SHEAR STRESSES IN BEAMS

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- The applied shear force will induce shear stress across transverse section of the beam.
- At each point on a section, the transverse shear stress will produce a complementary horizontal shear stress.
- The longitudinal shear stresses will balance the variation of bending stresses along the beam.
- If the bending moment is constant, there is no shear force and hence no shear stress.
- If there is no variation of bending stress between successive transverse sections, there can be no longitudinal shear stresses.

Shear stress, $\tau = \dfrac{F.(A\bar{y})}{I.b}$

F : Shear force across the section

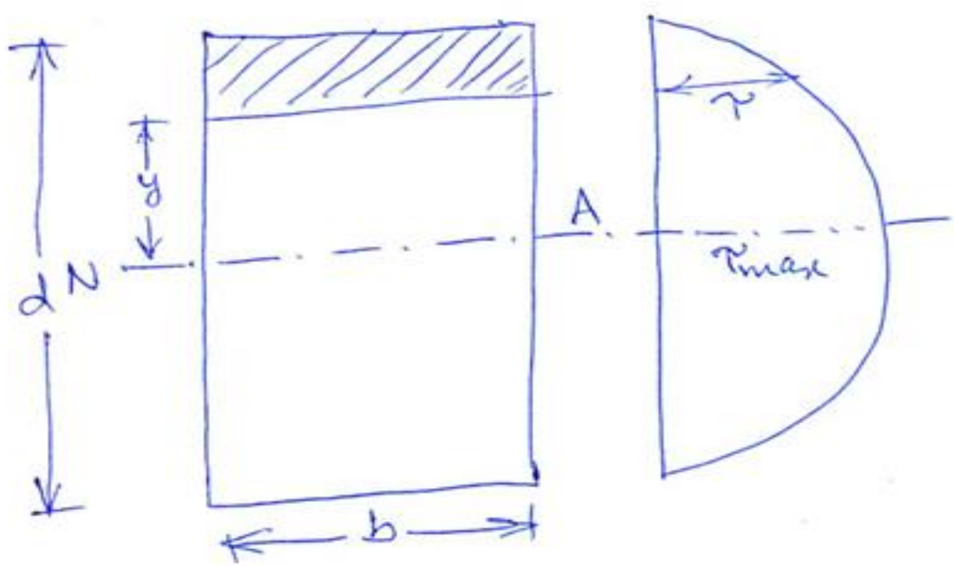
$A\bar{y}$: Moment of the sectional area above the point under consideration about the neutral axis.

I : Moment of inertia of the section about the neutral axis

b : Width of the section under consideration

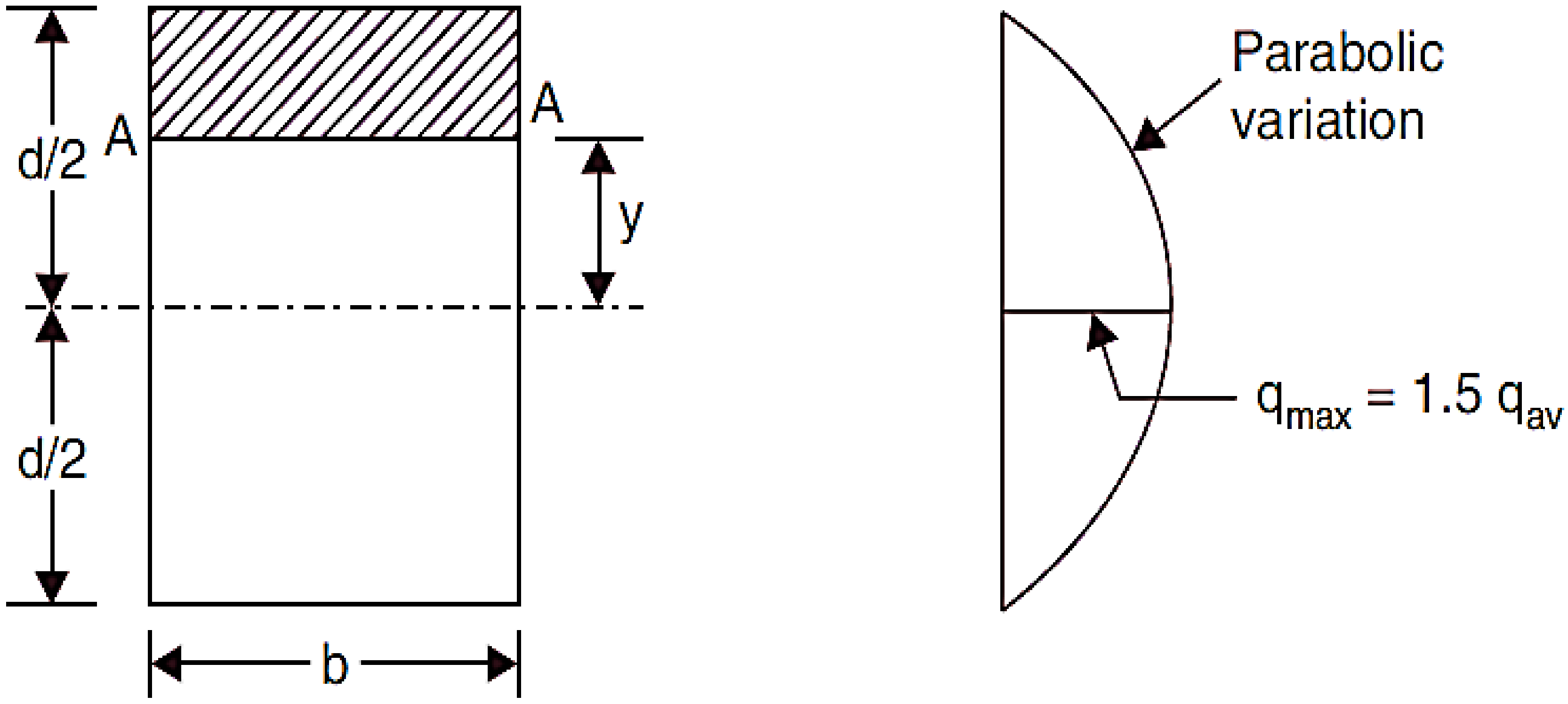
Variation of shear stress across the section:

1. Rectangular section



Maximum shear stress, $\tau_{\max} = \frac{3}{2} \cdot \frac{F}{bd}$

$$\tau_{\max} = \frac{3}{2} . \tau_{\text{mean}}$$



(a)

(b)

$$q = \frac{F}{bI}(a\bar{y})$$

where $(a\bar{y})$ is the moment of area above the section about the neutral axis. Now,

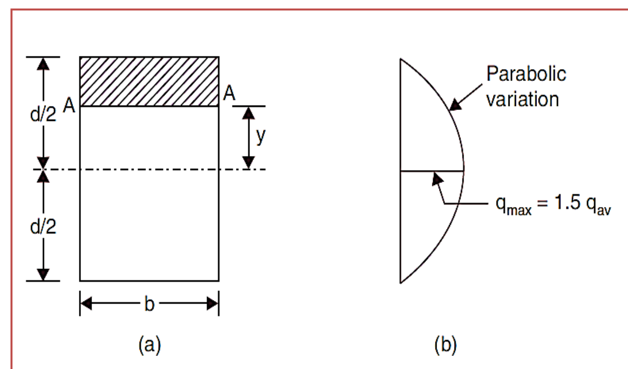
$$a = b(d/2 - y)$$

$$\bar{y} = y + \frac{1}{2}(d/2 - y) = \frac{1}{2}(d/2 + y)$$

$$\therefore \quad a\bar{y} = \frac{b}{2}(d/2 - y) \times \frac{1}{2}(d/2 + y)$$

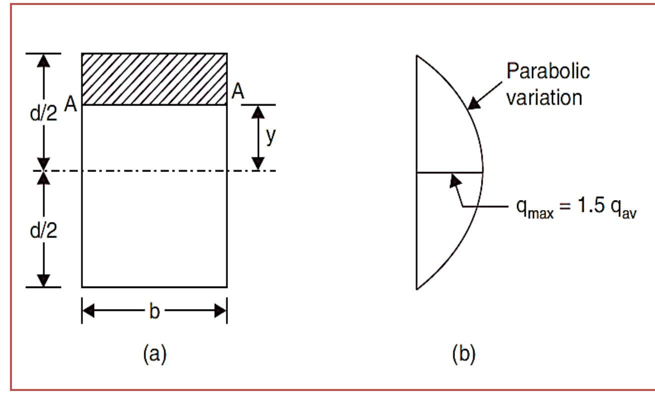
$$= \frac{b}{2}(d^2/4 - y^2)$$

$$I = \frac{1}{12} bd^3$$



$$\therefore \qquad q = \frac{F}{b\,\frac{1}{12}\,bd^3}\,\frac{b}{2}\,(d^2/4 - y^2)$$

$$= \frac{6F}{bd^3}\,(d^2/4 - y^2)$$



This shows shear stress varies parabolically.

