BUCKLING OF COLUMNS

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- Column, pillar or stanchion is a vertical member which carries an axial compressive load.
- **Slenderness ratio (λ)**: It is the ratio of unsupported length of the column to the minimum radius of gyration of the column.

$$\lambda = \frac{l}{r}$$

l: unsupported length of the column

r_{min} : least radius of gyration

$$\frac{P_e}{A} = \frac{\pi^2}{l^2} E \frac{I_{min}}{A}$$

$$\sigma_e = \frac{\pi^2}{l^2} E (r)^2$$

$$\sigma_e = \frac{\pi^2 E}{(l/r)^2} = \frac{\pi^2 E}{\lambda^2}$$

- **Radius of gyration:** It is property of cross section which influences the structural behaviour of the members.

The radius of gyration is the distance of the area squeezed into a strip frame parallel to the axis which produces the same moment of inertia as that of the plane area about that axis.

Radius of gyration, $k = \sqrt{\frac{I}{A}}$

Members tend to buckle when subjected to axial forces.

The load at which members will buckle is proportional to the square of the radius of gyration.

For circular cross section:

Cross sectional area, $A = \dfrac{\pi d^2}{4}$

Moment of inertia, $I = \dfrac{\pi d^4}{64}$

Radius of gyration, $k = \dfrac{I}{A} = \sqrt{\dfrac{\dfrac{\pi d^4}{64}}{\dfrac{\pi d^2}{4}}} = \sqrt{\dfrac{d^2}{16}} = \dfrac{d}{4}$

For rectangular cross section:

Cross sectional area, $A = bd$

Moment of inertia, $I = \dfrac{bd^3}{12}$

Radius of gyration, $k = \dfrac{I}{A} = \sqrt{\dfrac{\frac{bd^3}{12}}{bd}} = \sqrt{\dfrac{d^2}{12}} = \dfrac{d}{2\sqrt{3}}$

Classification of columns

The columns are classified based on the slenderness ratio or length to diameter ratio.

a. Short columns:

The columns are said to be short when

i. the length is less than 8 times its diameter. $l < 8d$ or

ii. the slenderness ratio is less then 32. $\left(\frac{l}{r_{\min}}\right) < 32$

Short columns are subjected to compressive load and the effect of buckling is neglected. Short columns are always subjected to direct compressive stress only. Short columns fail by pure compression.

b. Medium size columns: The columns are side to be medium size when

i. the length varies from 8 to 30 times its diameter. $8d \leq l < 30d$

ii. the slenderness ratio lying between 32 and 120. $32 \leq \left(\frac{l}{r_{\min}}\right) < 120$

These columns are subjected to both direct and buckling stresses both direct and buckling stresses are taken into account in the design of intermediate columns. Medium columns fail by combination of compression and buckling depending upon a slenderness ratio.

c. Long columns: The columns are said to be long when

i. the length is greater than 30 times its diameter. $l \geq 30d$

ii. the slenderness ratio is more than 120. $\left(\frac{l}{r_{\min}}\right) \geq 120$

Long columns are subjected to buckling stress only.

Direct compressive stress is very small as compared with buckling stress and hence it is neglected.

Long columns fail by buckling.

In long column, buckling takes place about the axis having minimum radius of gyration.

Buckling factor:

- It is the ratio between the equivalent length of the column to the minimum radius of gyration.
- For a given area, tubular section will have maximum radius of gyration. H section is more efficient than I section
- As the slenderness ratio increases, critical stress reduces and hence load carrying capacity of column reduces.

Buckling load (or crippling load):

- It is the maximum load at which the column tends to have lateral displacement or tends to buckle. The buckling takes place about the axis having minimum radius of gyration or least moment of inertia.

Safe load:

- It is the load to which a column is actually subjected. Safe load is obtained by dividing the buckling load by suitable factor of safety.

$$\text{Safe load} = \frac{\text{Buckling load}}{\text{Factor of safety}}$$

Strength of columns:

- The strength of a column depends upon the slenderness ratio and end conditions. If the slenderness ratio is increased, the compressive strength of column decreases as the tendency to buckle increases.

Effective length (l_e) or Equivalent length or Simple column length:

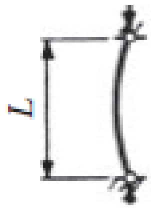
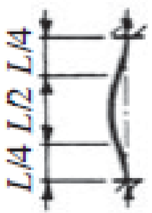
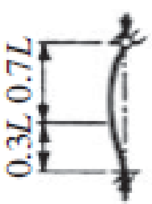
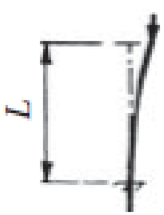
- Effective length is the distance between adjacent points of inflexion.
- A point of inflexion is found at
  - i. every column end that is free to rotate and
  - ii. Every point where there is a change of the axis.

