod, $\sigma = ?$



$\sigma = \alpha TE = 1\times10^{-5}\times250\times200\times10^{3} = 500$ N/mm² (Compressive)

20. If the Poisson's ratio of an elastic material is 0.4, the ratio of modulus of rigidity to Young's modulus is......... GATE ME4 2014

20. 0.357

Poisson's ratio, $\mu = 0.4$

$$\frac{\text{Modulus of rigidity}\,(G)}{\text{Youngs modulus}\,(E)} = ?$$

$$E = 2G(1+\mu) \Rightarrow E = 2G(1+0.4) = 2.8G$$

$$\frac{G}{E} = \frac{1}{2.8} = 0.357$$

21. The number of independent elastic constants required to define the stress-strain relationship for an isotropic elastic solid is…….. GATE ME4 2014

21. 2

For an isotropic elastic solid, two independent elastic constants required to define the stress strain relationship.

22. A rod is subjected to a uni-axial load within linear elastic limit. When the change in the stress is 200 MPa, the change in the strain is 0.001. If the Poisson's ratio of the rod is 0.3, the modulus of rigidity (in GPa) is…. GATE ME 2015

22. 76.9

Change in stress, $\delta\sigma = 200\,\text{MPa}$

Change in strain $\delta e = 0.001$

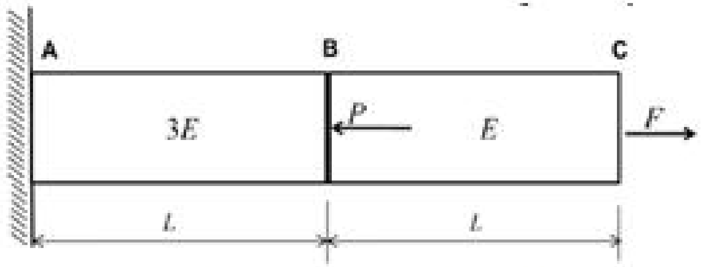
Poisson's ratio, $\mu = 0.3$

Modulus of rigidity, $G = ?$

Modulus of elasticity, $E = \dfrac{\sigma}{e} = \dfrac{\delta\sigma}{\delta e} = \dfrac{200}{0.001} = 200 \times 10^3\,\text{MPa} = 200\,\text{GPa}$

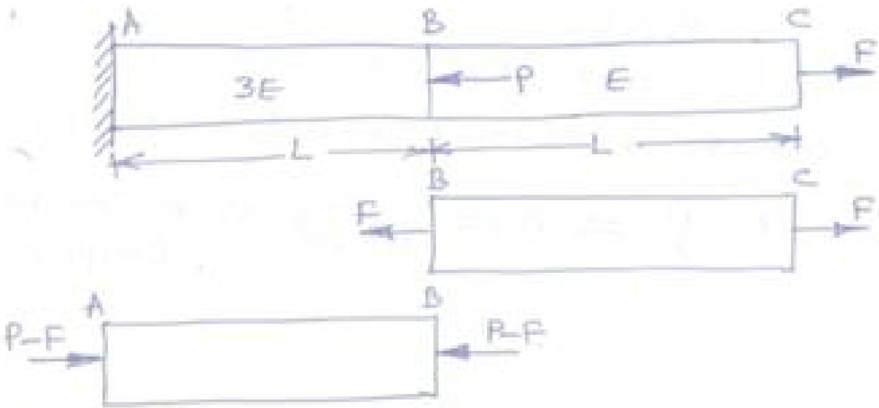
$E = 2G(1+\mu) \Rightarrow 200 = 2G(1+0.3) \Rightarrow 200 = 2.6G \Rightarrow G = 76.9\text{ GPa}$

23. A horizontal bar with a constant cross-section is subjected to loading as shown in the figure. The Young's moduli for the sections AB and BC are $3E$ and E, respectively.



For the deflection at C to be zero, the ratio P/F is _______ GATE ME1 2016

23. 4



Deflection of C, $\delta = \delta_{AB} + \delta_{BC}$

$$0 = -\frac{(P-F)L}{A \times 3E} + \frac{FL}{AE} \Rightarrow \frac{P-F}{3} = F \Rightarrow P-F = 3F \Rightarrow P = 4F \Rightarrow \frac{P}{F} = 4$$

24. A circular metallic rod of length 250 mm is placed between two rigid immovable walls as shown in the figure. The rod is in perfect contact with the wall on the left side and there is a gap of 0.2 mm between the rod and the wall on the right side. If the temperature of the rod is increased by 200^0 C, the axial stress developed in the rod is ___________ MPa.

Young's modulus of the material of the rod is 200 GPa and the coefficient of thermal expansion is 10^{-5} per ^{0}C. GATE ME2 2016

24. 240

Length of the rod, $L = 250$ mm

The rod is placed between two rigid immovable walls.