When $\quad y = \pm\, d/2, \qquad q = 0$

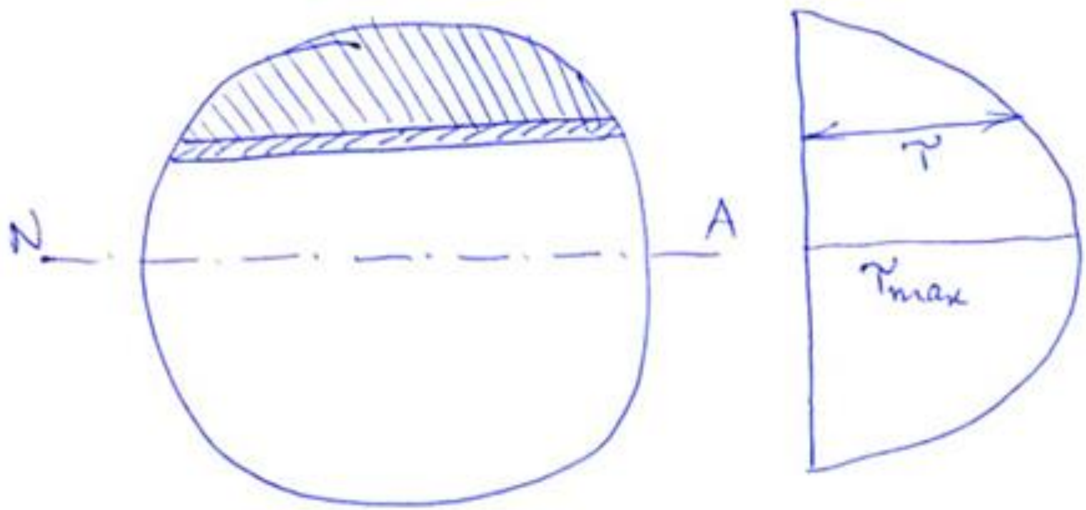
At $\quad y = 0, \qquad q_{max} = \frac{6F}{bd^3}\,\frac{d^2}{4} = 1.5\,\frac{F}{bd}$

$$= 1.5\ q_{av}$$

where $\quad q_{av} = \frac{F}{bd}$ is average shear stress.

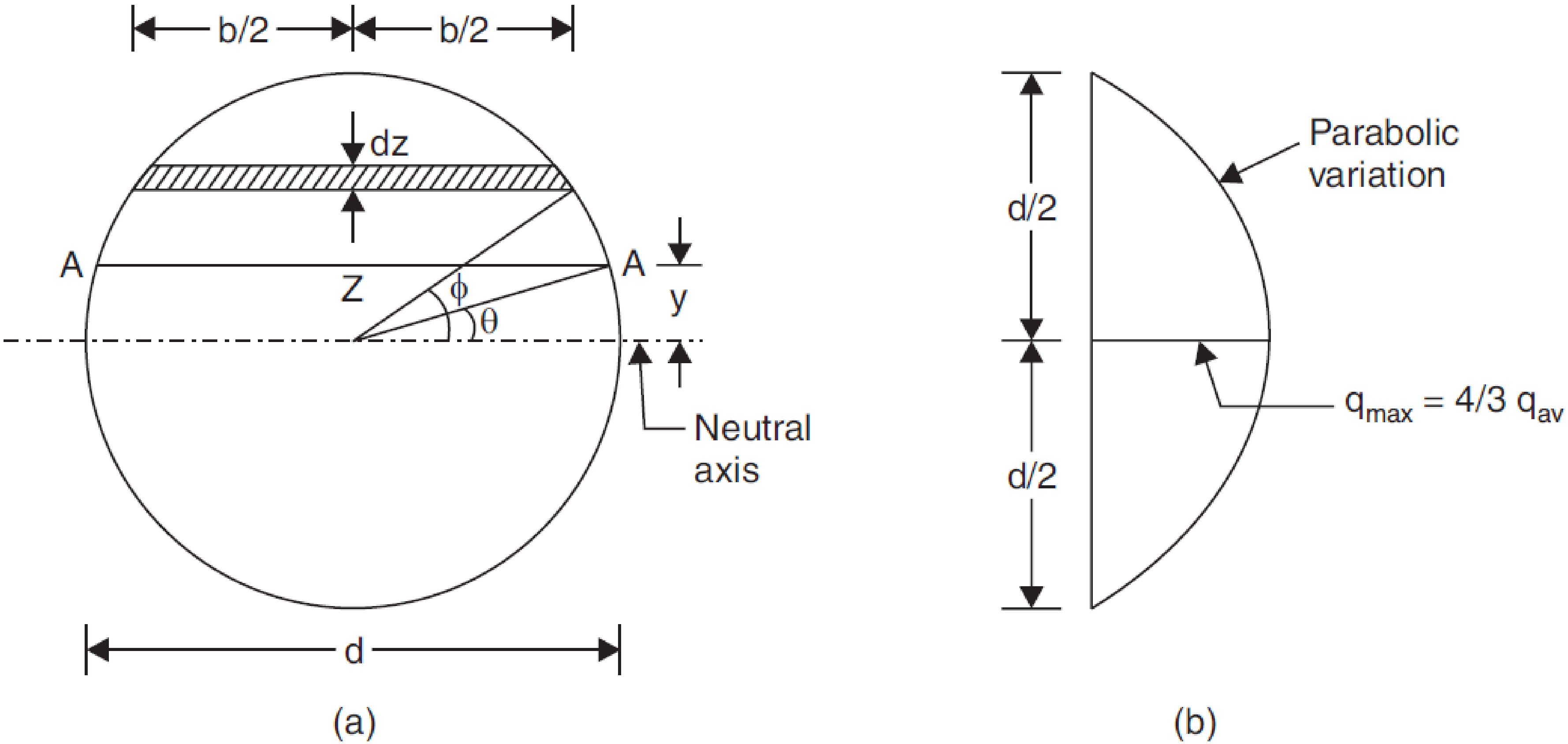
Thus in rectangular section maximum shear stress is at neutral axis and it is 1.5 times average shear stress. It varies parabolically from zero at extreme fibres to 1.5 q_{av} at mid depth as shown in Fig.

2. Solid circular section



Maximum shear stress, $\tau_{\max} = \frac{4}{3} \cdot \frac{F}{\pi R^2}$

$$\tau_{\max} = \frac{4}{3} . \tau_{\text{mean}}$$


b/2
b/2
dz
A
A
Z
ϕ
θ
y
Neutral axis
d
(a)
d/2
d/2
Parabolic variation
q_max = 4/3 q_av
(b)

Width of element $$b = 2 \cdot \frac{d}{2} \cos \phi$$

$$= d \cos \phi$$

$$z = \frac{d}{2} \sin \phi$$

$\therefore$ $$dz = \frac{d}{2} \cos \phi \; d\phi$$

$\therefore$ Area of the element

$$a = bdz = d \cos \phi \cdot \frac{d}{2} \cos \phi \; d\phi$$

$$= \frac{d^2}{2} \cos^2 \phi \; d\phi$$

Moment of this area about neutral axis

$$= \text{area} \times z$$

$$= \frac{d^2}{2} \cos^2 \phi \, d\phi \, \frac{d}{2} \sin \phi$$

$$= \frac{d^3}{4} \cos^2 \phi \sin \phi \, d\phi$$

$\therefore$ Moment of area about section A-A about neutral axis

$$(a\bar{y}) = \int_{\theta}^{\pi/2} \frac{d^2}{4} \cos^2 \phi \sin \phi \, d\phi$$

$$= \frac{d^3}{4} \left[\frac{-\cos^3 \phi}{3} \right]_{\theta}^{\pi/2}$$

[Since if $\cos \phi = t$, $dt = -\sin \phi \, d\phi$ and $-t^3/3$ is integration]

$$\therefore \qquad (a\bar{y}) = \frac{d^3}{4 \times 3}\left[-\cos^2\frac{\pi}{2} + \cos^3\theta\right]$$

$$= \frac{d}{12}\cos^3\theta$$

$$\text{Now} \qquad I = \frac{\pi d^4}{64}$$

$$\therefore \qquad q = \frac{F}{bI}(a\bar{y})$$

$$= \frac{F}{d\cos\theta\,\frac{\pi}{64}d^4} \times \frac{d^3}{12}\cos^3\theta$$

$$= \frac{64}{12} \frac{F}{\pi d^2} \cos^2 \theta$$

$$= \frac{16}{3} \frac{F}{\pi d^2} [1 - \sin^2 \theta]$$

$$= \frac{16}{3} \frac{F}{\pi d^2} \left[1 - \left(\frac{y}{d/2}\right)^2\right]$$

$$= \frac{16}{3} \frac{F}{\pi d^2} \left[1 - \frac{4y^2}{d^2}\right]$$

Hence shear stress varies parabolically.