a. Both ends hinged. $l_e = l$

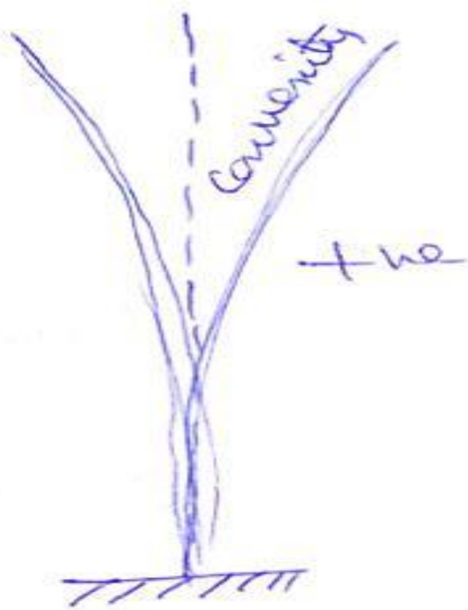
b. One end fixed and other end free. $l_e = 2l$

c. One end fixed and other pin jointed. $l_e = \frac{l}{\sqrt{2}}$

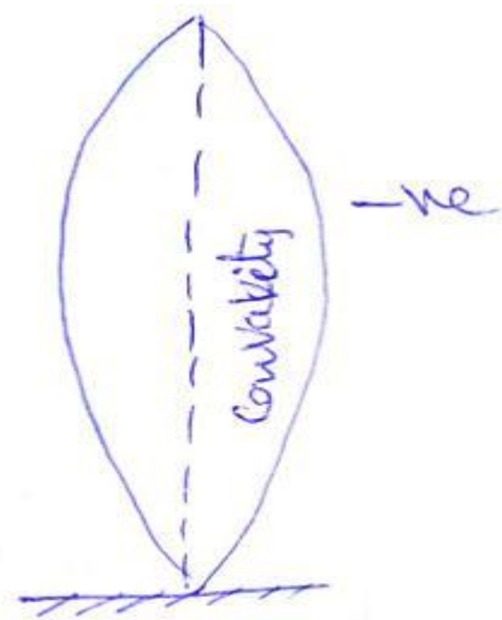
d. Both ends fixed. $l_e = \frac{l}{2}$

Type of column	Effective length	Critical buckling load
	L	$\frac{\pi^2 EI}{L^2}$
	$\frac{L}{2}$	$\frac{4\pi^2 EI}{L^2}$
	$\approx 0.7L$	$\approx \frac{2\pi^2 EI}{L^2}$
	$2L$	$\frac{\pi^2 EI}{4L^2}$

Sign conventions



Convexity towards the initial centre line of the member



Concavity towards initial centre line the member.

Euler's theory for long columns

Assumptions:

- The column is initially straight
- The columns have uniform cross sectional area throughout
- The material of the column is perfectly homogeneous and isotropic
- The columns carry perfectly, axial loads and it passes through the centroid of the column section.
- Self weight of the column is neglected
- The columns fail only due to buckling
- The pin joints are frictionless and fixed ends are perfectly rigid
- Shortening of columns due to direct compression is neglected
- The stresses do not exceed the limit of proportionality.

Euler's buckling load, $P_E = \frac{\pi^2 EI}{l_e^2}$

P_E or P_C : Buckling or crippling load

E : Modulus of Elasticity

I : Least moment of Inertia of the cross section

l_e : Equivalent length of column.

Euler's formula is applicable for long columns.

S.No	End condition	Equivalent length, l_e	Buckling load, P_E
1.	Both ends are hinged or pinned	$l_e = l$	$\frac{\pi^2 EI}{l^2}$
2.	One end fixed, other end free	$l_e = 2l$	$\frac{\pi^2 EI}{4l^2}$
3.	One end fixed, other end hinged	$l_e = \frac{l}{\sqrt{2}}$	$\frac{2\pi^2 EI}{l^2}$
4.	Both ends fixed	$l_e = \frac{l}{2}$	$\frac{4\pi^2 EI}{l^2}$

Strength of a column depends on slenderness ratio and end condition.

Limitations of Euler's formula:

Euler's crippling load, $P = \dfrac{\pi^2 EI}{l^2}$

$$I = A.k^2$$

$$P = \frac{\pi^2 EA}{\left(\dfrac{l}{k}\right)^2}$$

$$\sigma_c = \frac{P}{A} = \frac{\pi^2 E}{\left(\dfrac{l}{k}\right)^2}$$

The value of σ_c depends only on E and the slenderness ratio $\left(\dfrac{l}{k}\right)$ depends on no other factor.

For mild steel $\sigma_c \leq 320$ N/mm^2, $E = 2.1\times10^5$ N/mm^2

$$\frac{\pi^2 E}{\left(\frac{l}{k}\right)^2} \leq 320$$

$$\left(\frac{l}{k}\right) \geq \sqrt{\frac{\pi^2 \times 2.1\times10^5}{320}} \geq 80.5$$

For the Euler formula to be valid, the slenderness ratio for mild steel column should not be less than 80.5.

