GATE – CIVIL ENGINEERING

GEOTECHNICAL ENGINEERING

Prof.B.Jayarami Reddy
Professor and Head
Department of Civil Engineering
Y.S.R. Engineering College of
Yogi Vemana University,
Proddatur, Y.S.R.(Dt.), A.P-516360.
E.mail : bjrcivilgate@gmail.com

8.11 STRESS DISTRIBUTION IN SOILS

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STRESS DISTRIBUTION IN SOILS

- Stresses imposed on the soil by its over burden (self weight) and structural loads applied at or below the surface
- Stresses due to over burden increases with depth
- Stresses due to structural loads decreases with depth and distance from the point of application of load
- The stress distribution depends on physical properties of soil, rigidity of foundation, shape of foundation and type of loading.

Boussines'q theory:

- Load applied at the surface of infinite soil mass

Assumptions: The soil is elastic, homogeneous, isotropic, semi-infinite medium extending infinitely in all directions.

a. The soil medium obeys Hook's law
b. The self weight of the soil is ignored.
c. The soil is initially unstressed

d. Change in volume of soil due to loads is neglected

e. Top surface of soil is free from shear stresses and is subjected to only point load.

f. The stresses are distributed symmetrically with respect to z-axis

g. Continuity of stress is considered to exist in the soil medium

Vertical stress $\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$

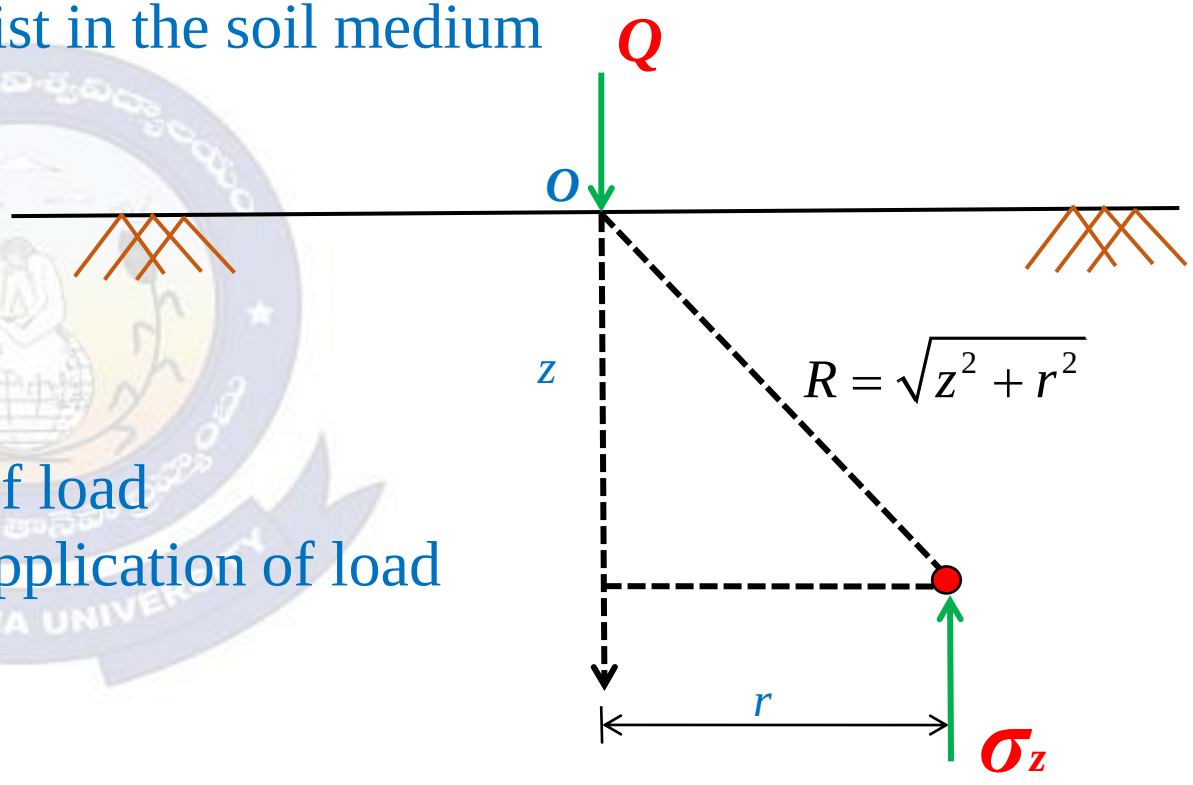
$$\sigma_z = K_B.\frac{Q}{z^2} \qquad \sigma_z \propto \frac{1}{z^2}$$

z : Vertical distance from the application of load

r : Horizontal distance from the point of application of load

K_B : Boussinesq's influence factor

$$K_B = \frac{3}{2\pi}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$



Vertical pressure within an earth mass

- Stress decreases as depth increases
- Stress decreases as moving away from the load
- Boussinesq's theory is best suited to shallow foundations
- σ_z does not depends on the projection of soil. Hence it can be applied for all types of soil except stratified or sedimentary soil.

Vertical stress distribution along a horizontal plane:

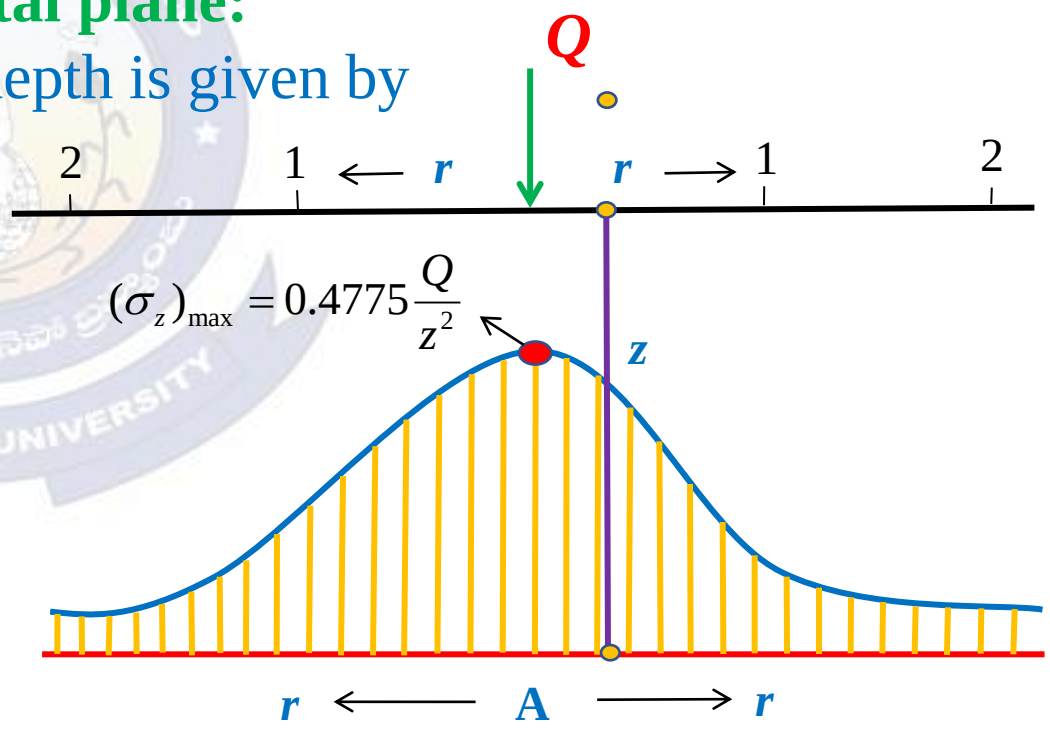
The vertical stress on a horizontal plane at a depth is given by

$$\sigma_z = K_B . \frac{Q}{z^2}$$

z : specified depth

For $r = 0$, σ_z is maximum.

The intensity of vertical stress directly below the point load, $(\sigma_z)_{max} = 0.4775 \frac{Q}{z^2}$



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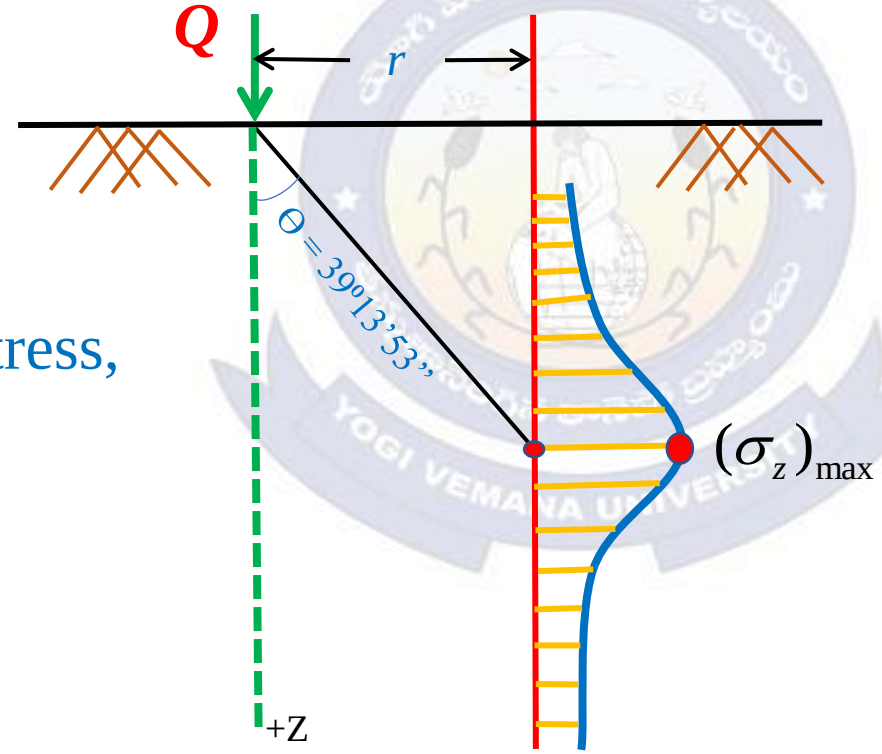
Vertical stress distribution along a vertical line:

- The variation of vertical stress with depth at constant radial distance from the axis of load σ_z first increases with depth attains a maximum value and then decreases with further increase in depth.
- Maximum value occurs when $\theta = 39^0 13' 53''$ makes with the polar ray.
- For maximum σ_z

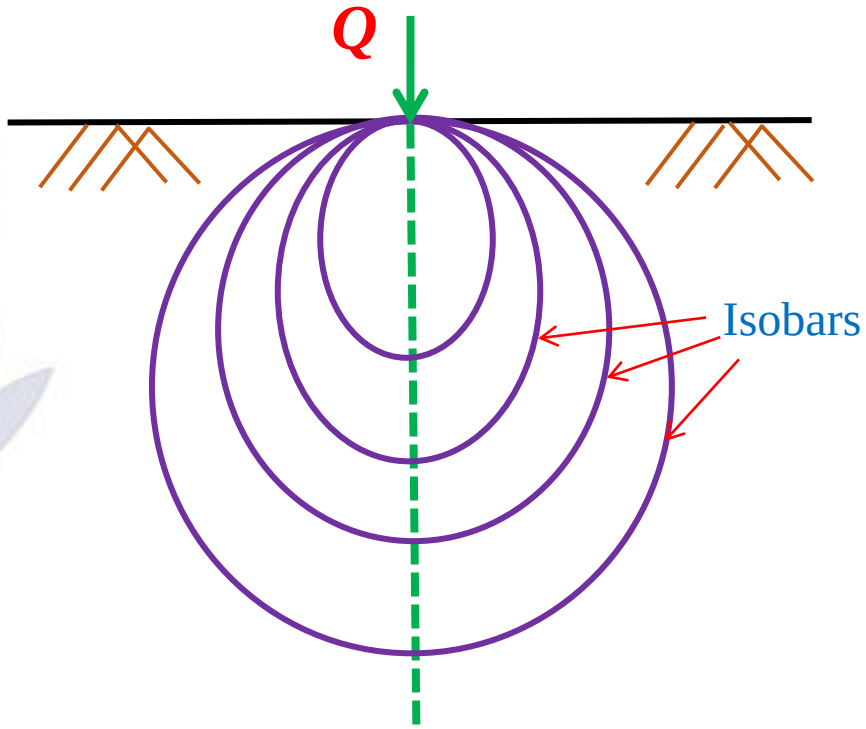
$$\frac{r}{z} = \sqrt{\frac{2}{3}} = 0.817$$

Maximum vertical stress,

$$(\sigma_z)_{max} = 0.0888\ Q$$



Vertical stress distribution along a vertical line at radial distance r



Isobar diagram (A system of pressure bulbs for a point load)

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- An isobar is a stress contour or a line which connects all points of same vertical pressure below the ground surface.
- An isobar is a special curved surface and resembles a bulb in shape, because the vertical pressure at all points in a horizontal plane at equal radial distances from the load is the same.
- Stress isobar is also called as bulb of pressure or pressure bulb.
- The vertical pressure at each point on the pressure bulb is the same.
- A system of isobars indicates the decrease in stress intensity from the inner to the outer ones and reminds onion bulb.
- Isobars are not circular curves
- The shape of isobar approaches to that of lemniscate.

Westergaurd's theory:

- The material is assumed to be laterally reinforced by numerous, closely spaced horizontal sheets of negligible thickness but of infinite rigidity, which prevents the medium from undergoing lateral strain.
- Non-isotropic condition.