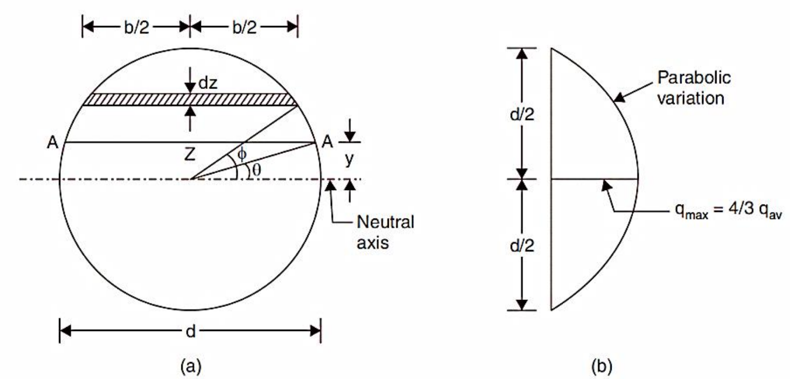
At $y = \pm\, d/2, \qquad q = 0$

$$y = 0, \qquad q = q_{max} = \frac{16}{3}\frac{F}{\pi d^2}$$

$$= \frac{4}{3}\frac{F}{\pi/4\, d^2}$$

$$= \frac{4}{3}\frac{F}{\text{Area}}$$

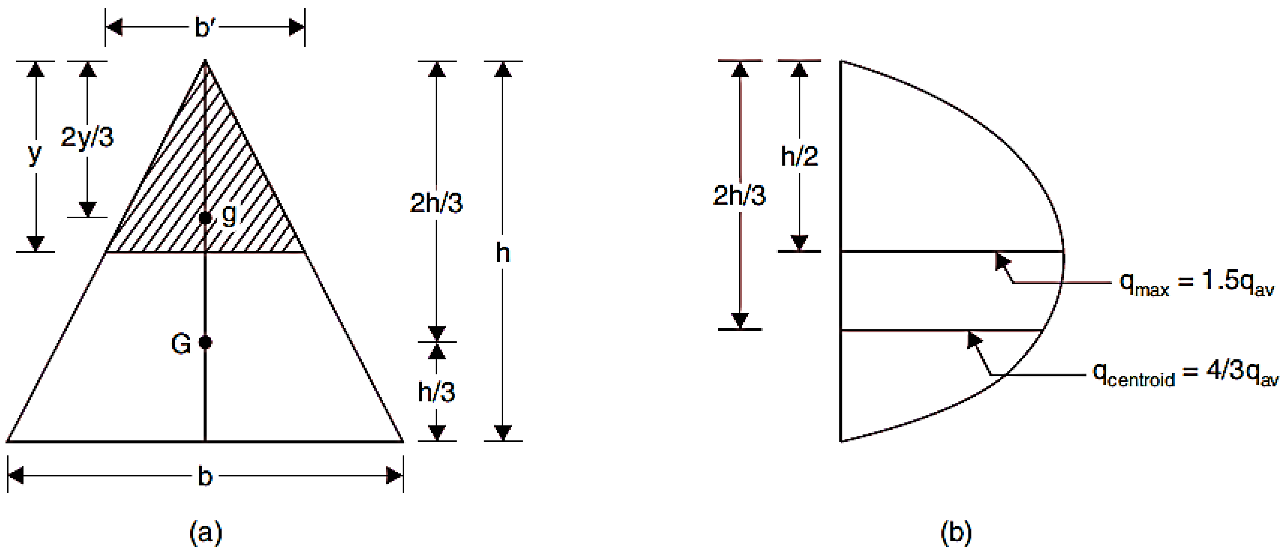
$$= \frac{4}{3}\, q_{av}.$$



where q_{av} = average shear stress.

Thus in circular sections also shear stress varies parabolically from zero at extreme edges to the maximum value of $\frac{4}{3}\, q_{av}$ at mid depth as shown in Fig.

3. Triangular Section

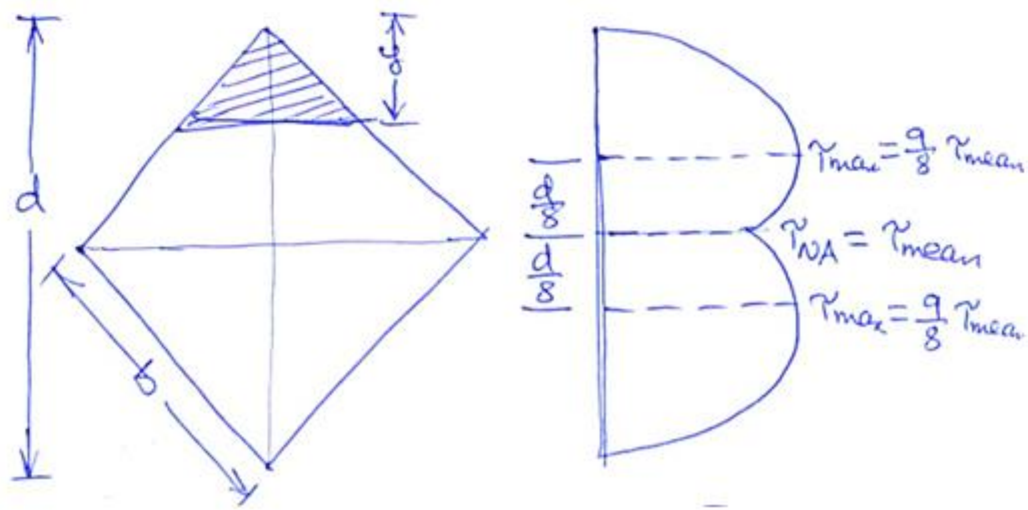


Shear stress at neutral axis, $\tau_{NA} = \frac{8}{3} \cdot \frac{F}{bh} = \frac{4}{3} \cdot \tau_{mean}$

Maximum shear stress, $\tau_{max} = 3 \cdot \frac{F}{bh} = \frac{3}{2} \cdot \tau_{mean}$

Maximum shear stress (τ_{max}) occurs at the center of the section.

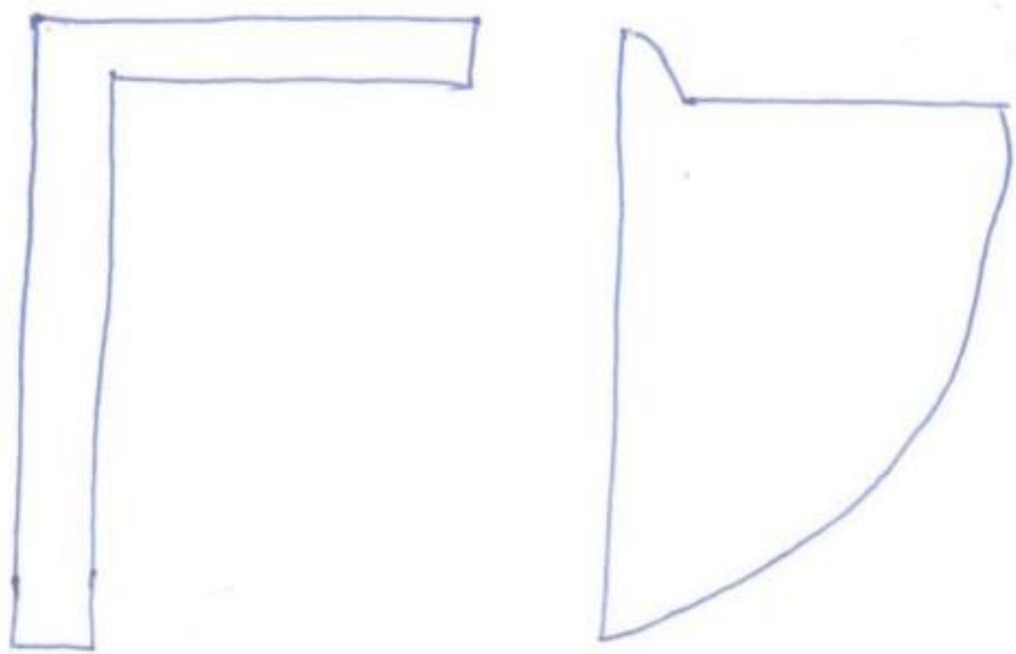
4. Diamond section



Shear stress at neutral axis, $\tau_{NA} = \dfrac{F}{b^2} = \tau_{\text{mean}}$

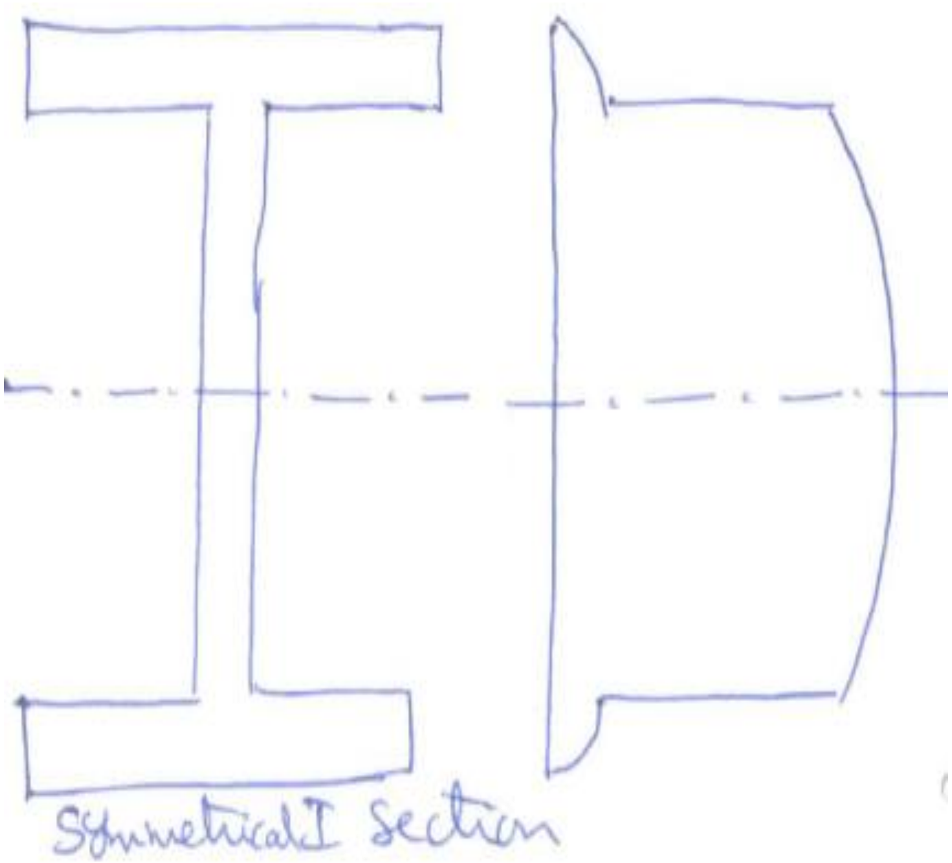
Maximum shear stress, $\tau_{\max} = \dfrac{9}{8} \cdot \dfrac{F}{b^2} = \dfrac{9}{8} . \tau_{\text{mean}}$