Rankine's formula:

Rankine proposed the following empirical relationship to cover all cases from a short to a very long column.

$$\frac{1}{P_R} = \frac{1}{P} + \frac{1}{P_E}$$

P_R : Rankine's crippling load for the column

P : Ultimate load for a short column $= \sigma_c . A$

P_E : Euler's crippling load for a long column $= \dfrac{\pi^2 EI}{l_e^2}$

If the column is very short, P_E becomes very large and $\frac{1}{P_E} = 0$ so that $P_R = P$.

If the column is very long, P_E becomes very small and $\frac{1}{P_E}$ becomes very large so that $P_R = P_E$.

Rankine's crippling load, $P_R = \frac{\sigma_c . A}{1 + \alpha . \left(\frac{l_e}{k} \right)^2}$

Rankine's constant, $\alpha = \frac{\sigma_c}{\pi^2 E}$

σ_c and α for different materials

Material	σ_c, N/mm^2	$\alpha = \dfrac{\sigma_c}{\pi^2 E}$
Mild steel	320	$\dfrac{1}{7500}$
Cast Iron	550	$\dfrac{1}{1600}$
Wrought Iron	250	$\dfrac{1}{9000}$

$$P_R = \frac{\text{Crushing load}}{1 + \alpha . \left(\frac{l_e}{k} \right)^2}$$

The factor $1 + \alpha . \left(\frac{l_e}{k} \right)^2$ is taken into account the buckling effect.

Rankine's formula takes into account the effect of direct compressive stress.

Straight line formula:

The approximate liner expression is of the form

$$\frac{P}{A} = \sigma_c - z\left(\frac{l}{k}\right)$$

where z = an empirical constant

Johnson's parabolic formula:

A simple expansion of the Rankine's formula will give a parabolic expression

$$\frac{P}{A} = \sigma_c \left[1 - \alpha . \left(\frac{l_e}{k} \right)^2 \right]$$

$$\frac{P}{A} = \sigma_c - b \left(\frac{l}{k} \right)^2$$

where $b = \dfrac{\sigma_c^2}{4\pi E^2}$

Johnson accepted true value of $b = \dfrac{\sigma_c^2}{64E}$ for pinned ends

Column subjected to eccentric loading:

a. Rankine's method:

Maximum compressive stress,

$$\sigma_{\max} = \frac{P}{A} + \frac{P.e}{I}.y = \frac{P}{A} + \frac{P.e}{Ak^2}.y = \frac{P}{A}\left(1 + \frac{ey}{k^2}\right)$$

$$P = \frac{\sigma_{\max}.A}{\left(1 + \frac{ey}{k^2}\right)}$$

When the effect of buckling is considered,

$$P = \frac{\sigma_{\max} .A}{\left(1+\frac{ey}{k^2}\right)\left(1+\alpha\left(\frac{l_e}{k}\right)^2\right)}$$

P : Eccentric load

e : Eccentricity fram the geometric axis

A : Cross sectional area of the member

σ_c : Safe stress in the column

l_e : Effective length of the column

y : Distance of extreme fibre from the geometric axis

k : Radius of gyration of the cross section

b. Euler's method : (Secant formula)

Maximum stress, $\sigma_{max} = \sigma_d + \sigma_b$

$$\sigma_{max} = \frac{P}{A} + \frac{P.e \sec\left(\frac{l_e}{2} . \sqrt{\frac{P}{EI}}\right)}{Z}$$

Maximum BM, $M = P.e \sec\left(\frac{l_e}{2}\sqrt{\frac{P}{EI}}\right)$

Secant formula is valid for long columns under eccentric loading.

Prof. Perry's formula:

It is used to determine the safe load that can be applied on a column at a given eccentricity.

Stress due to direct load, $\sigma_d = \frac{P}{A}$

$\sigma_{\max}$: Maximum permissible stress

l_e : Effective length of the column

$$\sigma_{\text{Euler}} = \frac{P_{\text{Euler}}}{A}$$

y_c : Distance of the extreme layer in compression from the neutral axis

a. Stress at critical load for cast Iron :

$$\frac{P}{A} = 23.8 - 0.6\left(\frac{l}{k}\right)\text{N/mm}^2$$

b. Stress at critical load for structural steel

$$\frac{P}{A} = 367.5 - 2\left(\frac{l}{k}\right)\text{N/mm}^2$$

c. Safe working stress for mild steel:

$$\frac{P}{A} = 150\left(1 - 0.0038\frac{l}{k}\right)\text{N/mm}^2$$

GATE CE

01. The axial load carrying capacity of a long column of given material, cross-sectional area A and length L is governed by CE 1992

a. strength of its material only

b. its flexural rigidity only

c. its slenderness ratio only

d. both flexural rigidity and slenderness ratio

01. d

Eulers buckling load, $P_E = \frac{\pi^2 EI}{L^2}$

$$P_E = \frac{\pi^2 E.Ak^2}{L^2} = \frac{\pi^2 EA}{\lambda^2}$$

λ : Slenderness ratio $= \frac{L}{k}$

k : Radius of gyration

For a given material, E is constant.

For a given cross sectional area, A is constant and for a given length, L is constant.

P_E depends on least radius of gyration, which in turn depends on slenderness ratio / flexural rigidity.