Gap between wall and rod, $\delta_0 = 0.2\,\text{mm}$

Rise of temperature, $T = 200^0$C

Axial stress developed in the rod, $\sigma = ?$

Young's modulus, $E = 200$ GPa

Coefficient of thermal expansion, $\alpha = 10^{-5}/^0 C$

Free expansion of the bar due to rise of temperature $\delta_T = L\alpha T$

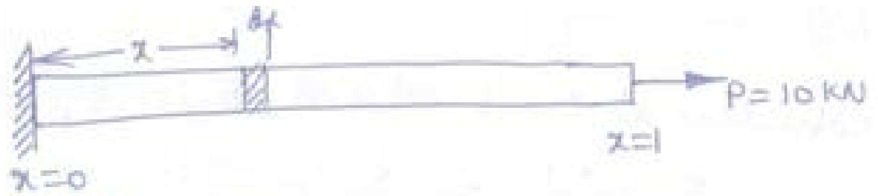
$$= 250 \times 10^{-5} \times 200 = 0.5 \text{ mm}$$

Expansion of the bar prevented, $\delta = 0.5 - 0.2 = 0.3\,\text{mm}$

$$\delta = \frac{PL}{AE} \Rightarrow 0.3 = \frac{\sigma \times 250}{200 \times 10^3} \Rightarrow \sigma = 240\,\text{MPa (Compressive)}$$

25. A horizontal bar, fixed at one end ($x = 0$), has a length of 1 m, and cross-sectional area of 100 mm². Its elastic modulus varies along its length as given by $E(x) = 100e^{-x}$ GPa, where x is the length coordinate (in m) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end ($x = 1$). The axial displacement of the free end is ….. mm. GATE ME 2017

25. 1.718



Length of bar, $L = 1\text{m}$

Cross sectional area, $A = 100\text{ mm}^2$

Elastic modulus, $E(x) = 100e^{-x}$ GPa

Axial displacement of the free end, $\delta L = ?$

Let us consider the elemental length dx which is at a distance of x from the free end.

$$\delta(dx) = \frac{pdx}{AEx}$$

$$\delta L = \int_0^L \frac{P.dx}{A.Ex} = \int_0^L \frac{p.dx}{A.100\,e^{-x}} = \frac{P}{100A}\int_0^L e^x dx = \frac{p}{100A}\left[e^x\right]_0^L = \frac{P}{100A}.(e^L - 1)$$

$$= \frac{10\times10^3}{100\times100}\left(e^1 - 1\right) = 1.718\,\text{mm}$$

26. The Poisson's ratio for a perfectly incompressible linear elastic material is

a. 1 **b.** 0.5 c. 0 d. infinity GATE ME 2017

27. A steel bar is held by two fixed supports as shown in the figure and is subjected to an increase of temperature $\Delta T = 100^0 C$. If the coefficient of thermal expansion and Young's modulus of elasticity of steel are $11\times 10^{-6}/^0C$ and 200 GPa, respectively, the magnitude of thermal stress (in MPa) induced in the bar is …

GATE ME 2017

27. 30

Steel bar is held by two fixed supports

Increase in temperature, $T = 100^0$ C

Coefficient of thermal expansion, $\alpha = 11\times10^{-6}\ /^0\ C$

Young's modulus of elasticity, E= 200 GPa

Thermal stress induced in the bar, $\sigma = ?$

$$\sigma = \alpha TE = 11 \times 10^{-6} \times 100\times200\times10^3 = 220 \text{ MPa}$$

28. A rod of length 20 mm is stretched to make a rod of length 40 mm. Subsequently, it is compressed to make a rod of final length 10 mm. Consider the longitudinal tensile strain as positive and compressive strain as negative. The total true longitudinal strain in the rod is GATE ME 2017

a. -0.5 b. -0.69 c. -0.75 d. -1.0

28.

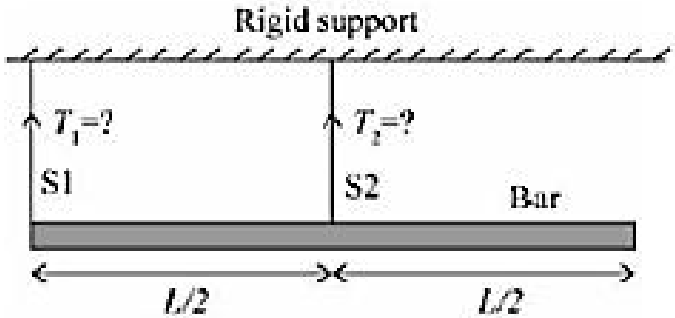
Initial length of rod, $L_1 = 20$ mm

Stretched length of rod, $L_2 = 40$ mm

Final length of rod, $L_3 = 10$ mm

Total true strain in the rod, $e = ?$

29. A bar of uniform cross section and weighing 100 N is held horizontally using two massless and inextensible strings S1 and S2 as shown in the figure.