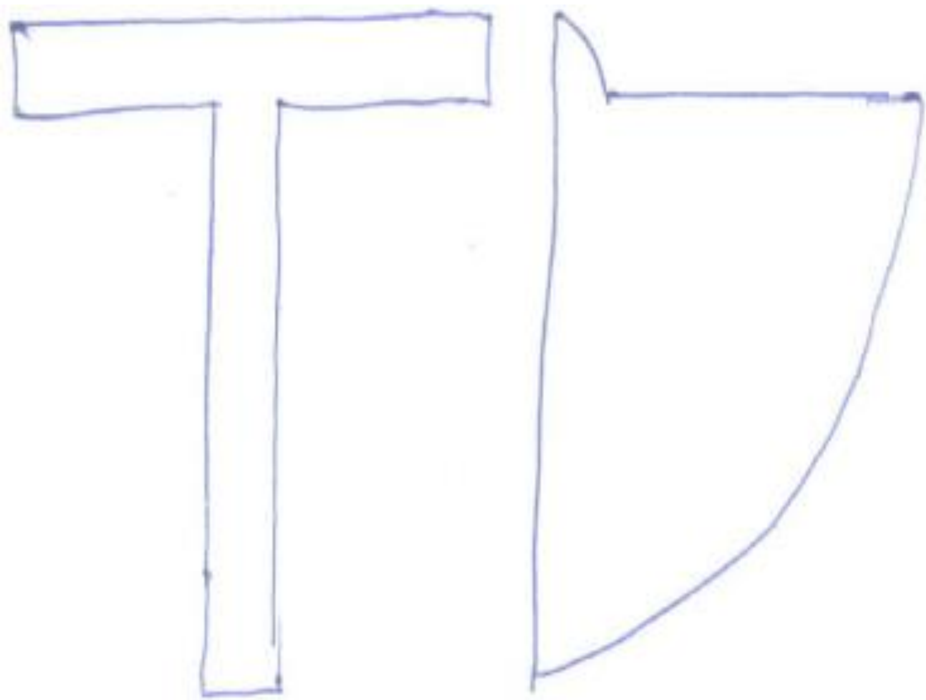
Shear stress distribution diagrams for the different shape of cross sections:



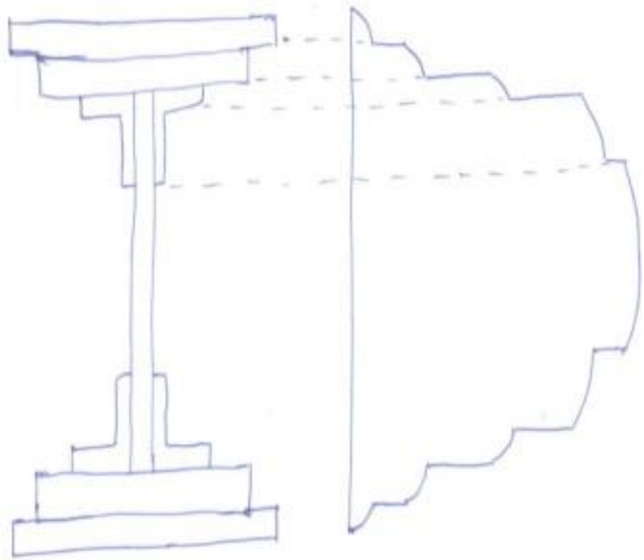
L Section

Shear stress distribution diagrams for the different shape of cross sections:

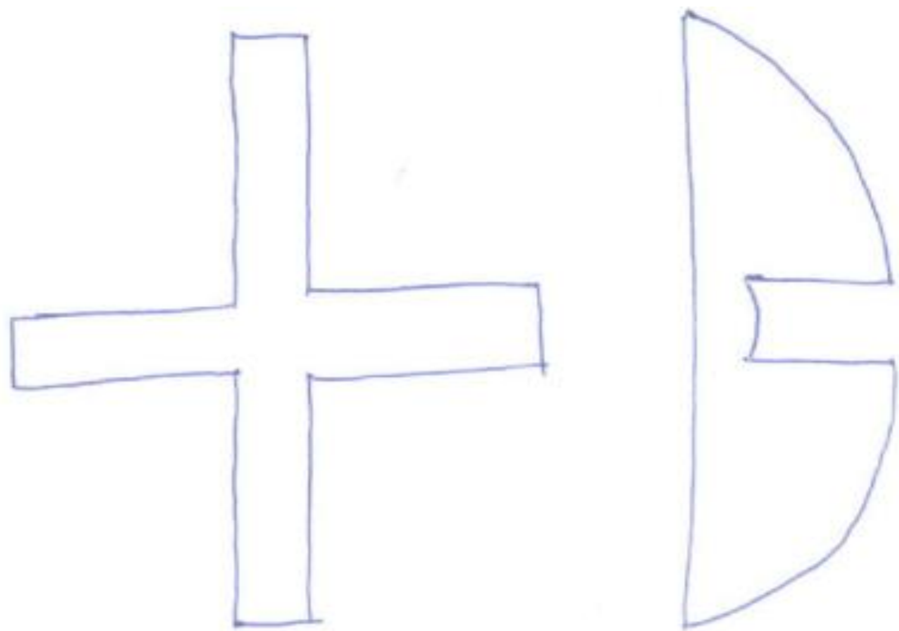




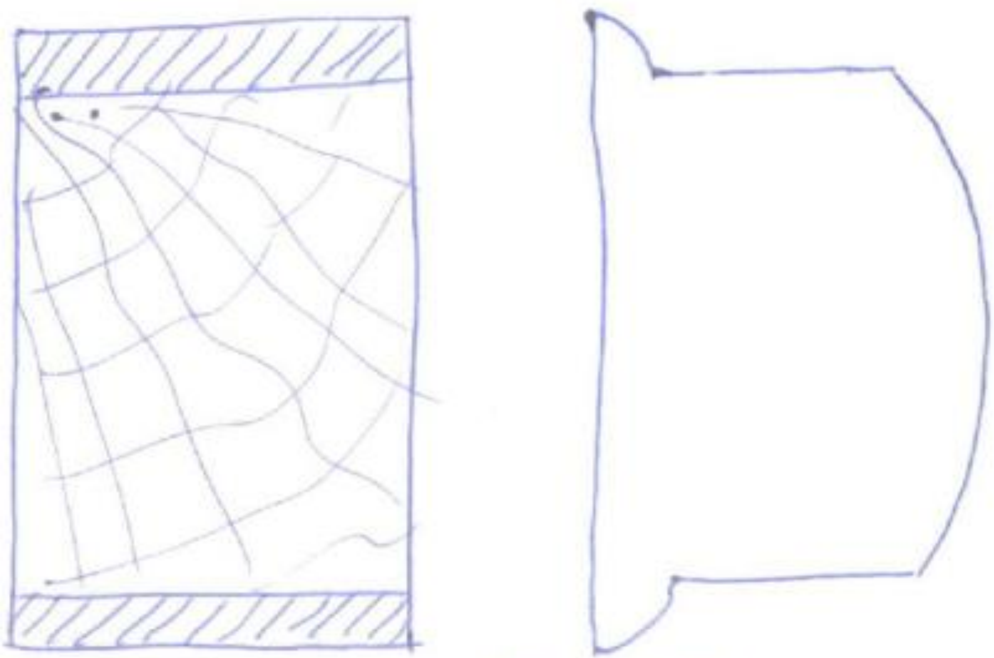
T section.



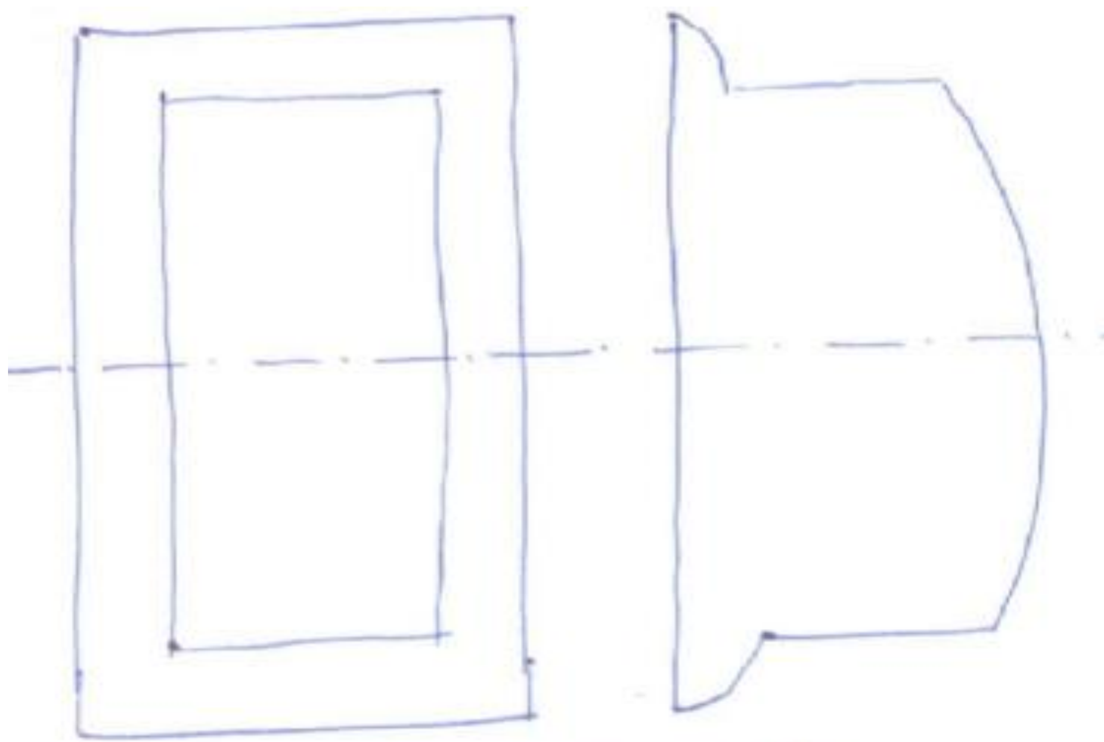
Built-up section.