02. When a column is fixed at both ends, corresponding Euler's critical load is

a. $\frac{\pi^2 EI}{L^2}$ b. $\frac{2\pi^2 EI}{L^2}$ c. $\frac{3\pi^2 EI}{L^2}$ **d.** $\frac{4\pi^2 EI}{L^2}$

where L is the length of the column. CE 1994

02. D

Eulers buckling load for a column hinged at both ends, $P_E = \frac{\pi^2 EI}{L^2}$

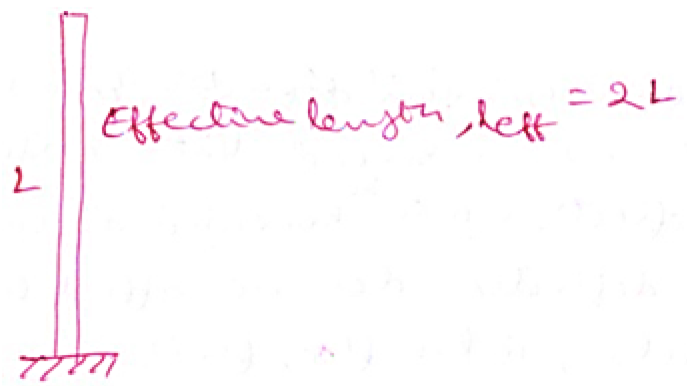
If both ends of the column are fixed, $L_{eff} = \frac{L}{2}$

Eulers buckling load for a column fixed at both ends, $P_E = \frac{4\pi^2 EI}{L^2}$

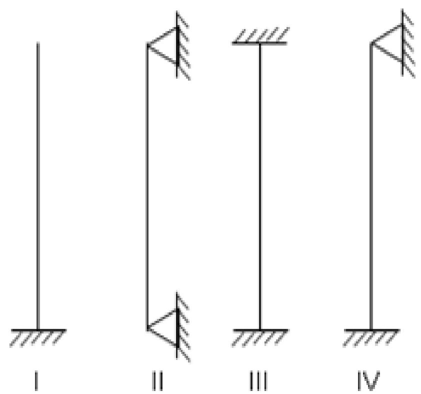
03. The effective length of a circular electric pole of length L and constant diameters erected on ground is CE 1996

a. 0.80 L b. 1.20 L c. 1.50 L **d.** 2.00 L

03. d

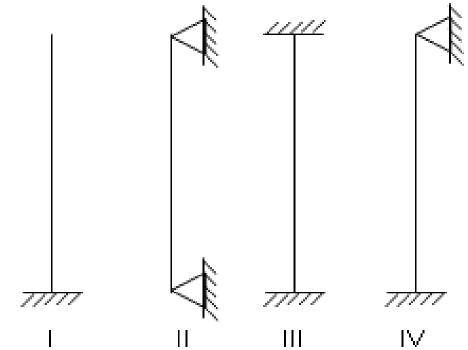


04. Four column of the same material and having identical geometric properties are supported in different ways as shown below : CE 2000



It is required to order these four beams in the increasing order of their respective first buckling loads. The correct order is given by

a. I, II, III, IV b. III IV, I, II c. II, I, IV, III **d.** I, II, IV, III



04. D

	End condition	Buckling load
I	One end fixed and the other free	$\frac{\pi^2 EI}{4L^2}$
II	Both ends hinged	$\frac{\pi^2 EI}{L^2}$
III	Both ends fixed	$\frac{4\pi^2 EI}{L^2}$
IV	One end fixed and the other hinged	$\frac{2\pi^2 EI}{L^2}$

Increasing order of buckling loads: I, II, IV and III

05. A long structural column (length = L) with both ends hinged is acted upon by an axial compressive load, P. The differential equation governing the bending of column is given by

$$EI\frac{d^2y}{dx^2} = -Py$$

where y is the structural lateral deflection and EI is the flexural rigidity. The first critical load on column responsible for its buckling is given by CE 2003

a. $\frac{\pi^2 EI}{L^2}$ b. $\frac{\sqrt{2}\pi^2 EI}{L^2}$ c. $\frac{2\pi^2 EI}{L^2}$ d. $\frac{4\pi^2 EI}{L^2}$

05. a.

Length of column = L

End condition: Both ends hinged

Compressive load = P

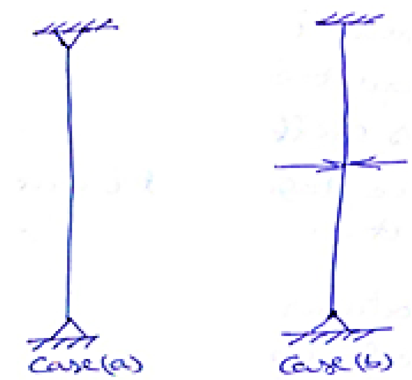
Flexural rigidity = EI

Critical load on column, $P_c = \frac{\pi^2 EI}{L^2}$

07. A steel column, pinned at both ends, has a buckling load of 200 kN. If the column is restrained against lateral movement at its mid height, its buckling load will be