Vertical stress, $\sigma_z = \frac{Q}{\pi z^2} \cdot \frac{1}{\left[1+2\left(\frac{r}{z}\right)^2\right]^{3/2}}$; $\sigma_z = K_w \cdot \frac{Q}{z^2}$

Westergaurd's influence coefficient $K_w = \frac{1}{\pi} \cdot \frac{1}{\left[1+2\left(\frac{r}{z}\right)^2\right]^{3/2}}$

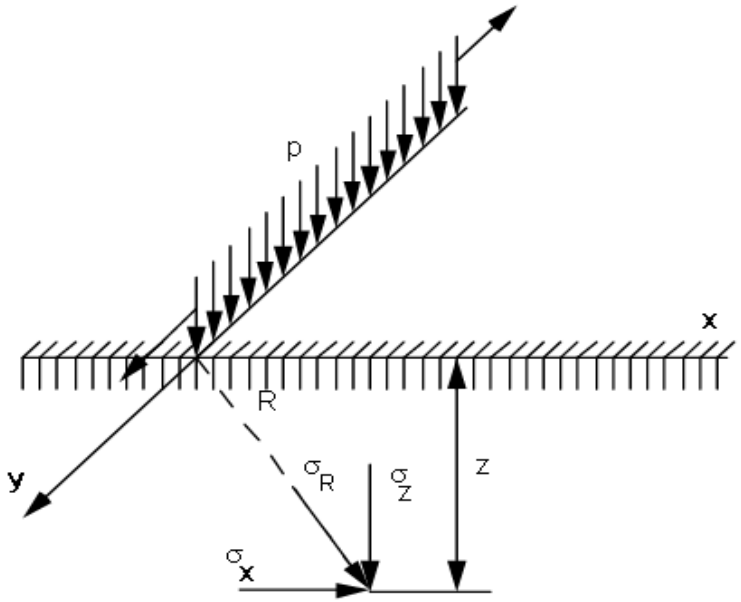
Line load

Vertical stress, $\sigma_z = \frac{2q}{\pi z} \cdot \frac{1}{\left[1+(x/z)^2\right]^2}$; $\sigma_z = K_l \cdot \frac{q}{z}$

Boussinesq's influence coefficient for line load, $K_l = \frac{2}{\pi} \cdot \frac{1}{\left[1+(x/z)^2\right]^2}$

If the point A is situated vertically below the line load, at a depth z and $x = 0$

$$\sigma_z = \frac{2q}{\pi z}$$



Geometry of the Problem Involving a Uniform Line Load of Infinite Surface at the Surface

Strip load:

Vertical stress, $\sigma_z = \sigma_1 = \frac{q}{\pi}(\theta + \sin\theta)$

Horizontal stress, $\sigma_x = \sigma_3 = \frac{q}{\pi}(\theta - \sin\theta)$

Maximum shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{q}{\pi}.\sin\theta$

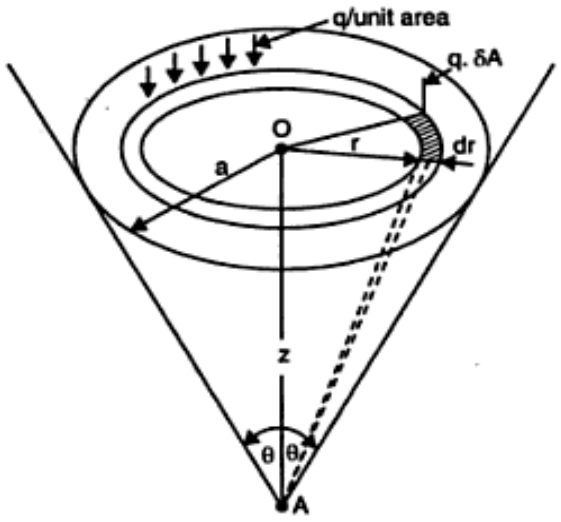
$$(\tau_{max})_{max} = \frac{q}{\pi}$$

Uniform load on circular area

Vertical stress, $\sigma_z = q.\left[1 - \frac{1}{[1+(a/z)^2]^{3/2}}\right]$; $\sigma_z = q.K_{B_C}$

$$\sigma_z = q(1-\cos^3\theta)$$

Boussinesq's influence eoefficient for uniform load on circular area, $K_{B_C} = \left[1 - \frac{1}{[1+(a/z)^2]^{3/2}}\right]$



Uniform load over circular area

θ = Angle made by the vertical line passing through centre of the periphery of the loaded area.

Uniform load on Rectangular area

The vertical stress at a point below the corner of a rectangular area loaded uniformly is

$$\sigma_z = q.I$$

$$\sigma_z = \frac{q}{4\pi}\left[\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right)\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\tan^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right)\right]$$

$$\sigma_z = \frac{q}{4\pi}\left[\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right)\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right)+\sin^{-1}\left(\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right)\right]$$

$$m = \frac{B}{z}, n = \frac{L}{z}$$

The principle of superposition may be used to compute the stress beneath any point either inside or outside a uniformly loaded rectangular area.

When the point is inside:

Let O be an interior point of a rectangular loaded area ABCD shown in figure.

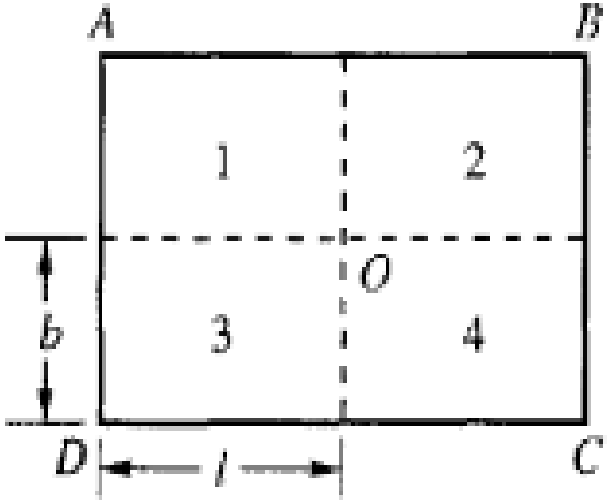
Total stress at O, $\sigma_z = q(I_1 + I_2 + I_3 + I_4)$

When the point is outside:

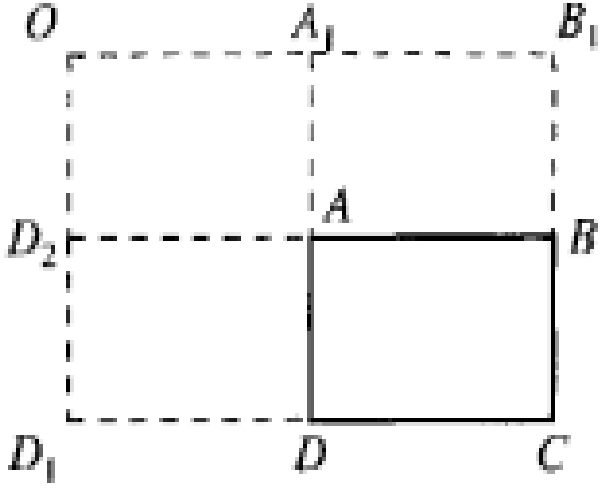
Let O be an exterior point of loaded rectangular area ABCD shown in figure.

Area ABCD = OB_1CD_1 - OB_1BD_2 - OD_1DA_1 + OA_1AD_2

Total stress at O, $\sigma_z = q(I_1 - I_2 - I_3 + I_4)$



When the point 'O' is within the rectangle



When the point 'O' is outside the rectangle

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Uniform load on irregular area: Newmarks chart

- Newmarks influence chart is a graphical procedure for computing the vertical stress inside the soil subjected to uniform load on irregular areas.
- Applicable to a semi-infinite, homogeneous, isotropic and elastic soil mass.
- Not applicable for a stratified soil.
- The vertical stress underneath the center of a uniformly loaded circular area is

$$\sigma_z = q.\left[1 - \frac{1}{[1+(a/z)^2]^{3/2}}\right] \; ; \; \frac{a}{z} = \sqrt{\left(1 - \frac{\sigma_z}{q}\right)^{-2/3} - 1}$$

a : Radius of the loaded area

z : Depth at which the vertical stress is required

q : Intensity of the uniform load

Total number of sectors = $c.s$

c : Number of circles,

s : Number of equally spaced rays

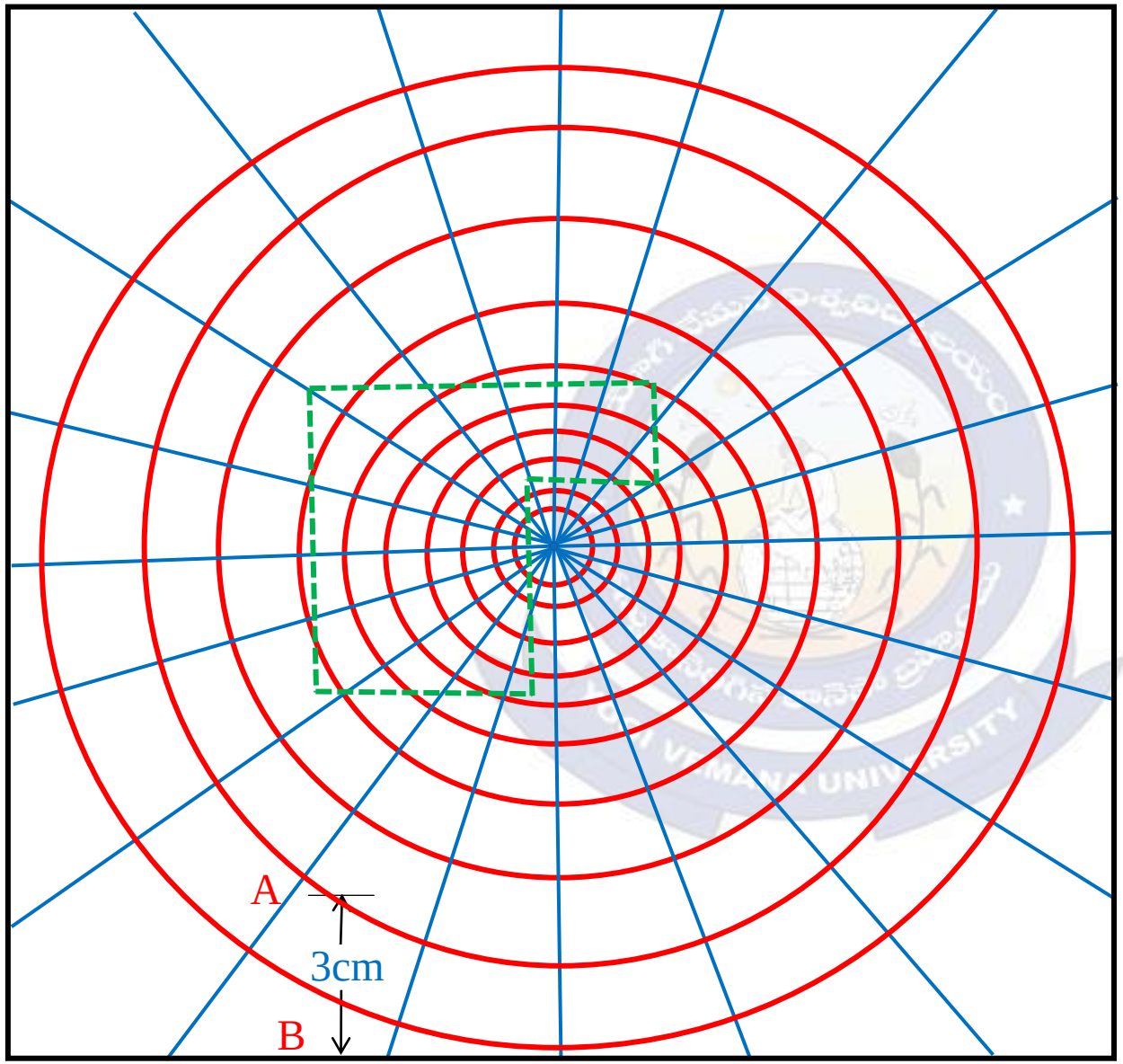
Vertical stress of each sector = $\frac{1}{c.s} \times$ total value at the centre

Influence value (influence factor) $I = \frac{1}{c.s}$

The stress at the specified depth at the specified point, $\sigma_z = I.N.q$

I : Influence value of the chart

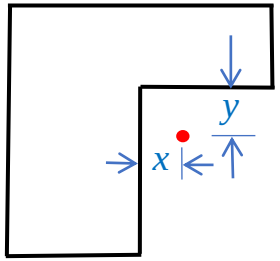
N: Number of influence units



Newmarks chart

Influence value,

$$I = \frac{1}{10 \times 20} = 0.005$$



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The graphical procedure for computing the vertical stress due to any surface loading is as follows.

- Select some definite scale to represent depth z
- For instance a suitable length AB in cm as shown in fig. to represent depth z in meters. In such a case, the scale is 1cm = z /AB meters.
- The length of the radius $R_{0.8}$ which corresponds to $\sigma_z / q = 0.8$ is then equal to $[(a/z).\text{AB}] = 1.387 \times \text{AB}$ cm, and a circle of that radius may be drawn.
- This procedure may be repeated for other ratios of $\sigma_z / q = 0.7, 0.6, ...$ etc. as shown in fig. The annular space between the circles may be divided into n equal blocks, and in this case n =20. The influence value I is equal to 1/10 20=0.005.
- A plan of the foundation is drawn on a tracing paper to a scale such that the distance AB on the chart corresponds to the depth z at which the stress σ_z is to be computed.