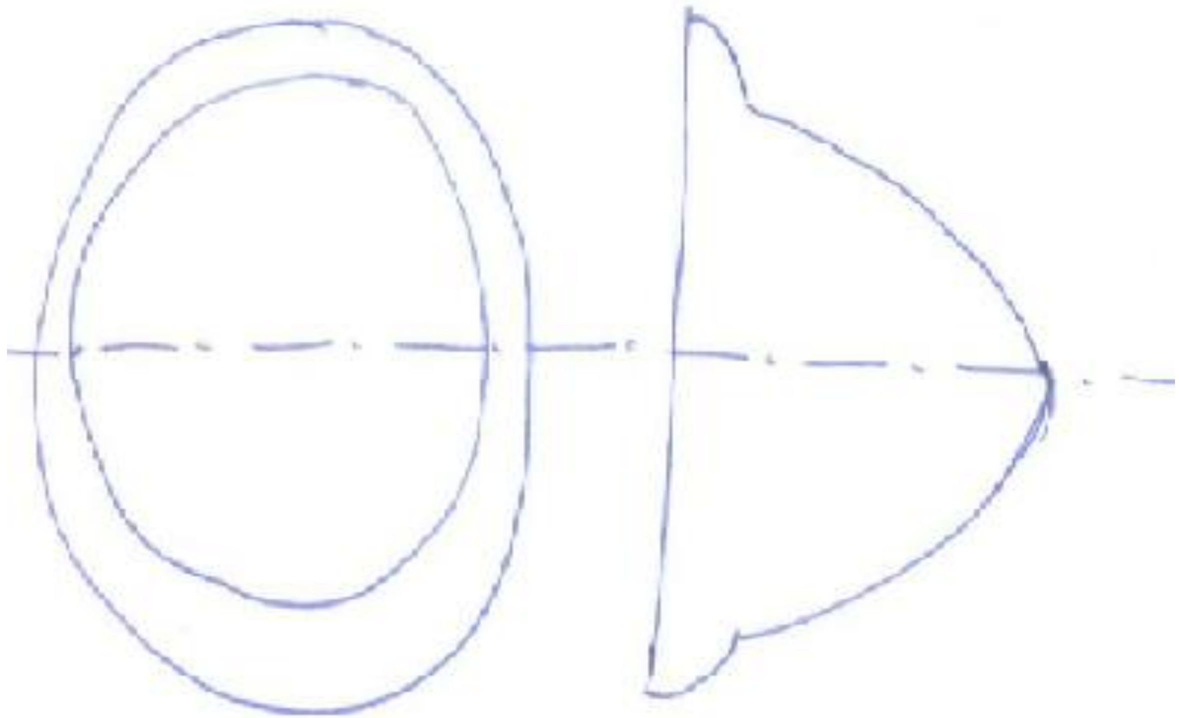
cross



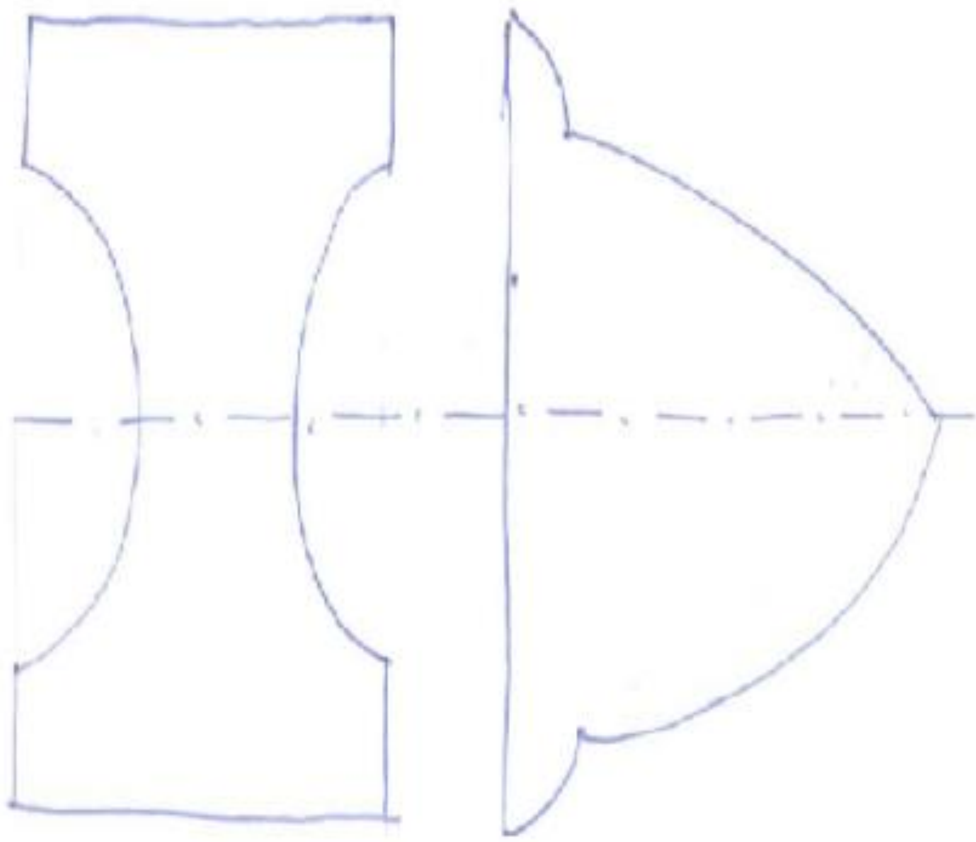
Composite section

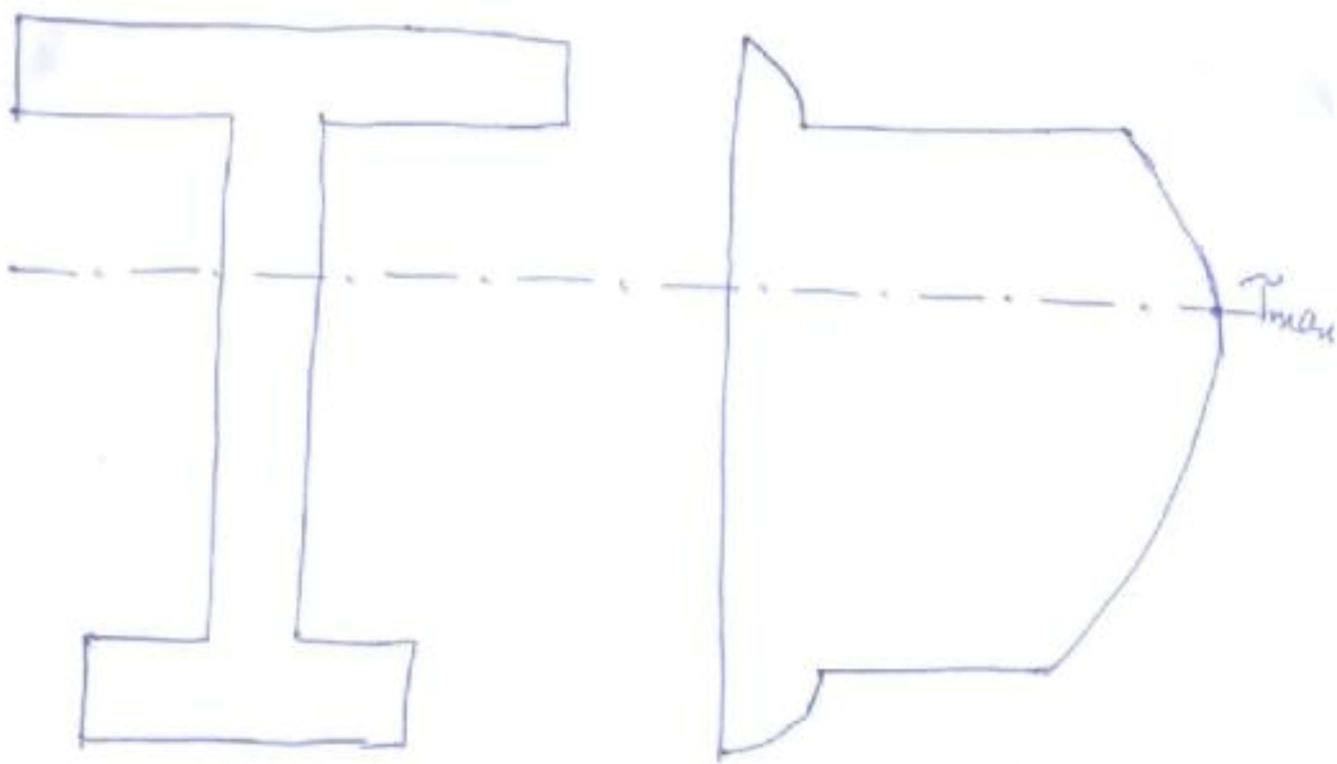


Hollow rectangular section.