a. 200 kN b. 283 kN c. 400 kN d. 800 kN CE 2007

07. D



Buckling load when the column ends hinged, $P_E = 200$ kN

For case a, When the ends of the column are hinged $l_e = L$

$$P_E = \frac{\pi^2 EI}{L^2} \quad \Rightarrow 200 = \frac{\pi^2 EI}{L^2}$$

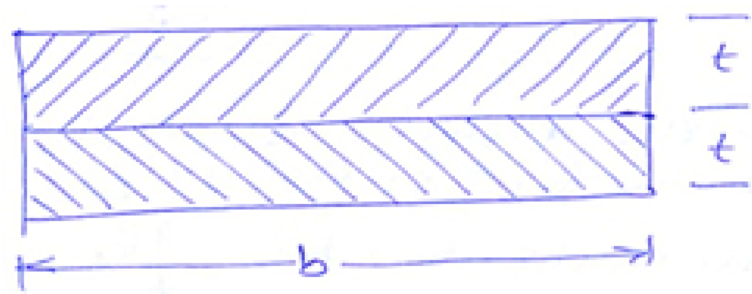
For case b, When the lateral movement at mid height is restrained, $l_e = \frac{L}{2}$

$$P_E = \frac{\pi^2 EI}{\left(\frac{L}{2}\right)^2} = \frac{4\pi^2 EI}{L^2} = 4 \times 200 = 800 \text{ kN}$$

08. Cross-section of a column consisting of two steel strips, each of thickness t and width b is shown in the figure below. The critical loads of the column with perfect bond and without bond between the strips are P and P_0 respectively. The ratio P/P_0 is CE 2008

a. 2 **b.** 4 c. 6 d. 8

08. b



Crippling load, $P_e = \dfrac{\pi^2 EI}{L^2}$

When the bond between the strips is perfect,

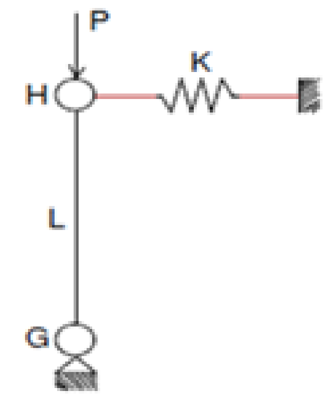
$$P = \frac{\pi^2 E}{L^2}.\frac{1}{12}b.(2t)^3 = \frac{\pi^2 E}{12L^2} \times 8bt^3$$

When the bond between the strips is not perfect,

$$P_0 = \frac{\pi^2 E}{L^2}2.\frac{1}{12}b.t^3 = \frac{\pi^2 E}{12L^2} \times 2bt^3$$

$$\frac{P}{P_o} = 4$$

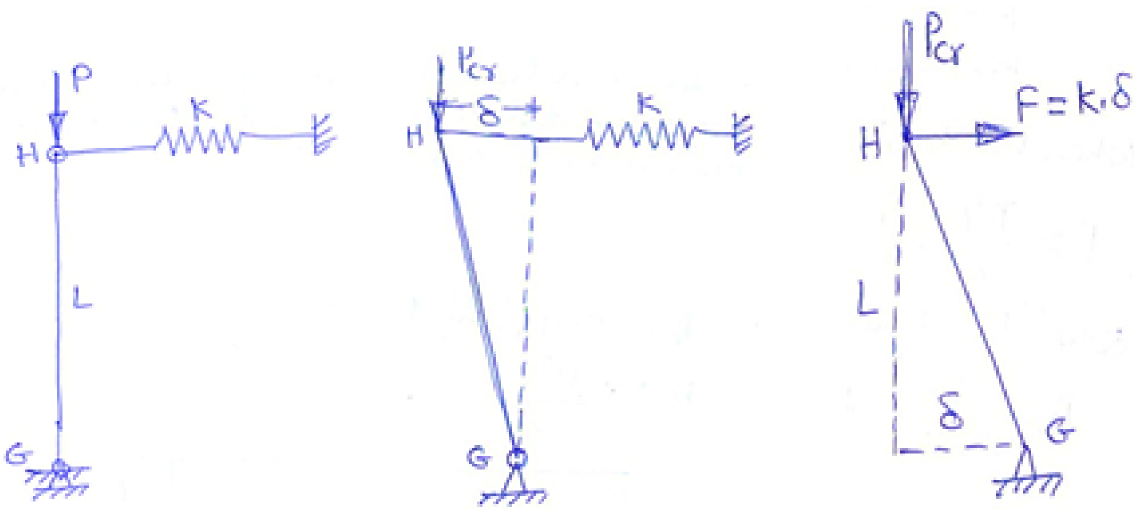
09. A rigid bar GH of length L is supported by a hinge and a spring of stiffness k as shown in the figure below. The buckling load, P_{cr}, for the bar will be CE 2008



a. 0.5 kL b. 0.8kL **c.** 1.0 kL

d. 1.2kL

09. c



Let δ be the deflection of the spring and F be the force in the spring.