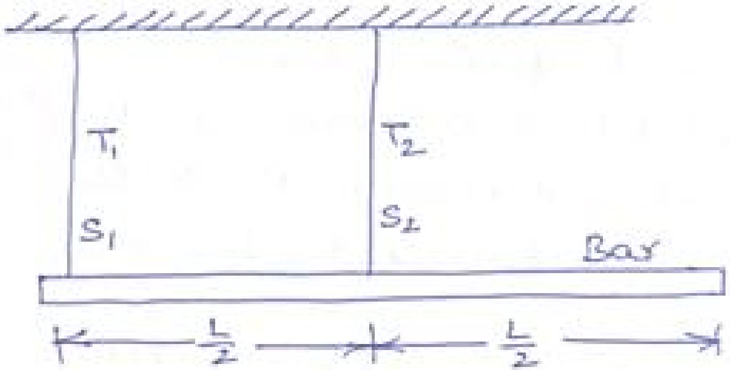


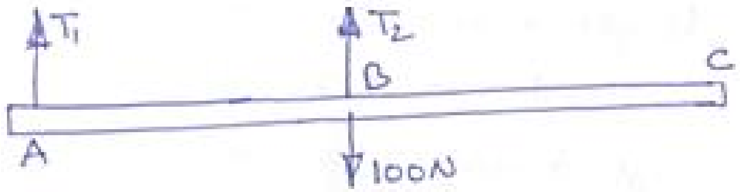
The tensions in the strings are GATE ME1 2018

a. $T_1 = 100$ N and $T_2 = 0$ N b. $T_1 = 0$ N and $T_2 = 100$ N

c. $T_1 = 75$ N and $T_2 = 25$ N d. $T_1 = 25$ N and $T_2 = 75$ N

29.



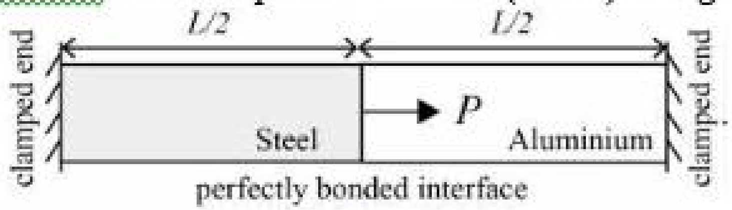


Free body diagram

$$\Sigma V = 0 \Rightarrow T_1 + T_2 = 100 \text{ ....i}$$

$$\Sigma M_A = 0 \Rightarrow (T_2 - 100)\frac{L}{2} = 0 \Rightarrow T_2 = 100\text{ N} \qquad T_1 = 0$$

30. A bimetallic cylindrical bar of cross sectional area 1 m2 is made by bonding Steel (Young's modulus = 210 GPa) and Aluminium (Young's modulus = 70 GPa) as shown in the figure. To maintain tensile axial strain of magnitude 10^{-6} in Steel bar and compressive axial strain of magnitude 10^{-6} in Aluminum bar, the magnitude of the required force P (in kN) along the indicated direction is



GATE ME2 2018

a. 70 b. 140 c. 210 **d.** 280

30. 210

Cross sectional area, $A = 1\ \text{m}^2$

Young's modulus of steel, $E_S = 210$ GPa

Young's modulus of Aluminum, $E_{AL} = 70$ GPa

Tensional axial strain in steel, $e_S = 10^{-6}$

Compressive axial strain in Aluminum, $e_{AL} = 10^{-6}$

$$e_S = e_{AL}$$

$$\left(\frac{P}{AE}\right)_S = \left(\frac{P}{AE}\right)_{AL} \Rightarrow \frac{P_S}{AE_S} = \frac{P_{Al}}{A.E_{Al}} \Rightarrow P_s = P_{Al}\frac{E_s}{E_{Al}} = P_{Al}\frac{210}{70} = 3P_{Al}$$

$$P_s + P_{Al} = P \Rightarrow 3P_{Al} + P_{Al} = P \Rightarrow P_{Al} = \frac{P}{4}, P_s = \frac{3}{4}.P$$

$$\left(\frac{P}{AE}\right)_S = 10^{-6} \Rightarrow \frac{P}{1\times10^6\times210\times10^3} = 10^{-6} \Rightarrow P = 210\times10^3\ \text{N} = 210\ \text{kN}$$

31. A solid cube of side 1 m is kept at a room temperature 32°C. The coefficient of linear thermal expansion of the cube material is $1\times10^{-5}/°C$ and the bulk modulus is 200 GPa. If the cube is constrained all around and heated uniformly to 42°C, then the magnitude of volumetric (mean) stress (in MPa) induced due to heating is… GATE ME1 2019

31. 60

Side of solid cube, a=1m

Initial temperature, $T_1 = 32^0 C$

Coefficient linear thermal expansion, $\alpha = 1\times10^{-5}/^0 C$

Bulk modulus, k=200GPa

Final temperature, $T_2 = 42^0 C$

Rise of temperature, T=42-32=10^0C

Strain in each direction due to temperatures,

$e_x = e_y = e_z = \alpha T = 1\times10^{-5}\times10 = 10^{-4}$

Volumetric strain, $e_v = e_x + e_y + e_z = 3\times10^{-4}$

Also, $e_v = \frac{3\sigma}{E}(1-2\mu) \Rightarrow e_v = \frac{3\sigma}{3k} \Rightarrow e_v = \frac{\sigma}{k}$

$3\times10^{-4} = \frac{\sigma}{200\times10^3} \Rightarrow \sigma = 60\,MPa$