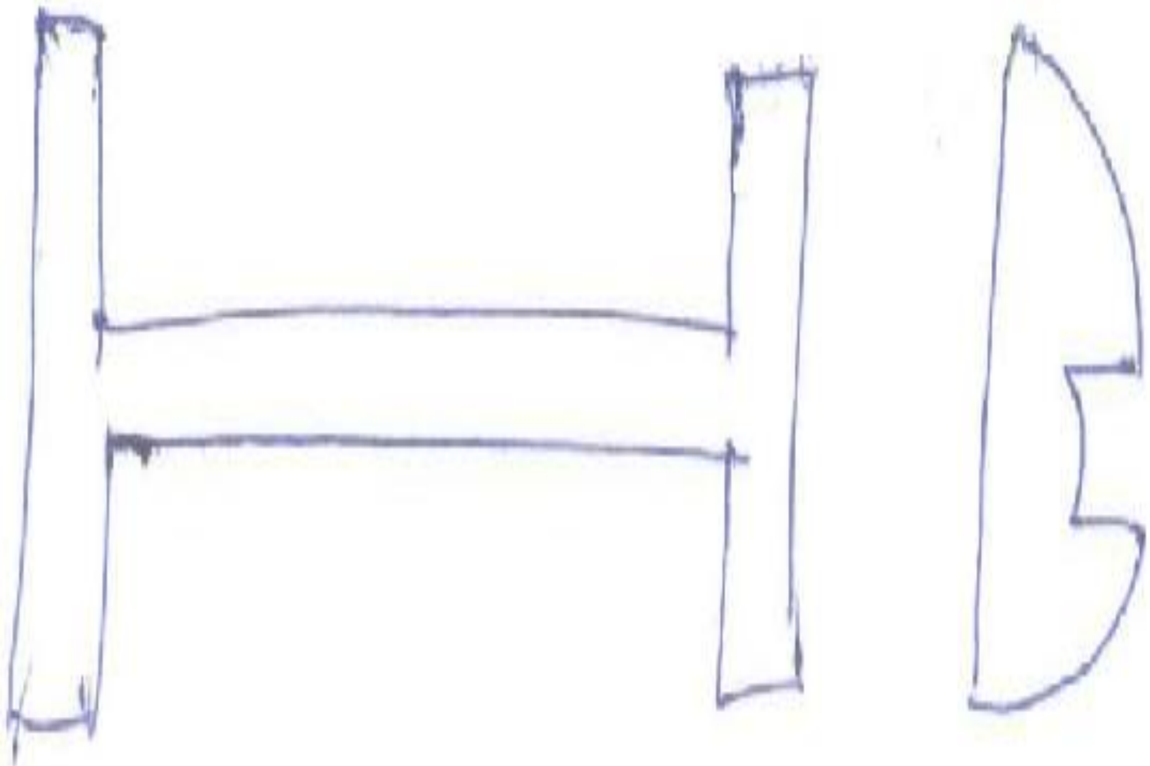


Hollow circular section.