Taking moments about the hinge G,

$$P_{cr}.\delta = F.L \Rightarrow P_{cr} = \frac{K\delta.L}{\delta} \Rightarrow P_{cr} = K.L$$

10. Consider the following statements for a compression member CE 2009

I. The elastic critical stress in compression increases with decrease in slenderest ratio

II. The effective length depends on the boundary conditions at its ends

III. The elastic critical stress in compression is independent of the slenderness ratio

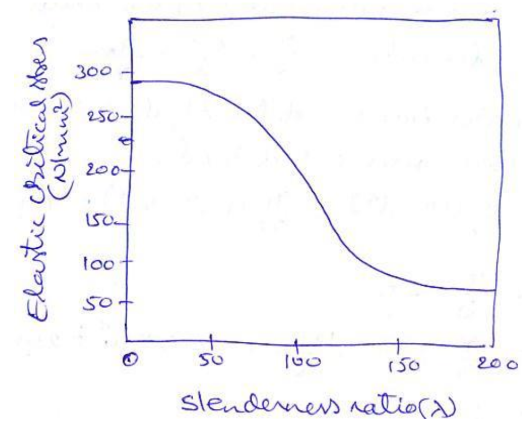
IV. The ratio of the effective length to its radius of gyration is called as slenderness ratio

The true statements are

a. II and III b. III and IV c. II, III and IV **d.** I, II and IV

10. D

Effective length of a compression member depends on the boundary condition at its ends.



Effective length, $l_e = L$ Both ends hinges

$= 2L$ One end fixed and other free

$= \frac{L}{2}$ Both ends fixed

$= \frac{L}{\sqrt{2}}$ One end fixed and other hinged.

Slenderness ratio is the ratio of the effective length to its radius of gyration.

$$\lambda = \frac{l_e}{r}$$

The elastic critical stress in compression increases with decrease in slenderness ratio.

11. The effective length of a column of length L fixed against rotation and transition at one end and free at the other end is CE 2010

a. 0.5 L b. 0.7 L c. 1.414 L d. 2L

11. D

End condition: Fixed at one end and free at the other end

Effective length, $l_e = 2L$

12. The ratio of the theoretical critical bucking load for a column with fixed ends to that of another column with the same dimensions and material, but with pinned ends, is equal to CE 2012

a. 0.5 b. 1.0 c. 2.0 d. 4.0

12. d

Column with Fixed ends, $P_{cr} = \dfrac{4\pi^2 EI}{l^2}$

Column with pinned ends, $P_{cr} = \dfrac{\pi^2 EI}{l^2}$

$$\frac{P_{cr}\,\text{fixed}}{P_{cr}\,\text{hinged}} = 4$$

13. Two steel columns P (length L and yield strength $f_y = 250$ MPa) and Q (length $2L$ and yield strength f_y =500 MPa) have the same cross-sections and end-conditions. The ratio of buckling load of column P to that of column Q is

a. 0.5 b. 1.0 c. 2.0 **d.** 4.0 CE 2013

13. D

Column P: Length = L, Yield strength, $f_y = 250\,\text{MPa}$

Column Q: Length = 2L, Yield strength, $f_y = 500\,\text{MPa}$

Columns P and Q have same cross sectional area and end conditions.

Buckling load, $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

$$\frac{(P_{cr})_{\text{column P}}}{(P_{cr})_{\text{column Q}}} = \frac{\left(\dfrac{\pi^2 EI}{L^2}\right)_P}{\left(\dfrac{\pi^2 EI}{L^2}\right)_Q} = \frac{(2L)^2}{L^2} = 4$$

08. Polar moment of inertia (I_p) in cm^4, of a rectangular section having width, $b = 2$ cm and depth, d= 6 cm is CE2 2014

08. 40

Width of the beam, b= 2 cm

Depth of the beam, d=6m

Polar moment of Inertia, $I_p = I_x + I_y$

$$I_p = \frac{bd^3}{12} + \frac{db^3}{12} = \frac{2\times 6^3}{12} + \frac{6\times 2^3}{12} = 36+4=40 \text{ cm}^4$$

42. Consider two axially loaded columns, namely 1 and 2, made of a linear elastic material with Young's modulus 2×10^5 MPa, square cross-section with side 10 mm and length 1 m. For column 1, one end is fixed and the other end is free. For column 2, one end is fixed and the other end is pinned. Based on the Euler's theory, the ratio (up to one decimal place) of the buckling load of column 2 to the buckling load of column is ………… CE1 2017

42. 8

Modulus of elasticity, $E = 2\times10^5$ MPa

Side of square cross section, $a = 10$ mm

Length of column, $l = 1$ m

Column 1: One end fixed and the other end is free

Column 2: One end fixed and the other end is pinned

$$\frac{\text{Buckling load of column 2} \left(P_{cr}\right)_2}{\text{Buckling load of column 1} \left(P_{cr}\right)_1} = \frac{\dfrac{2\pi^2 EI}{L^2}}{\dfrac{\pi^2 EI}{4L^2}} = 8$$