- For example, if the vertical stress at a depth of 9 m is required, and if the length AB chosen is 3cm, the foundation plan is drawn to a scale of 1cm = 9/3 = 3m.
- In case the vertical stress at a depth 12m is required, a new foundation plan on a separate tracing paper is required. The scale for this plan is 1cm = 12/AB = 12/3 = 4m.
- A different tracing has to be made for each different depth whereas the chart remains the same for all. It is required to determine the vertical stress σ_z at depth vertically below point O for foundation plan loaded with a uniformly distributed load per unit area.
- In order to determine σ_z , the foundation plan is laid over the chart in such a way that the surface point O coincides with the center O of the chart as shown in fig. The number of small blocks covered by the foundation plan is then counted.
- The vertical stress at a depth z below O, $\sigma_z = I.N.q$

8.11 STRESS DISTRIBUTION IN SOILS

Numerical Questions

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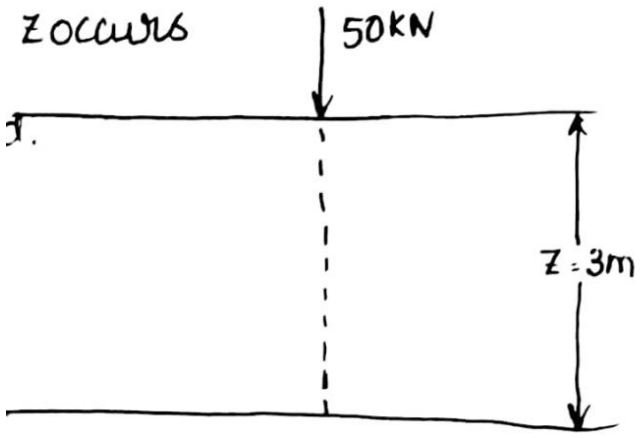
1. A Concentrated load of 50 kN acts on the surface of soil.

a. The maximum stress at a depth of 3 m below the load is ……

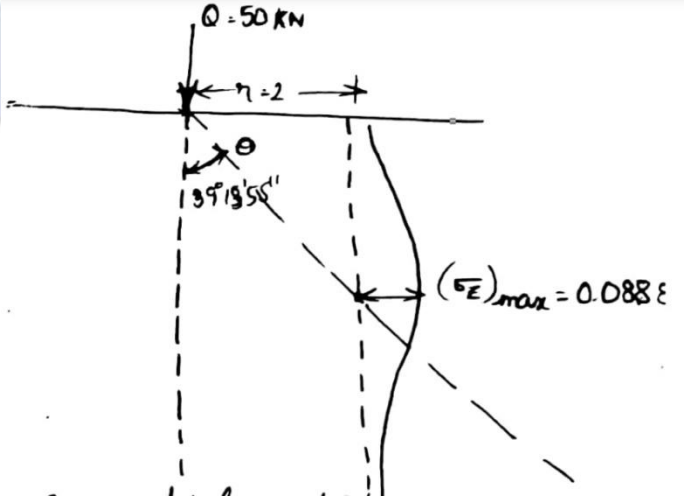
The maximum vertical stress at a given depth of z occurs at a point below the point of application of load.

$$(\sigma_z)_{max} = 0.4775.\frac{Q}{z^2} = 0.4775 \times \frac{50}{3^2} = 2.65\,\text{kN/m}^2$$



b. The maximum vertical stress at a radial distance of 2 m is ……

$$(\sigma_z)_{max} = 0.0888Q = 0.0888 \times 50 = 4.44\,\text{kN/m}^3$$



c. The vertical stress at a depth of 3 m in a radial distance of 2 m is……..

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}\right] = \frac{3\times 50}{2\times\pi\times 3^2}\left[\frac{1}{\left[1+\left(\frac{2}{3}\right)^2\right]^{5/2}}\right] = 1.04\,\text{kN/m}^2$$

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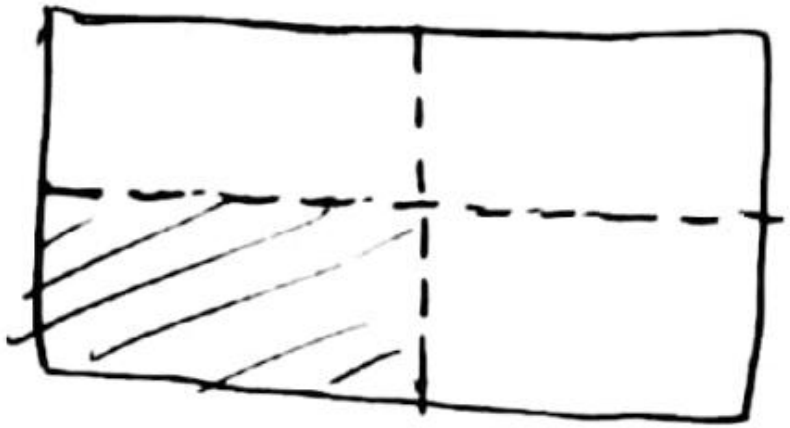
2. An elastic medium carries a uniform load of 250 kN/m^2 at its surface covering rectangular area 4 m×3 m. Assuming that the Boussinesq's influence factor for one quadrant is 0.0474, the vertical stress at a depth of 6 m below the centroid of area is …….



Solution:

Uniform load on rectangular area, $q = 250\,\text{kN/m}^2$

Influence factor for one quadrant, $I = 0.0474$

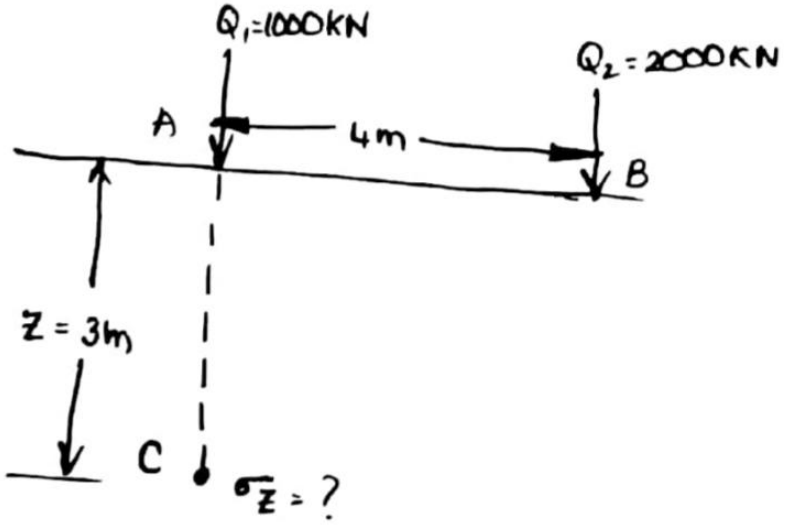
Vertical stress at a depth of 6 m below the centroid of area,

$$\sigma_z = 4\left(\sigma_z\right)_{\text{quater}} = 4.q.I = 4\times 250\times 0.0474 = 47.4\,\text{kN/m}^2$$

3. Two concentrated loads 1000 kN and 2000 kN acting at points A and B respectively at a spacing of 4 m. The increase in vertical stress at a depth of 3 m below the ground level under point A is …….

Solution:

Increase in vertical stress $\sigma_z = (\sigma_z)_1 + (\sigma_z)_2$



$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}\right]$$

$$\sigma_z = \frac{3\times1000}{2\pi\times3^2}\left[\frac{1}{(1+0)^{5/2}}\right] + \frac{3\times2000}{2\pi\times3^2}\left[\frac{1}{\left(1+\left(\frac{4}{3}\right)^2\right)^{5/2}}\right]_z = 61.30\,\text{kN/m}^2$$

4. A footing of size 4 m×3 m is constructed to carry an axial load of 1800 kN. The influence factor corresponding to the values m and n is shown in the table:

m	n	Influence Factor
0.25	0.333	0.0345
0.50	0.667	0.095

The vertical stress increased at a point 6 m below the centre of footing is ……..

Solution:

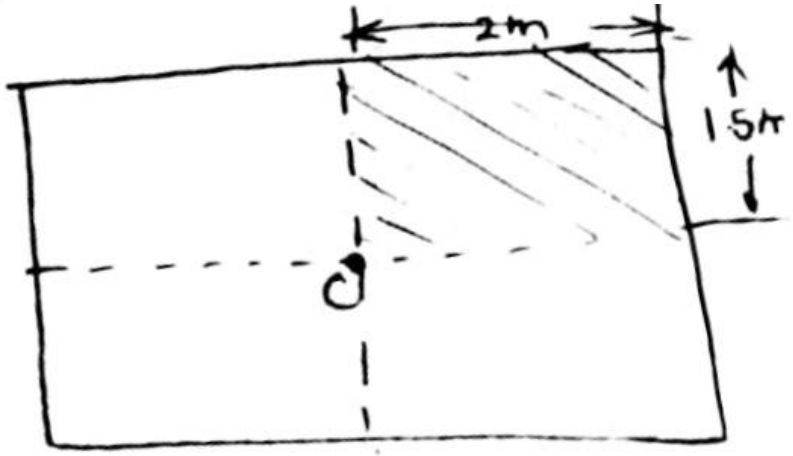
Vertical stress increased at a point 6 m below the centre of footing $(\sigma_z)_c = 4.q.I$

$$q = \frac{P}{A} = \frac{1800}{4\times 3} = 150\,\text{kN/m}^2$$

$$m = \frac{B}{z} = \frac{1.5}{6} = 0.25 \qquad n = \frac{L}{z} = \frac{2}{6} = 0.333$$

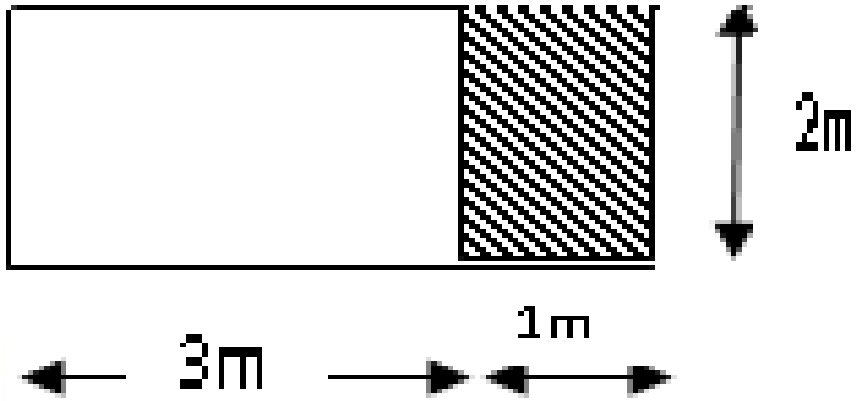
For $m = 0.25,\ n = 0.667,\ I = 0.0345$

$$\sigma_c = 4\times(\sigma_z) = 4\times q.I = 4\times 150\times 0.0345 = 20.68\,\text{kN/m}^2$$



5. The vertical stress increase below the point A at a depth of 4 m for the loaded area shown is …….. The influence values are given in the table below:

m	n	Influence factor
0.5	1.0	0.12
0.25	0.5	0.048
0.25	0.75	0.055



Solution:

Vertical stress increase below the point A, $\sigma_z = q\left[I_1 - I_2\right]$

I_1 : Influence factor for ABCD

I_2 : Influence factor for AEFD

For ABCD, $m = \frac{2}{4} = 0.5,\ n = \frac{4}{4} = 1,\ I_1 = 0.12$

$$\sigma_z = 150\left(0.12 - 0.048\right) = 10.8\,\text{kN/m}^2$$

for AEFD, $m = \frac{1}{4} = 0.25,\ n = \frac{2}{4} = 0.5,\ I_2 = 0.048$

6. A Structure of total weight 12000 kN is supported on 4 columns rests on piers located at the corners of a block 6 m×8 m of size. Vertical stress increment at one of the corner due to the load at a point 10 m beneath the structure is ……..

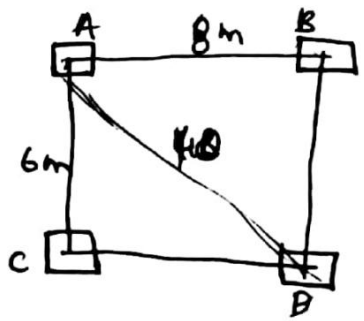
Solution:

Load on each column $= \dfrac{12000}{4} = 300\,\text{kN}$

$$(\sigma_z)_A = \sigma_A + \sigma_B + \sigma_C + \sigma_D$$

$$(\sigma_z)_A = \frac{3\times 3000}{2\pi\times 10^2}\left[\left[\frac{1}{1+\left(\frac{0}{10}\right)^2}\right]^{5/2} + \left[\frac{1}{1+\left(\frac{8}{10}\right)^2}\right]^{5/2} \left[\frac{1}{1+\left(\frac{6}{10}\right)^2}\right]^{5/2} + \left[\frac{1}{1+\left(\frac{10}{10}\right)^2}\right]^{5/2}\right]$$

$$= \frac{3\times 3000}{2\pi\times 10^2}\left[1+0.290+0.463+0.176\right] = 27.63\,\text{kN/m}^2$$



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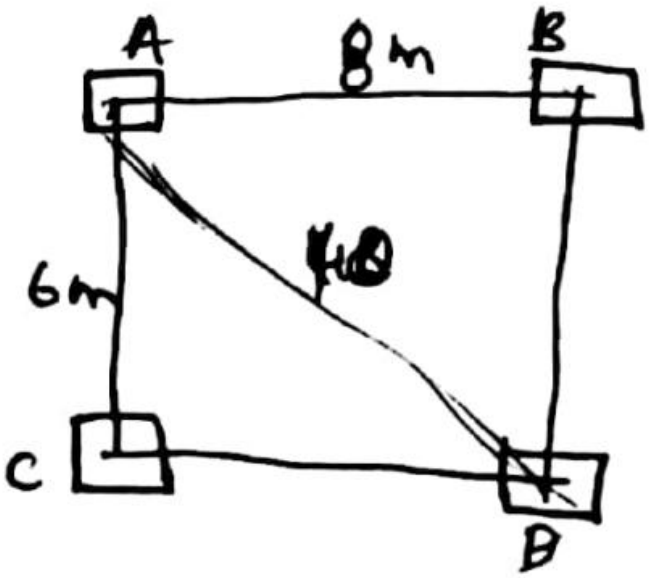
7. A Structure of total weight 12000 kN is supported on 4 columns rests on piers located at the corners of a block 6 m×8 m of size. Vertical stress increment due to the load at a point 10 m beneath the centre of structure is …….

Solution:

Load on each column $= \dfrac{12000}{4} = 300\,\text{kN}$

$$(\sigma_z)_A = \sigma_A + \sigma_B + \sigma_C + \sigma_D = 4 \times \sigma_A$$

$$(\sigma_z)_A = 4 \times \frac{3 \times 3000}{2\pi \times 10^2}\left[\frac{1}{1+\left(\frac{5}{10}\right)^2}\right]^{5/2} = 32.8\,\text{kN/m}^2$$



8. The plan of square footing has been plotted on the top of Newmark's chart to the scale and it is found that 48 sectors covering the area. The load acting through footing of size 3 m×3 m is 1000 kN. Influence value for the Newmark's chart is 0.005. The stress at a point A below 3 m from the edges of footing is …….