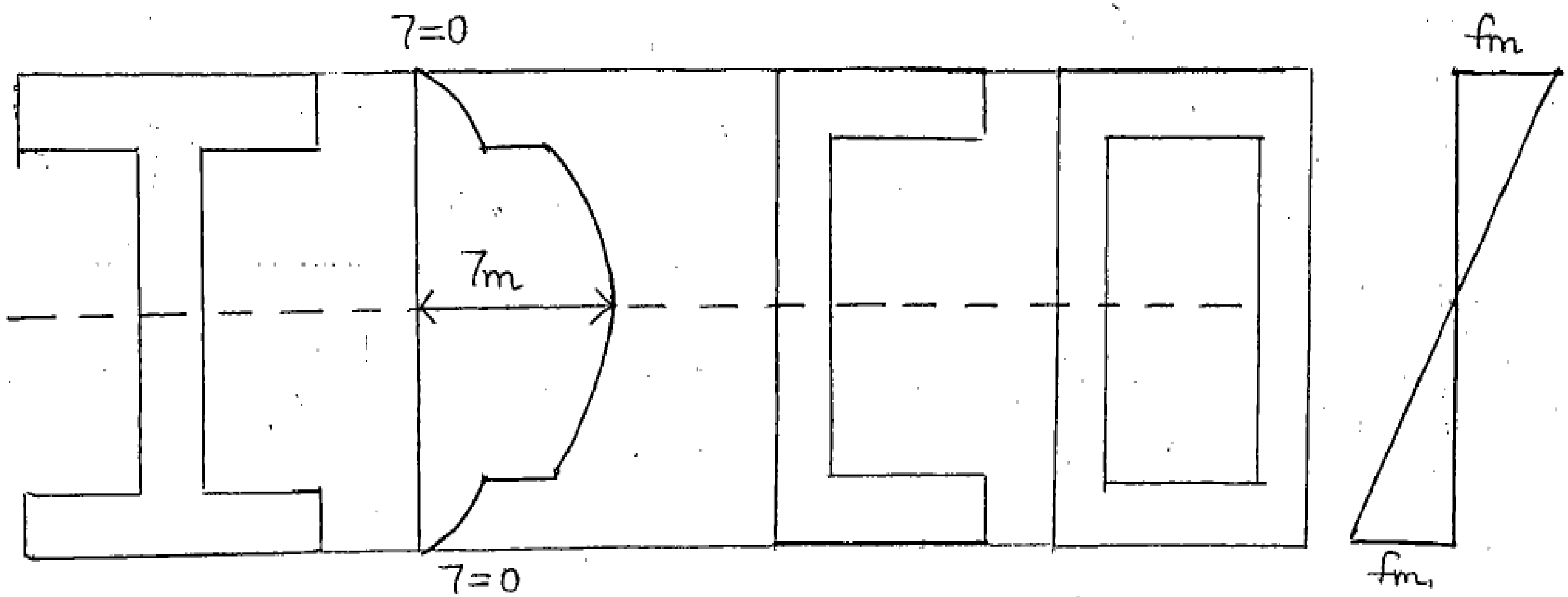


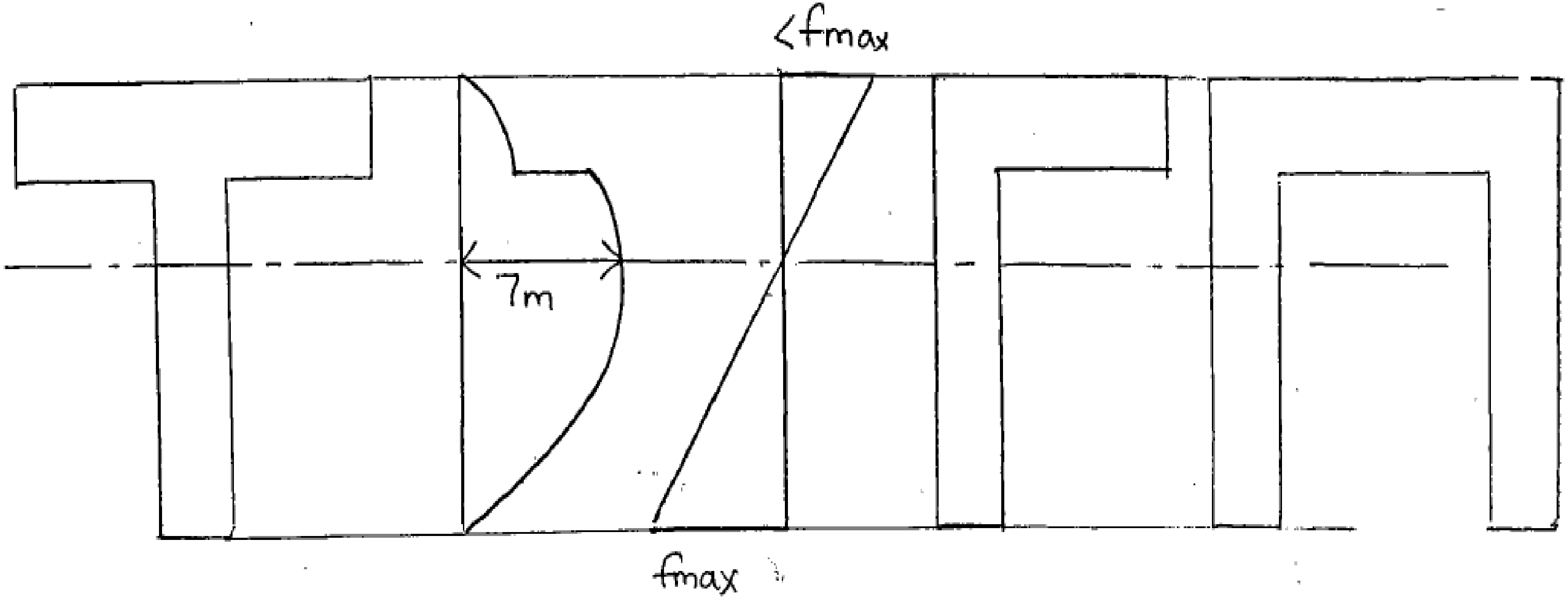


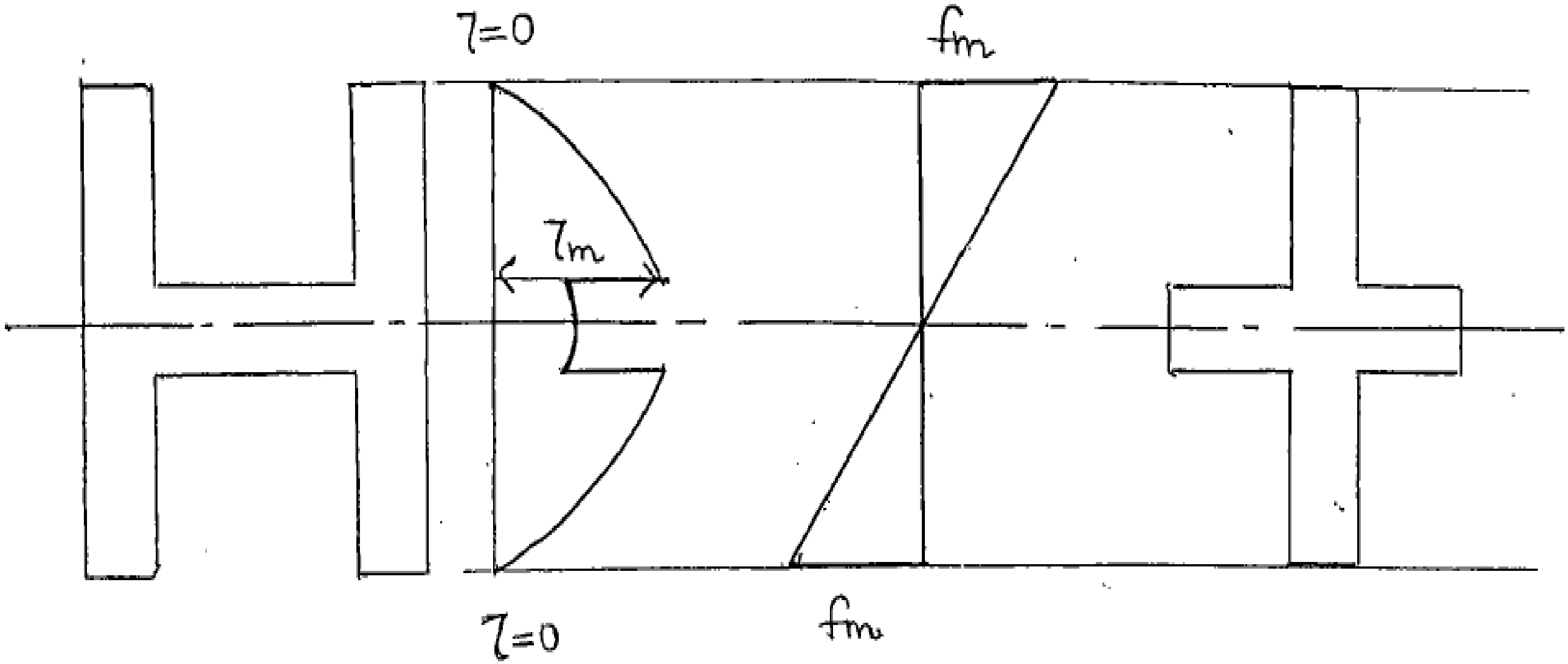
Unsymmetrical I section.



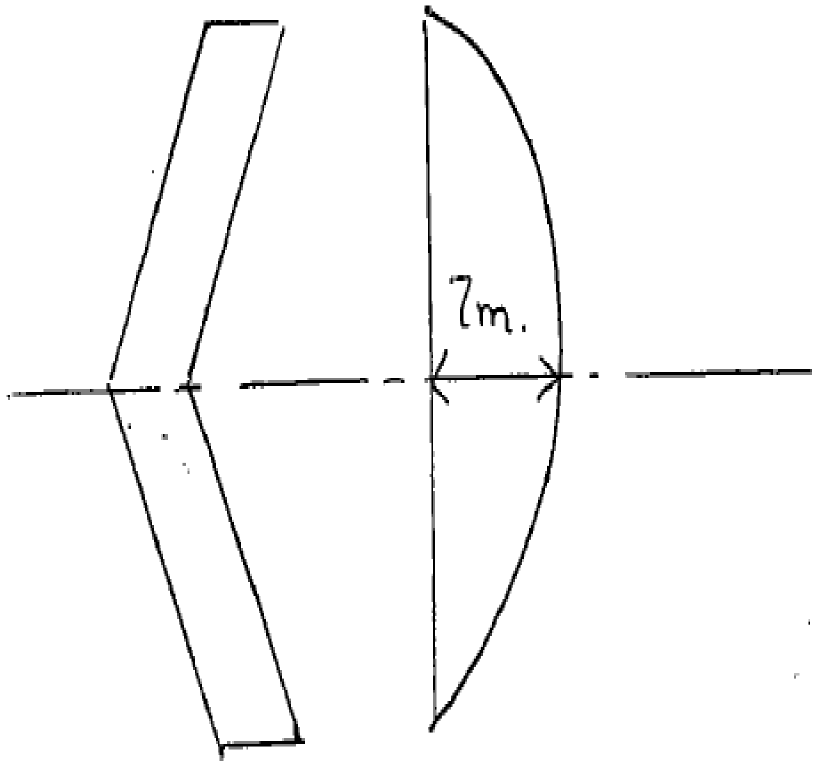
→ Flanged Beams




<fmax
7m
fmax



$\tau=0$
fm
τ_m
$\tau=0$
fm



$$\tau = \frac{V A \bar{y}}{I b} \Rightarrow \tau \propto \frac{1}{b}$$

ie

$b \uparrow$	$\tau \downarrow$
$b \downarrow$	$\tau \uparrow$

GATE CE

01. For a given shear force across a symmetrical '*I*' section, the intensity of shear stress is maximum at the CE 1991,1994

a. extreme fibres

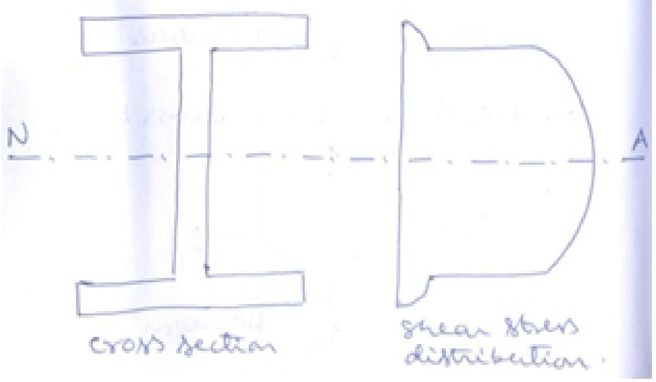
b. centroid of the section

c. at the junction of the flange and the web, but on the web

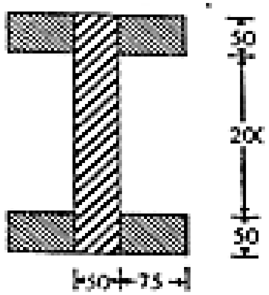
d. at the junction of the flange and the web, but on the flange

01. b

The shear stress distribution across the cross section of a symmetrical *I* section is shown in fig. The maximum shear stress occurs at centroid of the section.



02. T-section of a beam is formed by gluing wooden planks as shown in the figure below. If this beam transmits a constant vertical shear force of 3000 N, the glue at any of the four joints will be subjected to a shear force (in kN per meter length) of



CE 2006

a. 3.0 b. 4.0 c. 8.0 d. 10.7

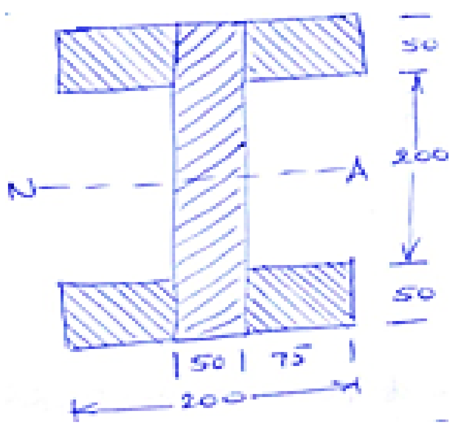
02. B

Shear stress, $\tau = \dfrac{F.A\bar{y}}{Ib}$

Shear flow, $q = \tau b = \dfrac{F.(A.\bar{y})}{I}$

Shear force, F = 3000N

Moment of inertia of the section about NA,



$$I = \frac{50\times300^3}{12} + 2\left[\frac{150\times50^3}{12} + 150\times50\times125^2\right]$$

$$= (1.125 + 2.375)\times10^8 = 3.5\times10^8\,\text{mm}^4$$

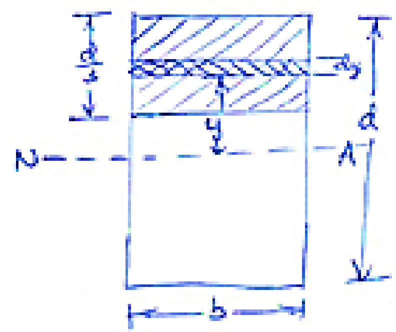
For any of the four joints, $A.\bar{y} = 75\times50\times125 = 468750$ mm^3

$$q = \frac{3000\times468750}{3.5\times10^8} = 4.0\text{ N/mm} = 4.0\text{ kN/m}$$

03. If a beam of rectangular cross-section is subjected to a vertical shear force V, the shear force carried by the upper one-third of the cross-section is CE 2006

a. zero b. $\frac{7V}{27}$ c. $\frac{8V}{27}$ d. $\frac{V}{3}$

03. b



Let y be the distance of point under consideration from the neutral axis.