04. A column of height h with a rectangular cross-section of size $a \times 2a$ has a buckling load of P. If the cross-section is changed to $0.5a \times 3a$ and its height changed to $1.5h$, the buckling load of the redesigned column will be CE1 2018

a. $\frac{P}{12}$ b. $\frac{P}{4}$ c. $\frac{P}{2}$ d. $\frac{3P}{4}$

04. a

For column 1: $L = h$, $A = a \times 2a$, $P_1 = P$

$$P_1 = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E \frac{2a \times a^2}{12}}{h^2} = \frac{\pi^2 E a^4}{6h^2} = \text{P}$$

For column 2: L=1.5h, $A = 0.5a \times 3a$

$$p_2 = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E 3a(0.5a)^3}{12(1.5h)^2} = \frac{1}{12} \cdot \frac{\pi^2 E a^4}{6h^2} = \frac{P}{12}$$

GATE
MECHANICAL

01. If the length of a Column is doubled, the critical load becomes GATE ME 1988

a. $\frac{1}{2}$ of the original value

b. $\frac{1}{4}$ of the original value

c. $\frac{1}{8}$ of the original value

d. $\frac{1}{16}$ of the original value

01. b

Euler's critical load, $P = \frac{\pi^2 EI}{L^2}$

$L_1 = 2L$

Euler's critical load, $P_1 = \frac{\pi^2 EI}{(2L)^2}$

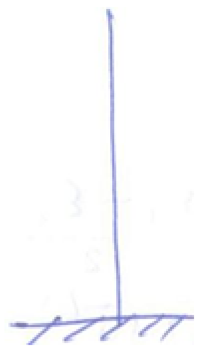
$$P_1 = \frac{1}{4} \cdot \frac{\pi^2 EI}{L^2} = \frac{P}{4}$$

02. For the case of a slender column of length L and flexural rigidity EI built-in at its base and free at the top , the Euler's critical buckling load is GATE ME 1994

a. $\frac{4\pi^2 EI}{l^2}$ b. $\frac{2\pi^2 EI}{l^2}$ c. $\frac{\pi^2 EI}{l^2}$ d. $\frac{\pi^2 EI}{4l^2}$

02. d.

Euler's crippling load



One end fixed and the other end free.

$$P_{cr} = P_E = \frac{\pi^2 EI}{4L^2}$$

03. The buckling load for a column pinned at both ends is 10KN .If the ends are fixed, the buckling load changes to GATE ME 1998

a. 40 kN b. 2.5 kN c. 5 kN d. 20 kN

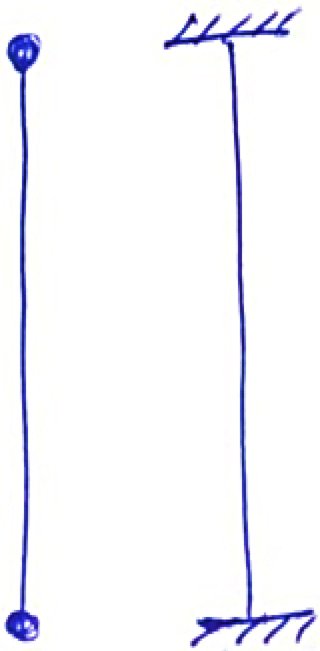
03. A.

End condition: pinned at both ends

$$P_1 = \frac{\pi^2 EI}{l^2} = 10 \text{ kN}$$

End condition: Both ends fixed

$$P_2 = \frac{4\pi^2 EI}{l^2} = 4 \times 10 = 40 \text{ kN}$$



04. The ratio of Euler's buckling loads of column with the same parameters having (i) Both ends fixed, and (ii) both ends hinged is GATE ME 2002

a. 2 b. 4 c. 6 d. 8

04. B

P_1 : Eulers buckling load of column both ends fixed $= \dfrac{4\pi^2 EI}{l^2}$

P_2 : Eulers buckling load of column both ends hinged $= \dfrac{\pi^2 EI}{l^2}$

$$\frac{P_1}{P_2} = 4$$

05. A pin-ended column of length L , modulus of elasticity E and second moment of the cross –sectional area I is loaded centrically by a compressive load P . The critical buckling load $((P_{cr}))$ is given by GATE ME 2006

a. $P_{cr} = \dfrac{EI}{\pi^2 L^2}$ b. $P_{cr} = \dfrac{\pi^2 EI}{3L^2}$ c. $P_{cr} = \dfrac{\pi EI}{L^2}$ d. $P_{cr} = \dfrac{\pi^2 EI}{L^2}$

05. D

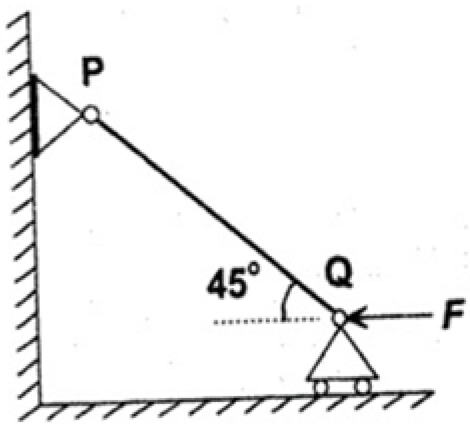
P_{cr} : Euler's critical load

$$= \frac{\pi^2 EI}{L^2}$$

06. The rod PQ of length L and with flexural rigidity EI is hinged at both ends. For what minimum force F is it expected to buckle? GATE ME 2008

a. $\left(\frac{(\pi^2 EI)}{L^2}\right)$ b. $\left(\frac{(\sqrt{2}\pi^2 EI)}{L^2}\right)$ c. $\left(\frac{(\pi^2 EI)}{\sqrt{2}L^2}\right)$ d. $\left(\frac{(\pi^2 EI)}{2L^2}\right)$