Solution:

Influence value for the Newmark's chart, $I = 0.005$

Intensity of loading, $q = \frac{1000}{3 \times 3} = 111.11 \text{kN/m}^2$

Number of sectors covering the area of Newmark's chart, $N = 48$

Stress at a point A, $\sigma_z = I.q.N = 0.005 \times 111.11 \times 48 = 26.66 \text{ kN/m}^2$

8.11 Stress distribution in soils

Previous GATE Questions

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01.A 2 m×4 m rectangular footing has to carry a uniformly distributed load of 120 kPa. As per the 2 V:1 H dispersion method of stress distribution, the increment in vertical stress (in kPa) at a depth of 2 m below the footing is …… CE2 2019

01. 40

Size of rectangular footing: 2m×4m

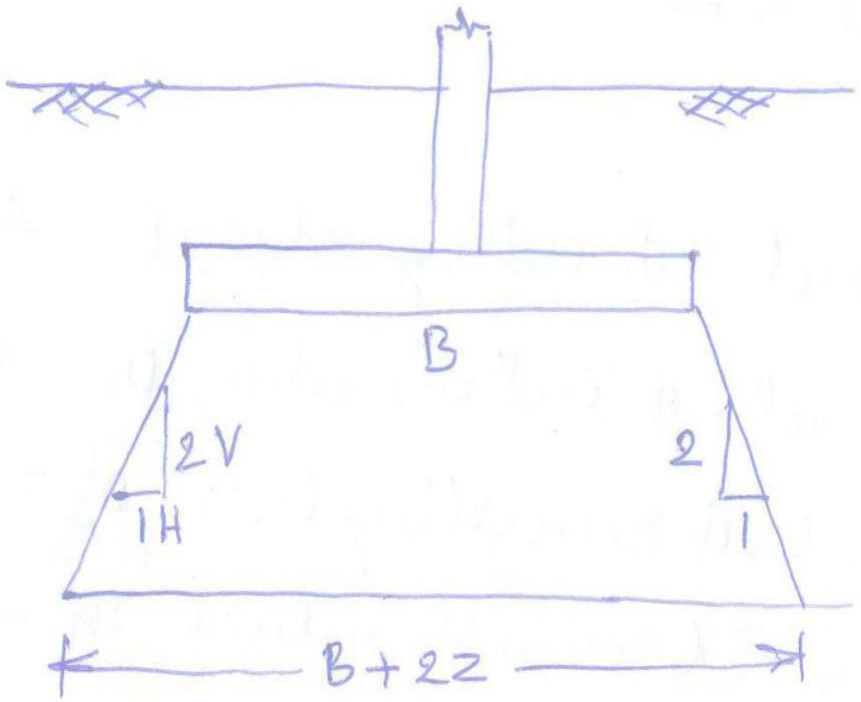
Uniformly distributed load on footing, q =120 kPa

Depth of footing, D = 2 m

Dispersion of stress distribution = 2:1 (2V:1H)

Vertical stress at 2 m depth, $\sigma_z = ?$

$$\sigma_z = \frac{q.B.L}{(B+2z)(L+2z)} = \frac{120\times 2\times 4}{(2+2\times 0.5)(4+2\times 0.5)} = 64 \text{ kPa}$$



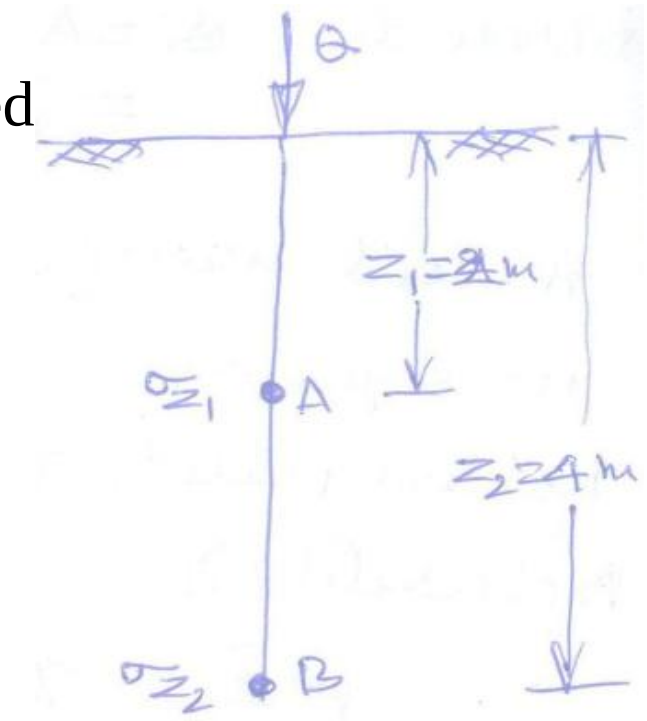
02.A concentrated load of 500 kN is applied on an elastic half space. The ratio of the increase in vertical normal stress at depths of 2 m and 4 m along the point of the loading, as per Boussinesq's theory, would be……. CE1 2019

02. 4

As per Boussinesq's theory, the vertical stress due to concentrated load is given by

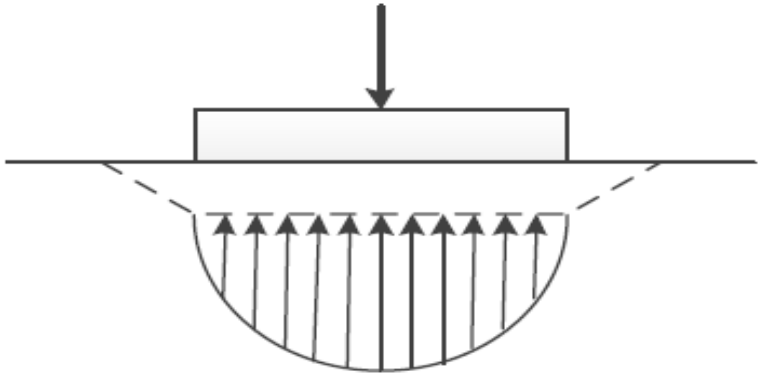
$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\sigma_z = K.\frac{Q}{z^2} \Rightarrow \sigma_z \propto \frac{1}{z^2}$$



$$\frac{\sigma_{z_1}}{\sigma_{z_2}} = \left(\frac{z_2}{z_1}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$$

03. The contact pressure and settlement distribution for a footing are shown in the figure. CE2 2018

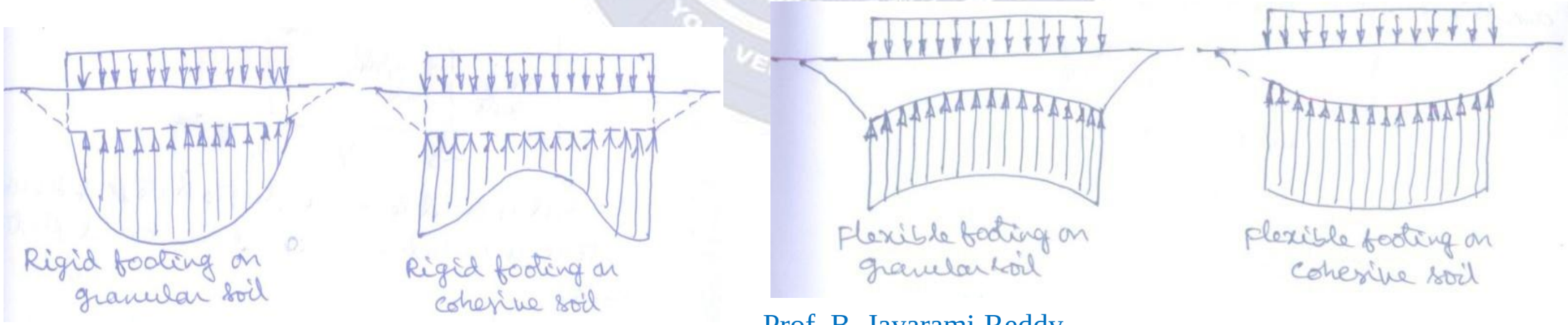


The figure corresponds to a

a. Rigid footing on granular soil b. Flexible footing on granular soil

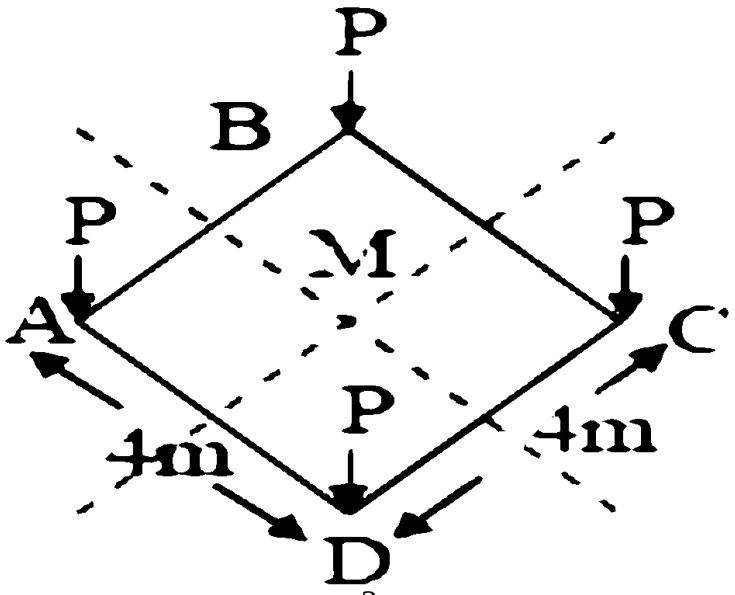
c. Flexible footing on saturated clay d. Rigid footing on cohesive soil.

03. a



Prof. B. Jayarami Reddy

04. Consider a square-shaped area ABCD on the ground with its centre at M as shown in the figure. Four concentrated vertical loads of P = 5000 kN are applied on this area, one at each corner. The vertical stress increment (in kPa, up to one decimal place) due to these loads according to the Boussinesq's equation, at a point 5 m right below M is …….. CE2 2017



04. 190.84

Radial distance AM = $2\sqrt{2}$ = 2.828 m

Magnitude of each corner load, P = 5000 kN

Depth of the point below M, z = 5 m

Vertical stress at centre due to loads using Bousiness's equation is given by

$$\sigma_z = 4\times\frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = 4\times\frac{3\times5000}{2\pi\times5^2}\left[\frac{1}{1+\left(\frac{2.828}{5}\right)^2}\right]^{5/2} \Rightarrow \sigma_z = 190.84\,\text{kN/m}^2$$

05. A uniformly distributed line load of 500 kN/m is acting on the ground surface. Based on Boussinesq's theory, the ratio of vertical stress at a depth 2 m to that at 4 m, right below the line of loading, is CE1 2017

a. 0.25 b. 0.5 **c.** 2.0 d. 4.0

05. c

Line load, q = 500 kN/m

Vertical stress as per Boussinesq's theory is given by $\sigma_v = \frac{2q}{\pi z}\left[\frac{1}{1+\left(\frac{x}{z}\right)^2}\right]^2$

For stress below the line of loading, $x = 0$

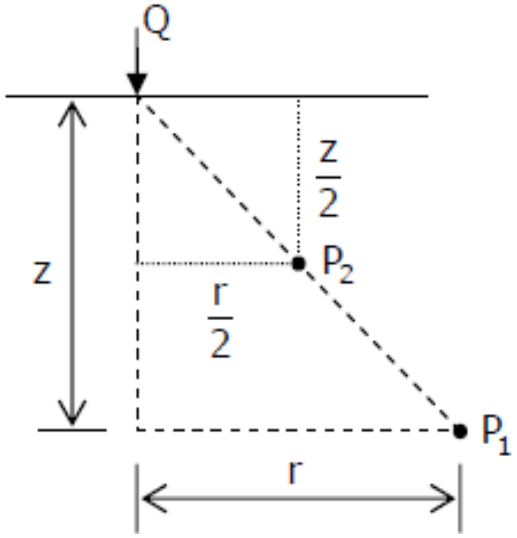
$$\sigma_z = \frac{2q}{\pi z} \Rightarrow \sigma_z \propto \frac{1}{z}$$

$z_1 = 2\,\text{m}$; $\sigma_1 = \sigma_{z1}$ $z_2 = 4\,\text{m}$; $\sigma_2 = \sigma_{z2}$

$$\frac{\sigma_1}{\sigma_2} = \frac{z_2}{z_1} = \frac{4}{2} = 2$$

06. The vertical stress at point P_1 due to the point load Q on the ground surface as shown in figure is σ_z . According to Boussinesq's equation, the vertical stress at point P_2 shown in figure will be 2010

a. $\sigma_z/2$ b. σ_z c. $2\sigma_z$ **d.** $4\sigma_z$



According to Boussinesq's equation, the vertical stress at a point located at a depth of r and a horizontal distance z from the point of application of point load is

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

The vertical stress at a depth of $z_1 = z/2$ and a horizontal distance of $r_1 = r/2$ will be

$$\sigma_1 = \frac{3Q}{2\pi\left(\frac{z}{2}\right)^2}\left[\frac{1}{1+\left(\frac{r/2}{z/2}\right)^2}\right]^{5/2} = 4.\frac{3Q}{2\pi.z^2}.\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = 4\sigma_z$$

07. The vertical stress at some depth below the corner of a 2 m× 3 m rectangular footing due to a certain load intensity is 100 kN/m^2. What will be the vertical stress in kN/m^2 below the centre of a 4 m ×6 m rectangular footing at the same depth and same load intensity ? 2007

a. 25 b. 100 c. 200 **d.** 400

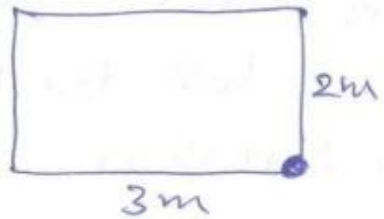
07. d

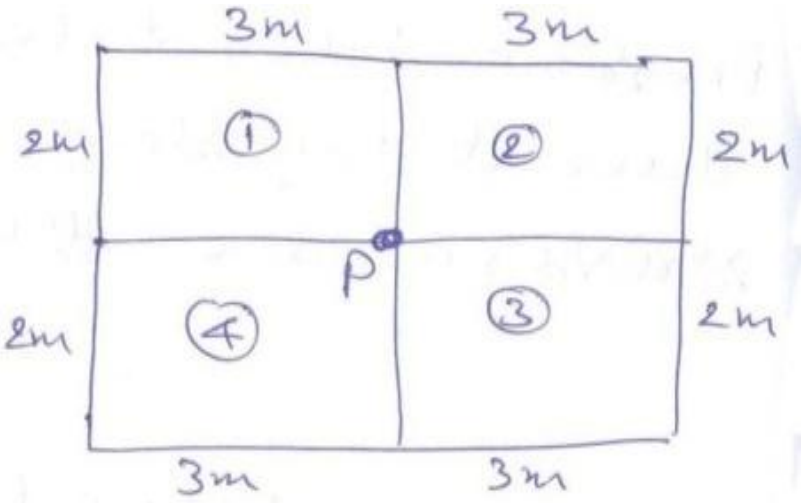
Area of rectangular footing, A_1 = 2×3 = 6 m^2

Intensity of stress at corner of 2×3 m, σ_1 = 100 kN/m^2

Area of rectangular footing, A_2 = 4×6 m

Intensity of stress at centre of 4×6 m, σ_2 = ?