Shear stress, $\tau = \frac{F.A\bar{y}}{I\,b}$

$$\tau = \frac{V.\left(\frac{d}{2} - y\right)b\left(\frac{d/2+y}{2}\right)}{Ib} = \frac{V}{2I}.\left(\frac{d^2}{4} - y^2\right)$$

Shear force carried beyond y distance from NA, $dF = \tau.b.dy$

$$= \frac{V.b}{2I}.\left(\frac{d^2}{4} - y^2\right)dy$$

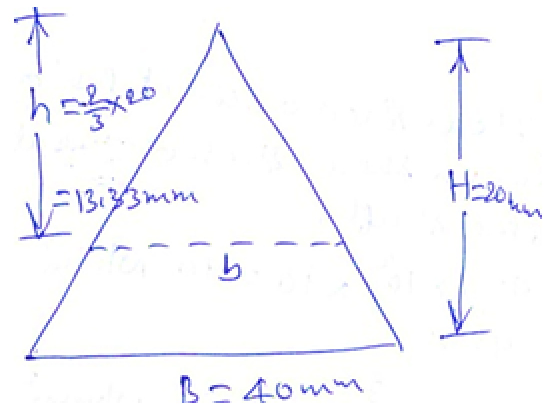
$$F = \frac{Vb}{2I}\int_{d/6}^{d/2}\left(\frac{d^2}{4} - y^2\right)dy = \frac{Vb}{2I}.\left[\frac{d^2}{4}.y - \frac{y^3}{3}\right]_{d/6}^{d/2} = \frac{Vb}{2I}\left[\frac{d^3}{8} - \frac{d^3}{24} - \frac{d^3}{24} + \frac{d^3}{648}\right]$$

$$= \frac{V\,b}{2\,bd^3}\times 12.\frac{28}{648}.d^3 = \frac{7}{27}V$$

04. The shear stress at the neutral axis in a beam of triangular section with a base of 40 mm and height 20 mm, subjected to a shear force of 3 kN is CE 2007

a. 3 MPa b. 6 MPa c. 10 MPa d. 20 MPa

04. c



Shear stress at neutral axis, $\tau_{NA} = ?$

Base of triangular section, B = 40 mm

Height of triangular section, H = 20 mm

Shear force, F = 3 kN

Distance of centroid from the apex $= \frac{2}{3} \times 20 = 13.33$ mm

Width of the section at the level where shear stress is desired is given by

$$\frac{40}{20} = \frac{b}{13.33}; \quad b = 26.67 \text{ mm}$$

Shear stress, $\tau = \dfrac{F.A\bar{y}}{I.b}$

Moment of the area above the section under consideration about NA,

$$A\bar{y} = \frac{1}{2} \times 26.67 \times 13.33 \times \frac{1}{3} \times 13.33 = 789.8 \text{ mm}^3$$

Moment of Inertia of the section about NA, $I = \dfrac{40 \times 20^3}{36} = 8888.9 \text{ mm}^3$

$$\tau_{NA} = \frac{3 \times 10^3 \times 789.8}{8888.9 \times 26.67} = 10 \text{ N/mm}^2$$

(or)

$$\tau_{NA} = \frac{4}{3}.\tau_{avg} = \frac{4}{3}.\frac{F}{A} = \frac{4}{3}.\frac{3 \times 10^3}{\frac{1}{2} \times 40 \times 20} = 10 \text{ N/ mm}^2$$

05. A symmetric I-section (with width of each flange =50 mm, thickness of each flange =10 mm, depth of web =100 mm, and thickness of web =10 mm) of steel is subjected to a shear force of 100 kN. Find the magnitude of the shear stress (in N/mm^2) in the web at its junction with the top flange……. CE 2013

05. **71.12**

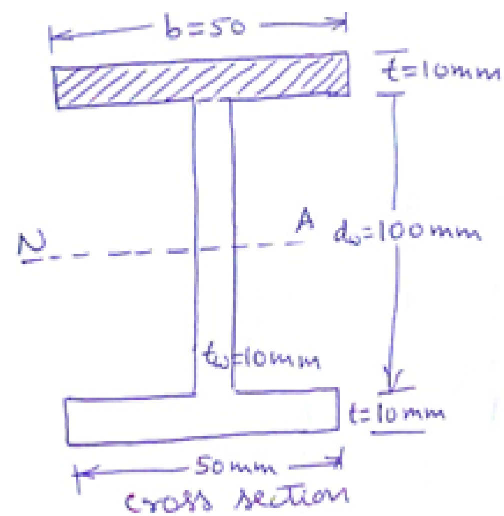
Width of flange,b= 50 mm

Thickness of flange, t_f = 10 m

Depth of web, d_w =100 mm

Thickness of web, $t_w = 10\,\text{mm}$

Shear force, $F = 100\,\text{kN}$



Shear stress at any distance from neutral axis, $\tau = \dfrac{F.(A\bar{y})}{I.b}$

Moment of the area above the point considered about neutral axis,

$$(A\bar{y}) = 50\times10\times55 = 275\times10^2\ \text{mm}^3$$

Moment of inertia of the section about neutral axis,

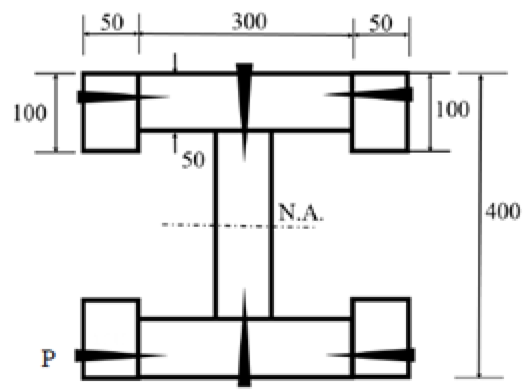
$$I = \frac{10\times100^3}{12} + 2\left[\frac{50\times10^3}{12} + 50\times10\times55^2\right]$$

$$=833.3\times10^3+3033.3\times10^3= 3866.6\times10^3\ \text{mm}^4$$

Width of section under consideration, b=10 mm

$$\tau = \frac{100\times10^3\times275\times10^2}{3866.6\times10^3\times10} = 71.12\ \text{N/mm}^2$$

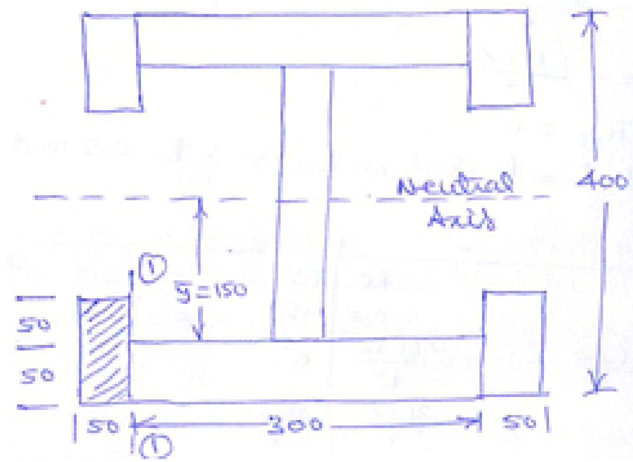
31. The cross-section of a built-up wooden beam as shown in the figure (not drawn to scale) is subjected to a vertical shear force of 8 kN. The beam is symmetrical about the neutral axis (N.A) shown, and the moment of inertia about N.A. is 1.5×10^9 mm^4. Considering that the nails at the location P are spaced longitudinally (along the length of the beam) at 60 mm, each of the nails at P will be subjected to the shear force of CE1 2019



All dimensions are in *mm*

a. 60 N b. 120 N c. 240 N d. 480 N

31. C



Shear flow at section 1-1, $q = \tau . b = \dfrac{F . A\bar{y}}{I}$

Shear force, F= 8 kN

Moment of inertia about NA, I= 1.5×10^9 mm^4

Spacing of nails along the length, l = 60 mm

Moment of the area beyond the section under consideration about the neutral axis,

$$A\bar{y} = 50\times100\times150$$

$$q = \frac{8\times10^3\times50\times100\times150}{1.5\times10^9} = 4 \text{ N/mm}$$

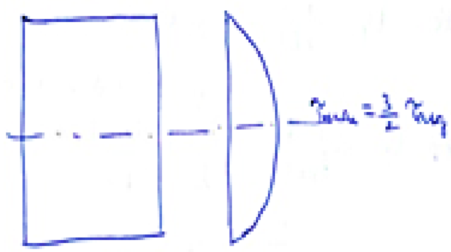
Shear force resisted by each nail $= q\times l$ = 4×60 = 240 N

GATE
MECHANICAL
ENGINEERING

01. The ratio of average shear stress to the maximum shear stress in a beam with a square cross-section is : GATE ME 1998

a. 1 b. $\frac{2}{3}$ c. $\frac{3}{2}$ d. 2

01. b



Maximum shear stress, $\tau_{\max} = \frac{3}{2} \times \tau_{avg}$

$$\frac{\tau_{avg}}{\tau_{\max}} = \frac{2}{3}$$

02. The transverse shear stress acting in a beam of rectangular cross-section, subjected to a transverse shear load, is

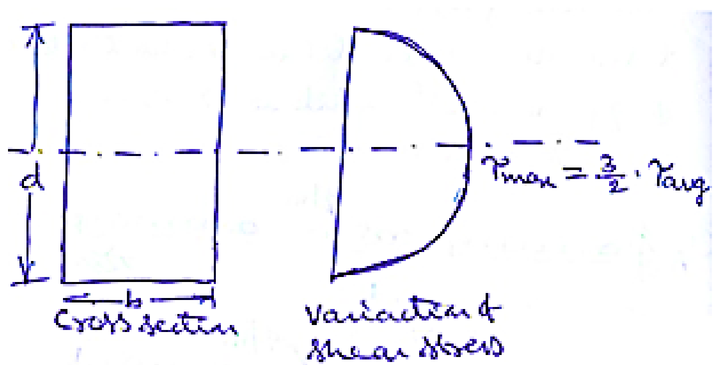
a. Variable with maximum at the bottom of the beam

b. Variable with maximum at the top of the beam

c. uniform

d. Variable with maximum of the neutral axis GATE ME 2008

02. d



The shear stress across the rectangular cross section varies parabolically with zero stress at top and bottom fibre and maximum at the neutral axis.