06. B



Euler's buckling load for both ends hinged, $P = \frac{\pi^2 EI}{L^2}$

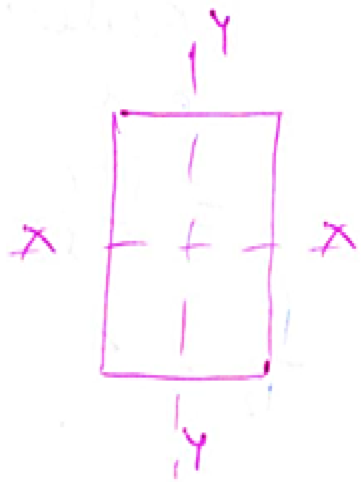
$$F\cos 45^0 = \frac{\pi^2 EI}{L^2}$$

$$\frac{F}{\sqrt{2}} = \frac{\pi^2 EI}{L^2} \Rightarrow F = \frac{\sqrt{2}.\pi^2 EI}{L^2}$$

07. A column has a rectangular cross-section of 10mm $\times$ 20mm and a length of 1m. The slenderness ratio of the column is close to GATE ME 2010

a. 200 b. 346 c. 477 d. 1000

07. B



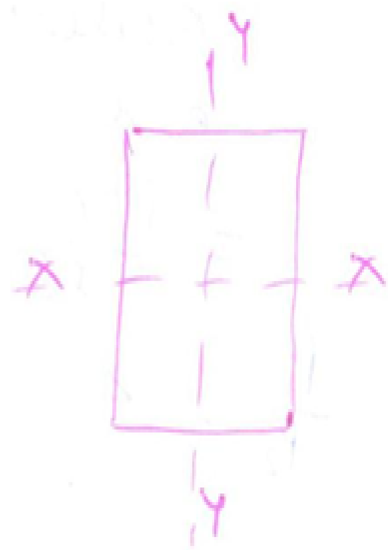
Size of column : $10 \text{ mm} \times 20 \text{ mm}$

Cross sectional area, $A = 200 \text{ mm}^2$

Movement of Inertia about XX axis, $I_x = \dfrac{10 \times 20^3}{12} = 6666.7 \text{ mm}^4$

Moment of inertia about YY axis $I_y = \dfrac{20 \times 10^3}{12} = 1666.7 \text{ mm}^4$

Radius of gyration about x axis, $r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{6666.7}{200}} = 5.77$ mm



Radius of gyration about Y axis, $r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{1666.7}{200}}$ =2.89 mm

r_{min}:least of r_x and r_y = 2.89 mm

l: length of the column =1 m

Slenderness ratio, $\lambda = \frac{l}{r_{min}} = \frac{1000}{2.89} = 346$

08. For a long slender column of uniform cross section, the ratio of critical buckling load for the case with both ends clamped to the case with both ends hinged is

a. 1 b. 2 c. 4 d. 8 GATE ME 2012

08. C

Column 1: Both ends clamped

P_{E1} : Eulers buckling load $= \dfrac{4\pi^2 EI}{L^2}$

Column 2: Both ends hinged

P_{E2} : Eulers buckling load $= \dfrac{\pi^2 EI}{L^2}$

$$\frac{P_{E1}}{P_{E2}} = 4$$

09. Consider a steel (Young's modulus E = 200 GPa) column hinged on both sides. Its height is 1.0 m and cross-section is 10 mm × 20 mm. The lowest Euler critical buckling load (in N) is.... GATE ME 2015

Ans: 3285 to 3295

09. 3290

Young modulus, E= 200 GPa

End condition: column hinged on both ends

Height of column, L = 1.0 m

Cross section: 10 mm × 20 mm

Euler critical load, $P_E = ?$

I : Least moment of inertia (least of I_X and I_Y) $= \dfrac{20\times10^3}{12}$ 1666.7 mm^4

$$P_E = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200\times10^3 \times 1666.7}{(1\times10^3)^2} = 3290\,\text{N}$$

10. The minimum axial compressive load, P, required to initiate buckling for a inned-pinned slender column with bending stiffness EI and length L is

GATE ME2 2018

a. $P = \dfrac{\pi^2 EI}{4L^2}$ b. $P = \dfrac{\pi^2 EI}{L^2}$ c. $P = \dfrac{3\pi^2 EI}{4L^2}$ d. $P = \dfrac{4\pi^2 EI}{L^2}$

10. B

Euler's load for a long column with pinned ends, $P_E = \dfrac{\pi^2 EI}{L^2}$