The point P is at the corner of four equal areas of 1, 2, 3 and 4.

Vertical stress below point P due to same loading, $\sigma_2 = 4\sigma_1$ = 4 × 100 = 400 kN/m^2

08. A 25 kN point load acts on the surface of an infinite elastic medium. The vertical pressure intensity in kN/m^2 at a point 6.0 m below and 4.0 m away from the load will be 2003

a. 132 b. 13.2 c. 1.32 **d.** 0.132

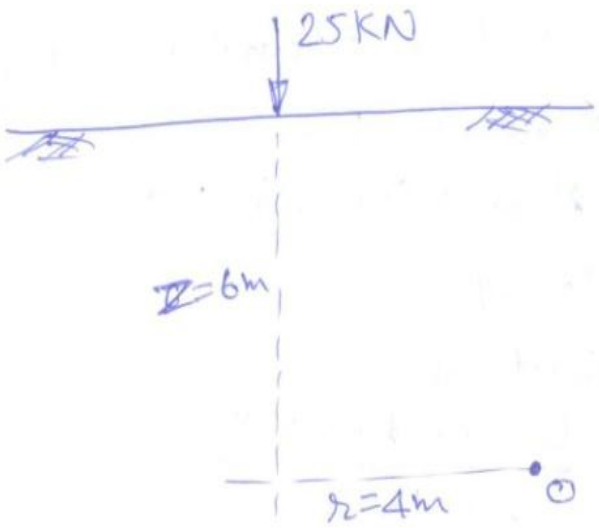
08. d

Vertical distance (depth) of the point from the ground surface, $z = 6\,\text{m}$

Horizontal distance to the point from the application of point load, $r = 4\,\text{m}$

As per Boussinesq's formula for point load, the vertical stress is given by

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = \frac{3\times 25}{2\pi(6)^2}\left[\frac{1}{1+\left(\frac{4}{6}\right)^2}\right]^{5/2} = 0.132\,\text{kN/m}^2$$



09. A point load of 700 KN is applied on the surface of thick layer of saturated clay. Using Boussinesq's elastic analysis, the estimated vertical stress at a depth of 2 m and a radial distance of 1.0 m from the point of application of the load is:

a. 47.5 kPa b. 47.6 kPa c. 47.7 kPa **d.** 47.8 kPa 1998

09. d

Point load, P = 700 kN

Depth, z = 2 m

Radial distance, r = 1m

The vertical stress according to Boussinesq's equation is given by

$$\sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} = \frac{3\times 700}{2\pi\times 2^2}\left[\frac{1}{1+(1/2)^2}\right]^{5/2} = 47.8 \text{ kN/m}^2 = 47.8 \text{ kPa}$$

10. The vertical stress at depth directly below the point load P is (k is a constant)

a. $k.\frac{P}{z}$ b. $k.\frac{P}{z^3}$ **c.** $k.\frac{P}{z^2}$ d. $k.\frac{P}{\sqrt{z}}$ 1997

10. c

As per Boussinesq's equation, the vertical stress at a depth z and radial distance r for a point load is given by

$$\sigma = \frac{3P}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

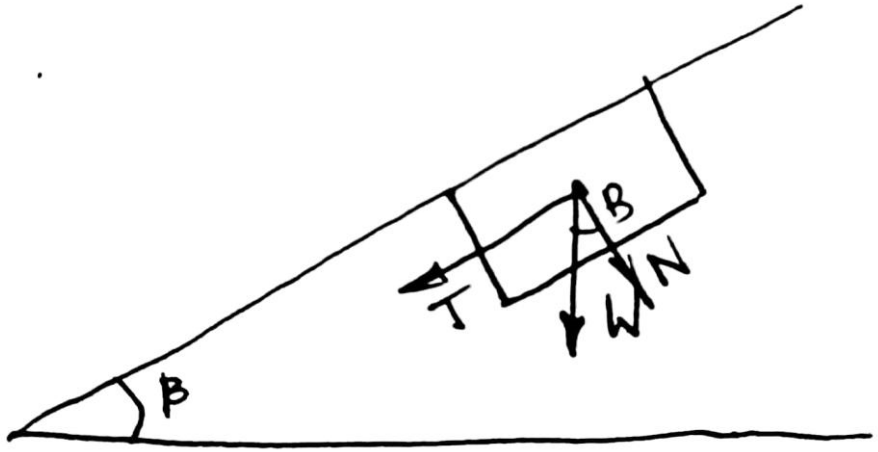
At, $r = 0$ $\sigma = \frac{3P}{2\pi z^2} = k.\frac{P}{z^2}$

Stability of Slopes

Prof. B. Jayarami Reddy

9.3 STABILITY OF SLOPES

Infinite slope in cohesionless soil:



W = Weight of soil mass

N=Normal component of soil mass

$=W.\cos\beta$

T= Tangential component of soil mass

$=W.\sin\beta$

μ= coefficeint of friction

$=\tan\phi$

Resisting force=$N.\mu=W.\cos\beta.\tan\phi$

Sliding force,T= $W\sin\beta$

Factor fo safety, $F=\dfrac{\text{Resisting force}}{\text{Sliding force}}=\dfrac{W.\cos\beta.\tan\phi}{W\sin\beta}=\dfrac{\tan\phi}{\tan\beta}$

$$F=\frac{\tan\phi}{\tan\beta}$$

Normal and Tangential Stresses at the failure surafces:

$$W = \gamma.b.z.1$$

$$\sigma = \frac{W}{A} = \frac{\gamma.b.z.1}{l.1} = \frac{\gamma.b.z.1}{\frac{b}{\cos\beta}} = \gamma z.\cos\beta$$

$$\sigma_n = \text{Normal stress}$$

$$= \sigma\cos\beta = \gamma z\cos\beta \times \cos\beta = \gamma z.\cos^2\beta$$

$$\tau = \text{Shear stress}$$

$$= \sigma\sin\beta = \gamma z.\cos\beta \times \sin\beta = \gamma z.\cos\beta\sin\beta$$

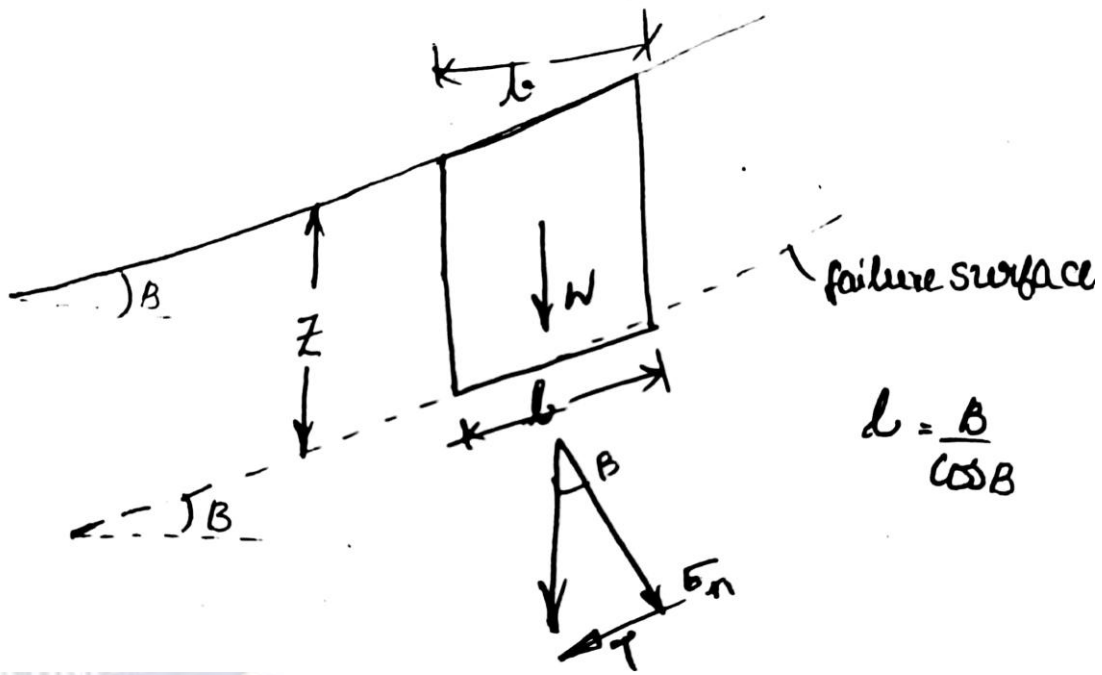
$$\tau = c + \sigma\tan\phi$$

For cohesionless soils, c=0

$$\tau = \sigma\tan\phi = \gamma z.\cos^2\beta.\tan\phi$$

$$F = \frac{\text{Shear Resistance of soil}}{\text{Shear stress induced}} = \frac{\gamma z.\cos^2\beta.\tan\phi}{\gamma z.\cos\beta\sin\beta} = \frac{\tan\phi}{\tan\beta}$$

$$F = \frac{\tan\phi}{\tan\beta}$$



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$$F = \frac{\text{Tan } \phi}{\text{Tan } \beta}$$

If F>1, the slope is stable . $\tan\phi > \tan\beta$

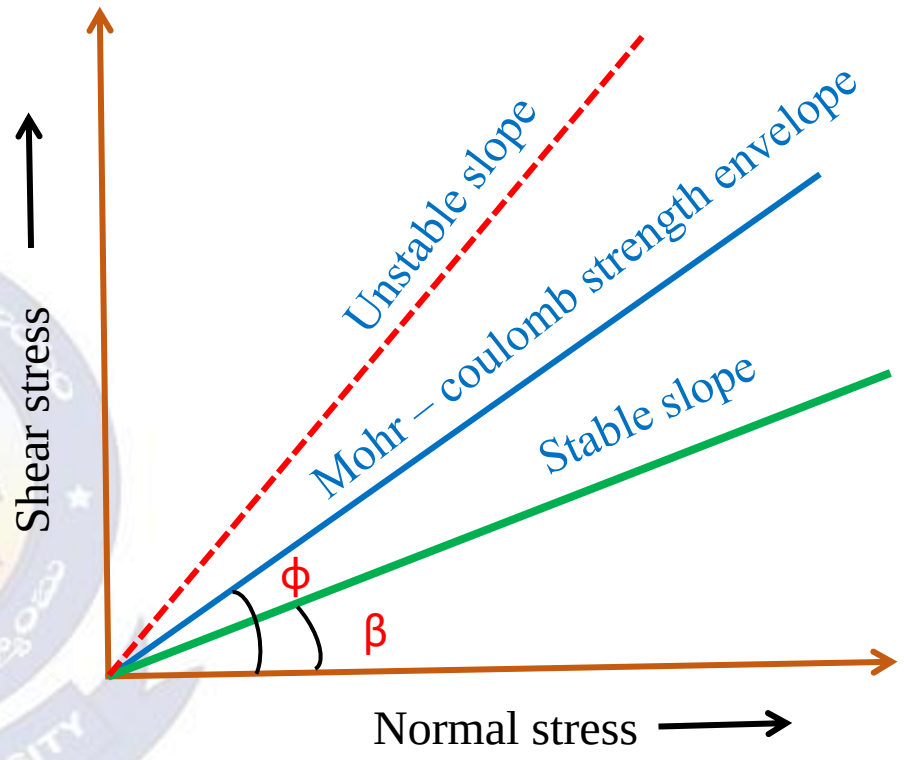
$\phi > \beta \Rightarrow$ Stable condition

$\phi = \beta \Rightarrow$ Neutral condition

$\phi < \beta \Rightarrow$ Unstable

When Shearing Resistance is fully mobilised

$$\text{F} = \frac{\tan\phi'}{\tan\phi'_{\text{m}}} = \frac{\tan\phi'}{\tan\beta}$$



Relation between strength envelope and angle of slope

F is independent of the depth.

F depends on ϕ and β

Seepage occurs parallel to the slope (or) water table parallel to the slope:

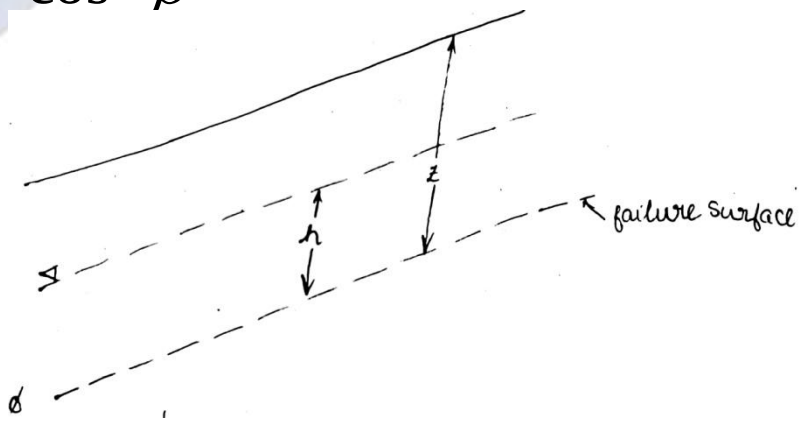
$$F=\frac{\sigma\tan\phi}{\tau}=\frac{(\sigma-u)\tan\phi}{\tau}=\frac{\left(\gamma z\cos^2\beta-\gamma_w h\cos^2\beta\right)\tan\phi}{\gamma z\cos\beta\sin\beta}=\left[1-\frac{\gamma_w h}{\gamma z}\right]\frac{\tan\phi}{\tan\beta}$$

If h=z, water level reaches upto ground level

$$F=\left[1-\frac{\gamma_w z}{\gamma z}\right]\frac{\tan\phi}{\tan\beta}=\left[1-\frac{\gamma_w}{\gamma}\right]\frac{\tan\phi}{\tan\beta}=\left[\frac{\gamma-\gamma_w}{\gamma}\right]\frac{\tan\phi}{\tan\beta}=\frac{\gamma'}{\gamma}.\frac{\tan\phi}{\tan\beta}$$

$$r_u=\text{Pore pressure ratio}=\frac{u}{\gamma z}=\frac{\gamma_w h\cos^2\beta}{\gamma z}\Rightarrow\frac{\gamma_w h}{\gamma z}=\frac{r_u}{\cos^2\beta}$$

$$F=\left[1-\frac{\gamma_w h}{\gamma z}\right]\frac{\tan\phi}{\tan\beta}=\left[1-\frac{r_u}{\cos^2\beta}\right]\frac{\tan\phi}{\tan\beta}$$



z : Depth from the top of the slope

h : Height of water above the failure plane

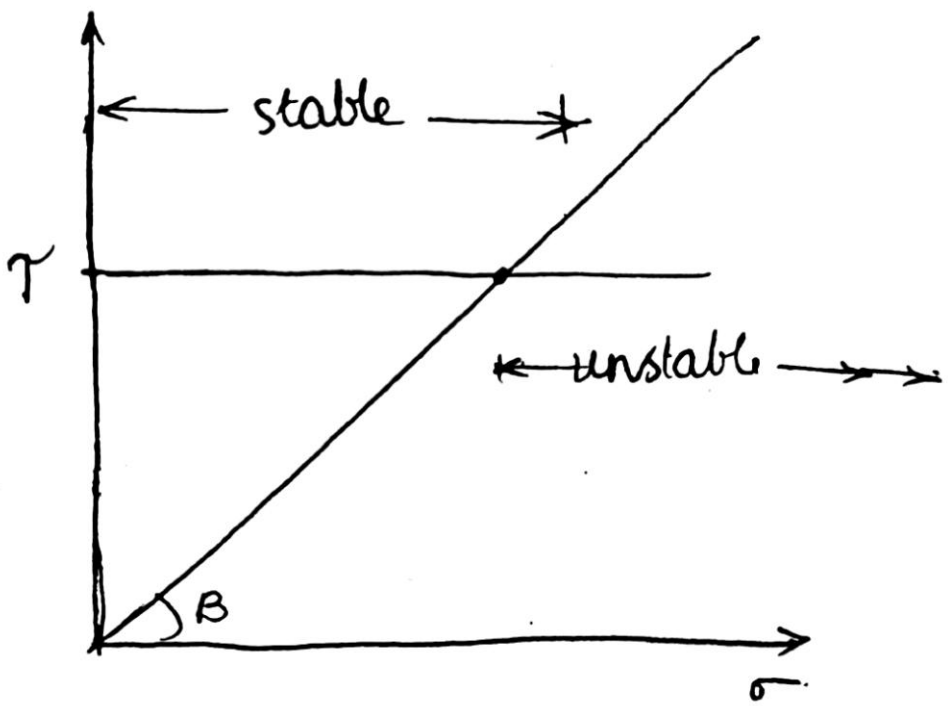
Infinite slope in purely cohesive soil

Factor of safety against slippage, $F = \frac{c}{\tau} = \frac{c}{\gamma z.\sin\beta.\cos\beta}$

If $F = 1$, $z = z_c$: critical height

$$z_c = \frac{c}{\gamma.\sin\beta.\cos\beta} \quad ; \quad \frac{c}{\gamma.z_c} = \sin\beta.\cos\beta$$

Stability number, $S_n = \frac{c}{\gamma.z_c}$



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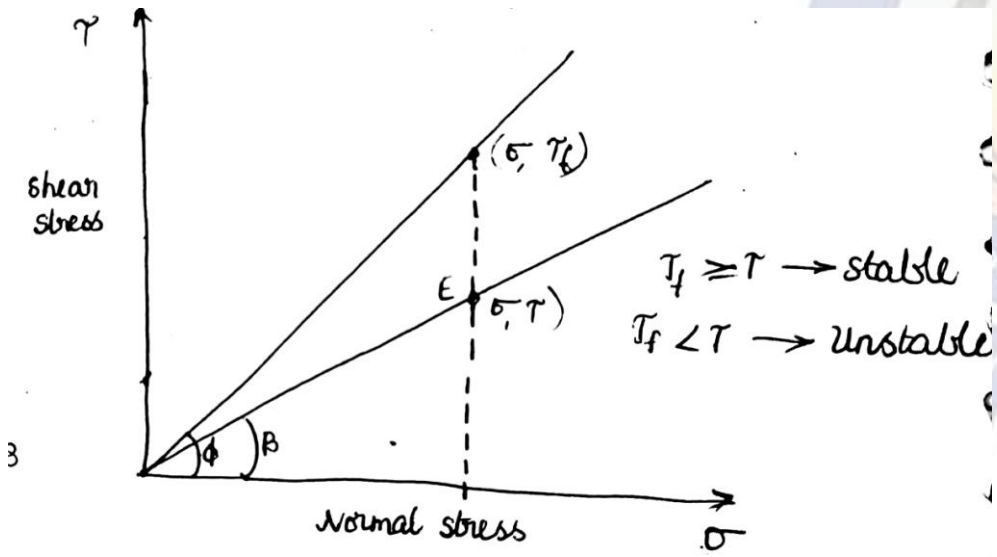
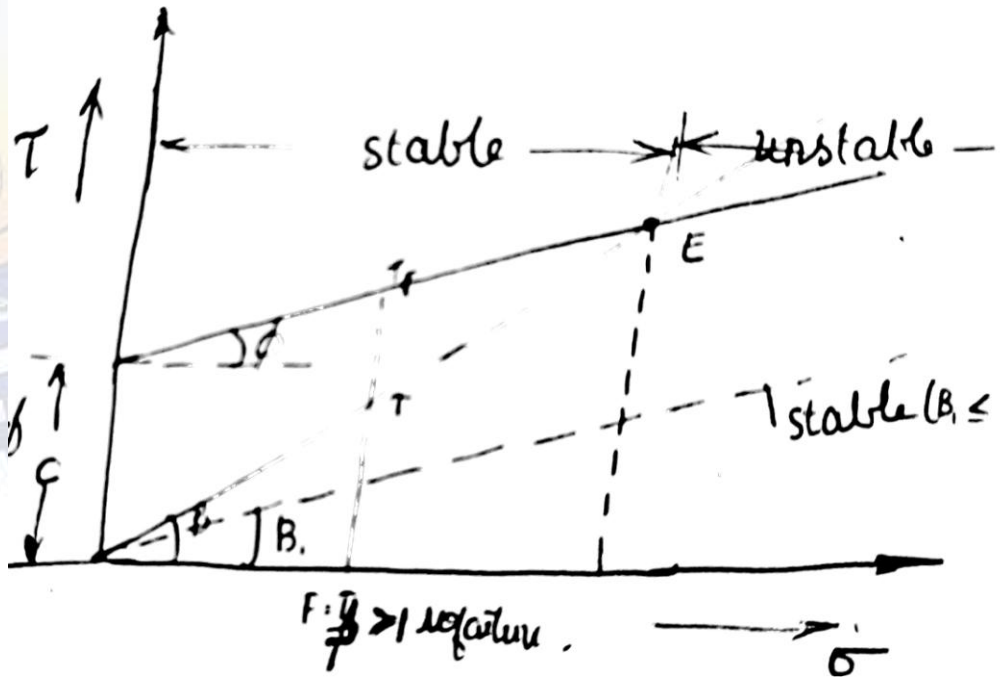
Infinite slope in c- ϕ soil:

Shear resisitance of soil $\tau_f = c' + \sigma' \tan\phi = c' + \gamma z \cos^2\beta . \tan\phi$

Shear stress, $\tau = \gamma z \cos\beta \sin\beta$

$$F = \frac{\tau_f}{\tau} = \frac{c' + \gamma z \cos^2\beta \tan\phi}{\gamma z \cos\beta \sin\beta} \quad ; \quad S_n = \frac{c}{F_c . \gamma . z}$$





Taylor's method:

$$S_n = \frac{c_m}{\gamma H} = \frac{c}{F_c.\gamma H}$$

H : Vertical height of the slope

If the slope is submerged use γ' instead of γ .

For the case of sudden drawdown, use γ_{sat} instead of γ and $\phi_w = \frac{\gamma'}{\gamma_{sat}}.\phi$

Finite slopes:

- Newly built embankment or newly cut slope in fully saturated clay soil.

$\phi = 0$ **Analysis:**

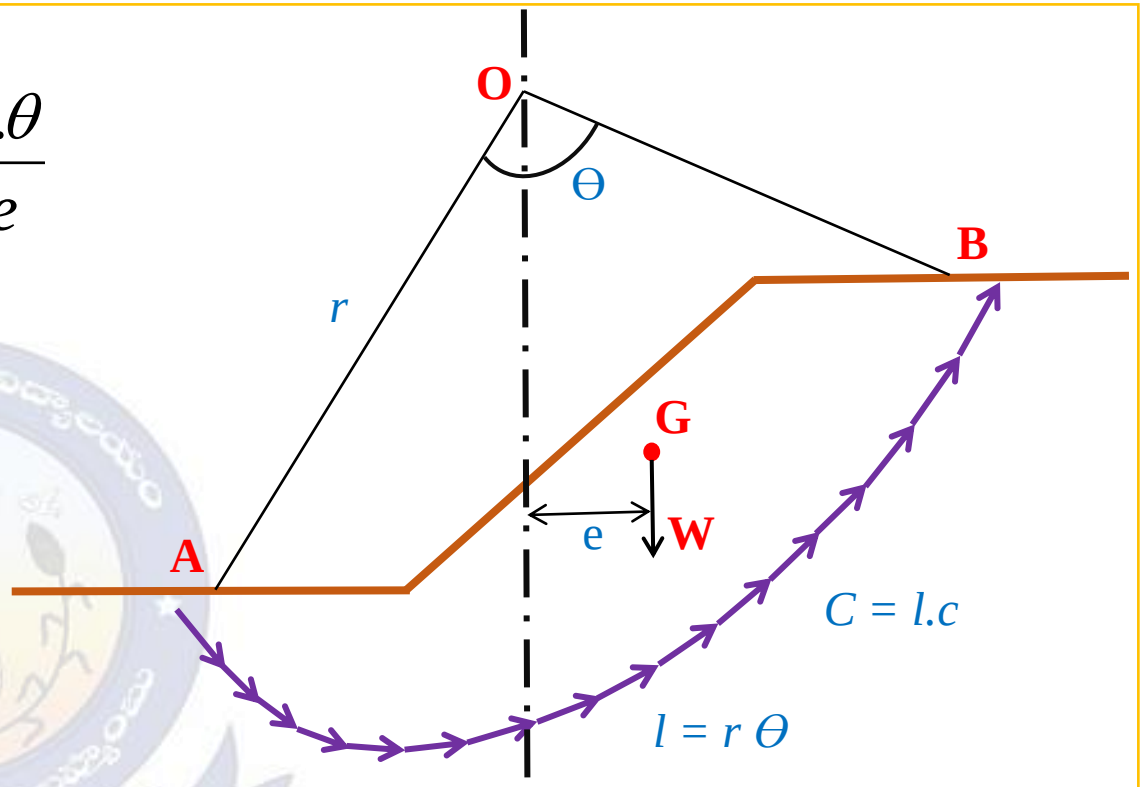
Shear strength parameters are obtained from either unconfined compression test or undrained triaxial test

$$W.e = c.l.r. = c.r\theta.r = cr^2.\theta$$

$$F = \frac{\text{Restraining moment}}{\text{sliding moment}} = \frac{c.l.r}{W.e} = \frac{c.r^2.\theta}{W.e}$$

Depth of tension crack, $z_c = \frac{2c}{\gamma}$

Unsupported depth of soil, $h_c = 2.z_c$



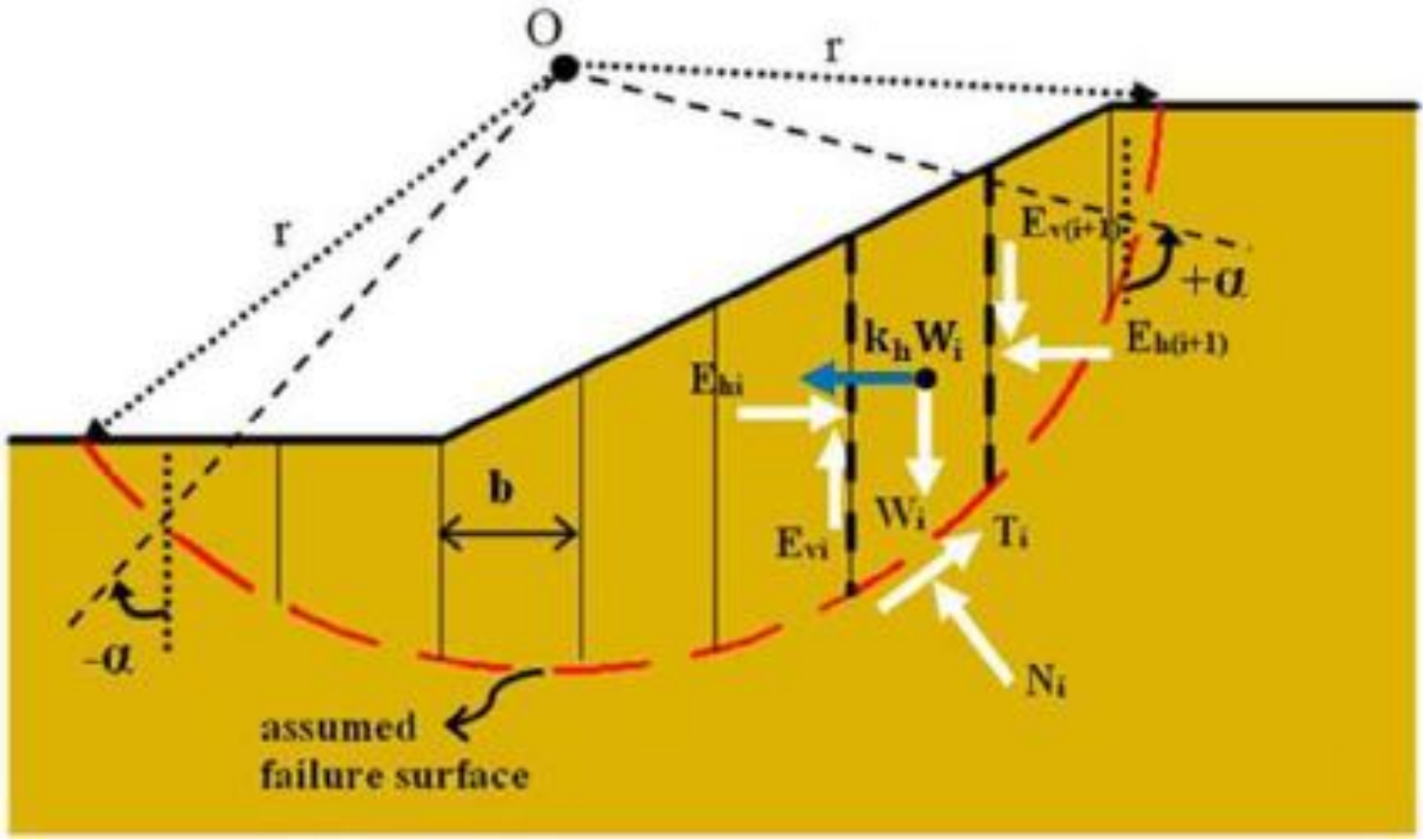
Total stress analysis for a purely cohesive soil

Swedish method of slices for soil:

Factor of safety, $F = \frac{\text{Resisting force}}{\text{Driving force}} = \frac{c.l + \Sigma N \tan\phi}{\Sigma T} = \frac{c.r.\theta + \Sigma N.\tan\phi}{\Sigma T}$

When steady seepage occurring through an embankment, critical condition is to be considered for the stability of the downstream slope.

Swedish method of slices for soil:



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$$F = \frac{c'.r.\theta + \Sigma(N - U).\tan\phi}{\Sigma T}$$

ΣN :Sum of normal components of the weights of slices

ΣT :Sum of tangential components of the weights of slices

ΣU :Total force due to pore pressure on the surface

Depth of tension crack, $z_c = \frac{2c}{\gamma}.\tan\left(45^0 + \frac{\phi}{2}\right)$

When rapid or sudden drawdown takes place in an embankment, critical condition is to be considered for the stability of the downstream slope.

- used for homogeneous or stratified soils
- used where seepage takes place and pore pressure present in the soil
- The most critical circle intersects the slope infront of the toe if and

Friction circle method

Factor of safety with respect to cohesion, assuming that the friction is mobilized in full.

$$F_c = \frac{c}{c_m}$$

The factor of safety with respect to friction, assuming that cohesion is mobilized in full.

$$F_\phi = \frac{Tan\,\phi'}{Tan\,\phi_m}$$

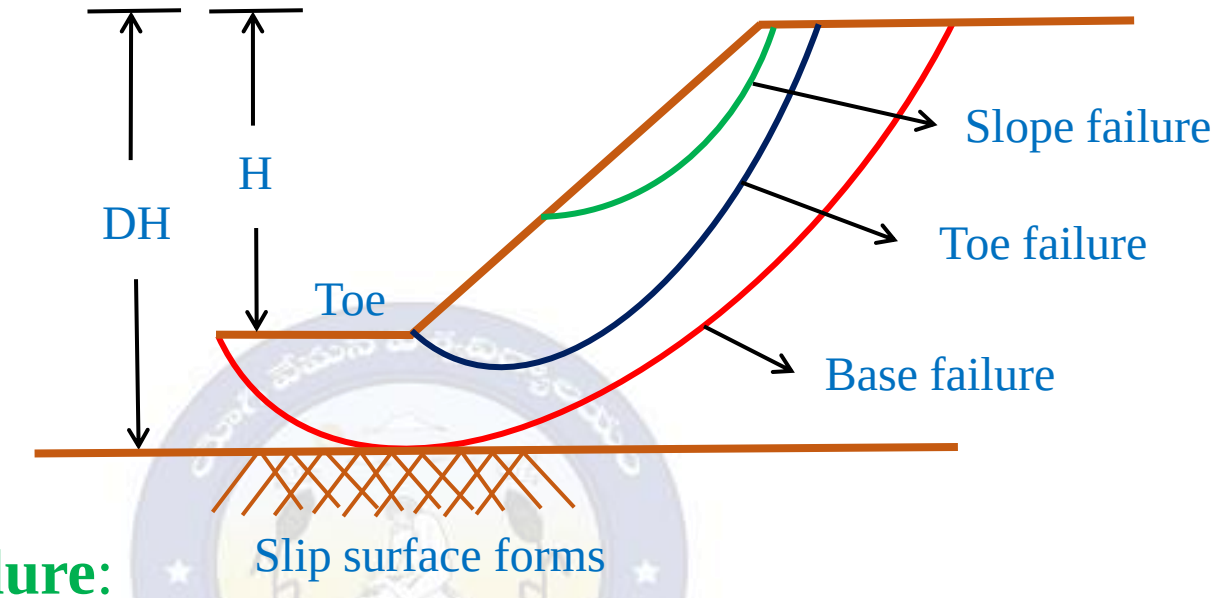
ϕ' : Effective friction angle

ϕ_m : Mobilized friction angle

The factor of safety with respect to the total shear strength,

$$F_s = \frac{s}{\tau} = \frac{\text{Shear strength}}{\text{Shear strength mobilized}} = \frac{c' + \bar{\sigma}.\tan\phi'}{c_m + \bar{\sigma}.\tan\phi_m}$$

Forms of slip surface



a. Slope failure or face failure:

- Slope failure occurs when the failure surface intersects the slope above the toe.
- It may occur when the slope angle is very high and the soil close to the toe is quite strong or the soil in the upper part of the slope is relatively weak.

b. **Toe failure**:

- Toe failure occurs when the failure surface passes through the toe.
- It may occur in steep slopes when the soil mass above the base and below the base is homogeneous
- It is the most common mode of failure

c. **Base failure**:

- Base failure occurs when the failure surface passes below the toe.
- It may occur when the soil below the toe is relatively weak and soft and the slope is flat.

Depth factor, $D = \frac{H_1}{H}$

H : Height of the slope

H_1 : Depth of the lowest point of the failure surface= $D.H$

Stability of Slopes
(Numerical Questions)

Prof. B. Jayarami Reddy

01. The following results were obtained in stability analysis by Swedish method of slices per running metre for a 10 m high embankment.

$L = 21m$

Total Shearing force : 430 kN

Total Normal force : 1650 kN

Total Neutral force : 240 kN

Length of arc : 21 m

If the properties of soil are c = 24 kN/m^2 and $\phi = 6^0$, then the factor of safety with respect to shear strength is

a. 1.20 **b.** 1.52 c. 1.58 d. 2.63

01. $\Sigma T = 430\,\text{kN}$ $\Sigma N = 1650\ \text{kN}$ $\Sigma U = 240\ \text{kN}$

$c = 24\ kN/m^2$ $\phi = 6^0$ $L = 21m$

Factor of safety, $F = \dfrac{c.L + \Sigma\left(N - U\right)\tan\theta}{\Sigma T}$, $F = \dfrac{24 \times 21 + \left(1650 - 240\right)\tan 6^0}{430} = 1.52$

02. An infinite slope of soil is likely to be subjected to seepage parallel to the slope with water level at the surface. The shear strength parameters are $c' = 0$ and $\phi = 32^0$ The saturated unit weight of soil is 20 kN/m^2 and unit weight of water is 10 kN/m^2. If the factor of safety is 1.5, then the maximum angle of slope is

a. 11.77^0 b. 19.9^0 c. 22.62^0 d. 39.8^0

02. a

Effective Cohesion, $c' = 0$

Effective angle of internal friction, $\phi' = 32^0$.

Saturated unit weight of soil, $\gamma_{sat} = 20\,\text{kN/m}^3$

When water level is at surface and seepage occurs parallel to surface,

$$\text{Factor of safety, } F = \left(\frac{\gamma'}{\gamma_{sat}}\right)\left(\frac{\tan\phi'}{\tan\beta}\right) \Rightarrow 1.5 = \frac{20-10}{20}.\frac{\tan 32^0}{\tan\beta}$$

$$\tan\beta = 0.2083 \quad \beta = 11.77^0$$

03. An infinite slope of soil having a slope angle of 12° to seepage parallel to the slope with water level well below the surface with parameters $c' = 0$ and $\phi' = 28^0$. The saturated unit weight of soil is 20 kN/m^2 and unit weight of water is 10 kN/m^2. The factor of safety for the slope is

a. 1.0 b. 1.25 **c.** 2.5 d. 5.0

03. c

Effective Cohesion, $c' = 0$

Effective angle of internal friction, $\phi' = 28^0$

Saturated unit weight of soil, $\gamma_{sat} = 20\,\text{kN/m}^2$

Unit weight of water, $\gamma_w = 10\,\text{kN/m}^2$

When the water table is well below the surface,

Factor of safety, $F = \dfrac{Tan\,\phi'}{Tan\beta} = \dfrac{Tan\ 28^0}{Tan\ 12^0} = 2.50$

04. A soil mass has slope 1 in 1.5 with a height of 6 m has the properties: $c = 24\text{kN/m}^2, \phi = 15^0$ and γ=18kN/m^3 . The Taylors stability number is given in the following table.

β / ϕ	10^0	15^0
30^0	0.075	0.046
45^0	0.108	0.083

i. The factor of safety of slope with respect to cohesion is

a. 2.68 b. 2.96 **c.** 4.04 d. 4.83

Slope angle: β $\tan\beta = \frac{1}{1.5} \Rightarrow \beta = 33.69^0$

For $\beta = 33.69$ and $\phi = 15^0$, Taylor's stability number,

$$S_n = 0.046 + \frac{0.083 - 0.046}{45 - 30}(33.69 - 30) = 0.055$$

$$S_n = \frac{c}{F_c.\gamma.H} \Rightarrow F_c = \frac{24}{0.055 \times 18 \times 6} = 4.04$$

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05. A slope 1 in 2 with a height of 6 m has the soil properties: c = 26 kN/m^2, ϕ = 10^0, γ = 18 kN/m^3. If the Taylor's stability number is 0.064, then the critical height of slope is

a. 33.9 m **b.** 22.6 m c. 11.3 m d. 6 m

05. B

c = 26 kN/m^2, ϕ = 10^0, γ = 18 kN/m^3 $S_n = 0.064$

Critical height of slope is given by $F_c = \frac{H_c}{H}$

$$H_c = F_c.H = \frac{c}{S_n.\gamma.H}.H = \frac{c}{S_n.\gamma}$$

$$H_c = \frac{26}{0.064 \times 18} = 22.6 \text{ m.}$$

06. A slope is to be laid at angle of 30^0 with the horizontal in a soil with parameters $C = 20$ kN/m^2, $\phi = 22^0$, $\gamma = 18$ kN/m^3. The Taylor's stability number is given by

β \ ϕ	15^0	22^0
30^0	0.046	0.041
45^0	0.083	0.055

If the factor of safety is 1.5, then the safe height of slope is

a. 18.06 m **b.** 16.1 m c. 19.25 m d. 8.92 m

06. b

Since factor of safety is with respect to shear strength, it is applicable to both C and ϕ.

Mobilized frictional angle is given by

$$F_\phi = \frac{\tan\phi}{\tan\phi_m} \Rightarrow \tan\phi_m = \frac{\tan\phi}{F_\phi} \Rightarrow \tan\phi_m = \frac{\tan 22^0}{1.5} \Rightarrow \phi_m = 15^0$$

For $\phi_m = 15^0$ and $\phi_m = 15^0$ $S_n = 0.046$

Taylor's stability number, $S_n = \frac{c}{F.\gamma.H} \Rightarrow H = \frac{20}{0.046 \times 1.5 \times 18} = 16.1$ m

07. A 4 m deep canal has side slope of 1:1. The properties of soil are: c_u = 24 kN/m^2, $\phi_u = 10^0$, e = 0.8 and G = 2.7. The Taylor's stability number is 0.108. When the canal runs full, the factor of safety with respect to cohesion is

a. 2.86 m b. 3.12 m c. 5.55 m **d.** 5.88 m

07. d

When canal runs full, the side slopes are submerged.

Taylor's stability number, $S_n = \dfrac{c}{F_c.\gamma'.H}$

c_u = 24 kN/m^2, $\phi_u = 10^0$, G = 2.7

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.7+0.8}{1+0.8}\right)10 = 19.44 \text{ kN/m}^3$$

$$\gamma' = \gamma_{sat} - \gamma_w = 19.44 - 10 = 9.44 kN/m^3$$

$$F_c = \frac{24}{0.108 \times 9.44 \times 4} = 5.88$$

08. A canal with a depth of 4 m has banks with slope 1:1. The properties of soil are c = 18 kN/m^2, ϕ =15^0, e = 0.7, G = 2.7. The stability number for slope angle of 45° is varied with ϕ as shown below.

ϕ	5^0	7.27^0	15^0
S_n	0.136	0.122	0.083

i. When the canal runs full, the factor of safety of slope with respect to cohesion is

a. 1.90 b. 2.79 c. 3.92 **d.** 5.76

08. i. d

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.6+0.7}{1+0.7}\right)10 = 19.41\,\text{kN/m}^3$$

$$\gamma' = \gamma_{\text{sat}} - \gamma_{\text{w}} = 19.41 - 10 = 9.41\,\text{kN/m}^3$$

For β = 45^0 and ϕ = 15^0, S_n = 0.083

Stability number, $S_n = \dfrac{c}{F_c.\gamma'.H}$ $\qquad F_c = \dfrac{18}{0.083 \times 9.41 \times 4} = 5.76$

ii. When the canal is suddenly and completely emptied, the factor of safety with respect to Cohesion is

a. 1.90 b. 2.79 c. 3.92 d. 4.76

08. ii. a

For sudden drawdown condition, ϕ_w is to be obtained for slope angle and weighted frictional angle .

$$\phi_w = \frac{\gamma'}{\gamma_{sat}}.\phi = \frac{9.41}{19.41} \times 15^0 = 7.27^0$$

For $\beta = 45^0$ and $\phi_w = 7.27^0$, $S_n = 0.122$

$$S_n = \frac{c}{F_c.\gamma_{sat}.H} \Rightarrow F_c = \frac{c}{S_n.\gamma_{sat}.H} \Rightarrow F_c = \frac{18}{0.122 \times 19.41 \times 4} = 1.90$$

09. Fig shows as slip surface with a radius of 22 m. The slope is 1:1 with a height of 12 m. The soil properties are c = 30 kN/m^2, ϕ = 0^0 and γ = 20 kN/m^3. Using friction circle method, the factor of safety with respect to cohesion is…….

$Slope = 1:1$ $\quad H = 12$ m $\quad r = 22$ m

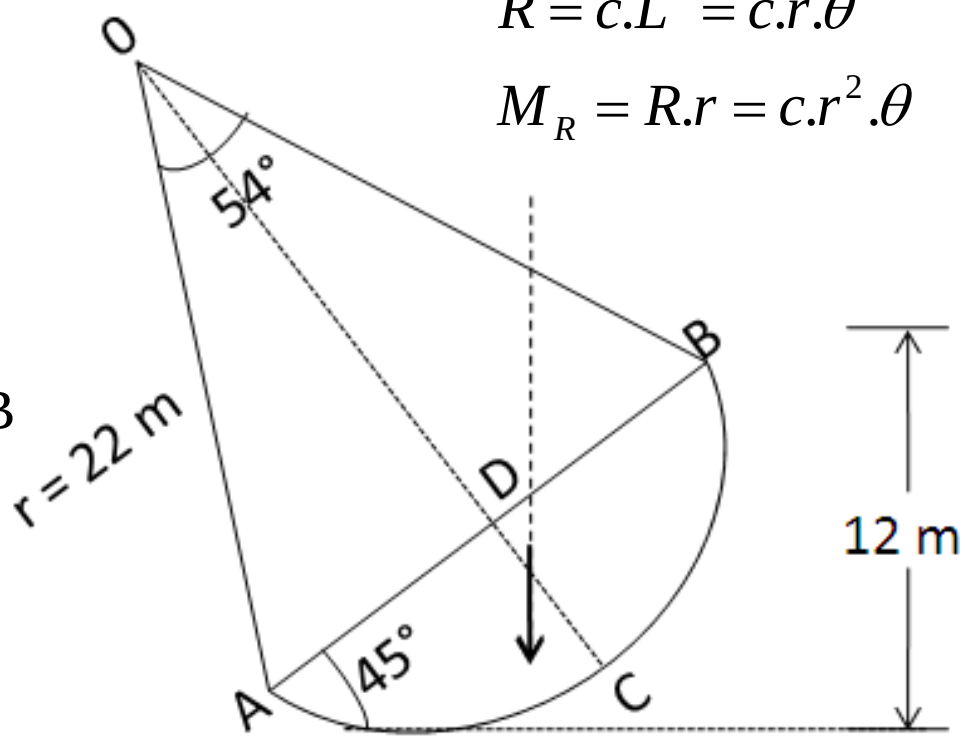
$\phi = 0^0$ $\quad c = 30$ kN/m^2 $\quad \gamma = 20$kN/m^3

$$F = \frac{\text{Resisting moment}}{\text{Sliding moment}} = \frac{c.r^2.\theta}{W.e}$$

$$\theta = 54^0 \times \frac{\pi}{180}$$

$$R = c.L = c.r.\theta$$

$$M_R = R.r = c.r^2.\theta$$



Area ABC = Area of section OACB - Area of Δle OAB

$$\sin 45^0 = \frac{12}{AB} \Rightarrow AB = 16.97m$$

$$AD = \frac{1}{2} \times 16.97 = 8.48 \qquad OD = 22 \times \cos 27^0 = 19.6m$$

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$$\text{Area ABC} = \pi \times 22^2 \times \frac{54}{360} - \frac{1}{2} \times 16.97 \times 19.6 = 161.77 m^2$$

$$\text{Weight of soil mass, W} = \text{Area} \times 1 \times \gamma = 61.77 \times 1 \times 20 = 1235.4\,\text{kN}$$

$$F = \frac{c.r^2.\theta}{W.e} \text{ (or) } \frac{c.r^2.\theta}{W.\sin\beta.r} = \frac{c.r.\theta}{W.\sin\beta}$$

$$= \frac{30 \times 22 \times 54 \times \frac{\pi}{180}}{1235.4 \times \sin 45^0}$$

$$F = 0.71$$

$$F = \frac{c.r^2.\theta}{W.e}$$

$$= \frac{30 \times 22^2 \times 54 \times \frac{\pi}{180}}{1235.4 \times 95}$$

$$F = 1.166$$

10. A new canal is to be excavated to a depth of 4 m with banks having slope 1:1. The properties of soil are c = 16 kN/m², ϕ =20°, e = 0.65 and G = 2.65. The Taylor's stability number is given in the following table for different values of β for ϕ = 20°.

β	30^0	45^0	60^0	75^0	90^0
S_n	0.052	0.062	0.097	0.134	0.182

When the canal is running full, the factor of safety with respect to Cohesion is

a. 7.69 **b.** 6.45 c. 4.12 d. 2.62

10. b

When the canal is running full, $\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.65+0.65}{1+0.65}\right)\times 10 = 20 \text{ kN/m}^3$

$\gamma' = 20 - 10 = 10 \text{ kN/m}^3$ For $\beta = 45^0$ $S_n = 0.062$

Factor of safety, $F_c = \frac{c}{S_n.\gamma'.H}$ $F_c = \frac{16}{0.062\times 10\times 4} = 6.45$

11. What will be the change in the factor of safety if the slope is changed to be at 30^0 to the vertical?

a. 7.69 b. 6.45 **c.** 4.12 d. 2.62

11. c

β = Angle of slope with respect to horizontal = 90 – 30 = 60

For $\beta = 60^0$, $S_n = 0.097$

$$F = \frac{16}{0.097 \times 10 \times 4} = 4.12$$

Stability of Slopes
(Previous GATE Questions)

Prof. B. Jayarami Reddy

01. A fully submerged infinite sandy slope has an inclination of 30° with the horizontal. The saturated unit weight and effective angle of internal friction of sand are 18 kN/m^3 and 38°, respectively. The unit weight of water is 10 kN/m^3. Assume that the seepage is parallel to the slope. Against shear failure of the slope, the factor of safety (round off to two decimal places) is…… GATE CE 2020

01. 0.60

Inclination of slope, $\beta = 30^o$

Saturated unit weight, $\gamma_{sat} = 18 KN/m^3$

Angle of internal friction, $\phi = 38^o$

Unit weight of water , $\gamma_w = 10 KN/m^3$

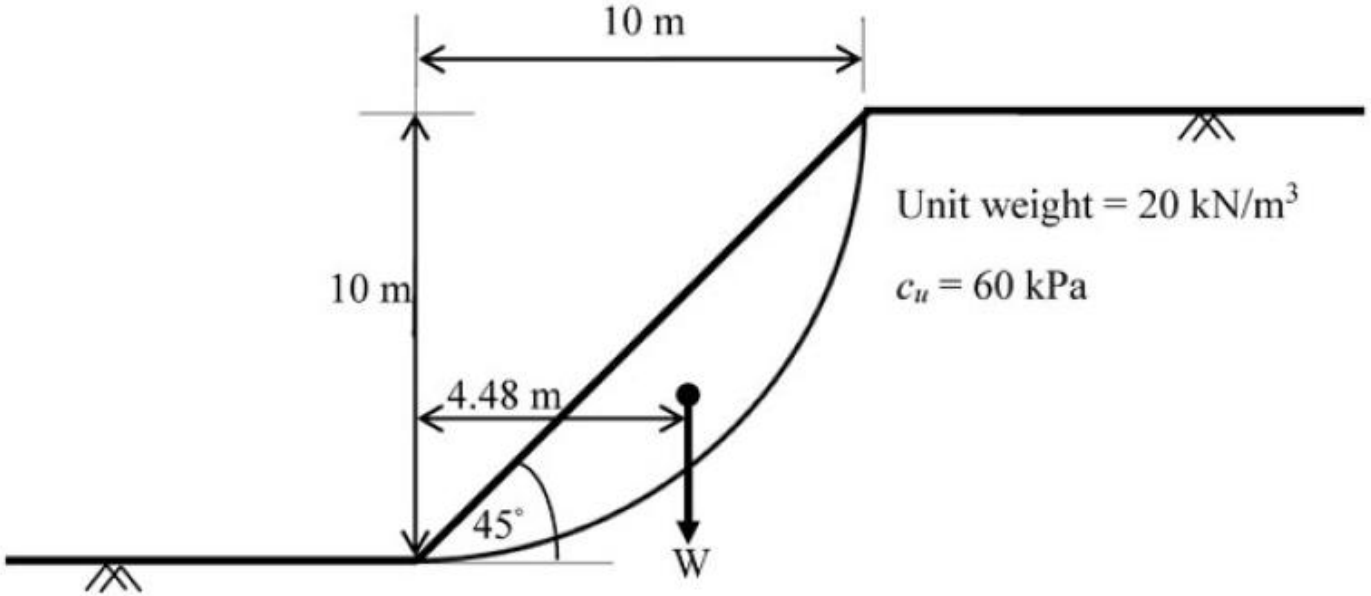
Factor of safety against shear failure of slope, F = ?

When see page is parallel to the slope, factor of safety against shear failure,

$$F = \frac{\gamma'}{\gamma_{sat}} . \frac{\tan\phi}{\tan\beta} \qquad F = \frac{18-10}{18} . \frac{\tan 38^o}{\tan 30^o} = 0.601$$

02. A 10 m high slope of dry clay soil (unit weight = 20 kN/m^3), with a slope angle of 45° and the circular slip surface, is shown in the figure (not drawn to the scale). The weight of the slip wedge is denoted by W. The undrained unit cohesion (c_u) is 60 kPa. The factor of safety of the slope against slip failure, is GATE CE 2020

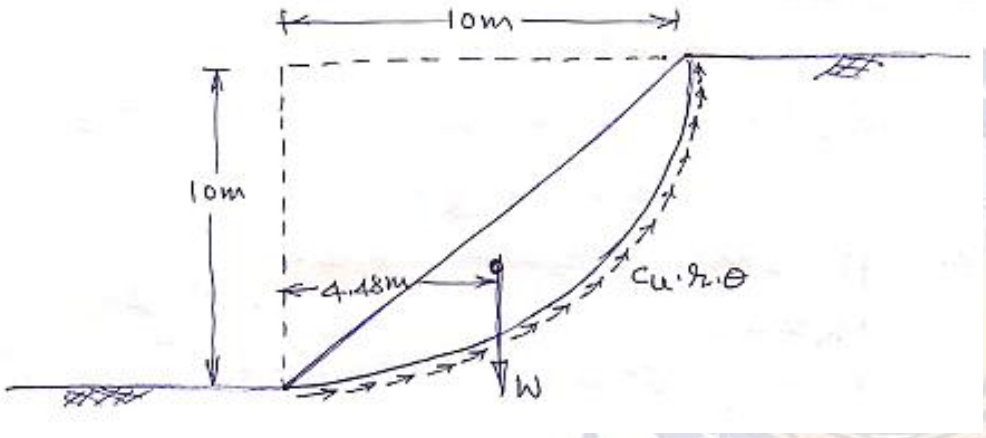
a. 1.84 b. 1.57 c. 0.58 d.1.67



Unit weight of soil, $\gamma = 20 KN/m^3$

Undrained unit cohesion, $C_u = 60\,KPa$

Factor of safety of the slope against the slip failure, F = ?



Resisting moment per unit width, $M_R = c_u.L.r = c_u.r.\theta.r$

$$M_R = 60 \times \frac{\pi \times 10}{2} \times 10 = 9424.8 KN/m$$

Weight of wedge, W = Area.1.r = $\left(\frac{\pi \times 10^2}{4} - \frac{1}{2} \times 10 \times 10\right) \times 20 = 570.8 KN$

Overturning moment, $M_0 = W.\bar{x} = 570.8 \times 4.48 = 2557.2 KNm/m \Rightarrow F = \frac{M_R}{M_0} = \frac{9424.2}{2557.2} = 3.01$

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03.An earthen dam of height H is made of cohesive soil whose cohesion and unit weight are c and γ, respectively. If the factor of safety against cohesion is , the Taylor's stability number (S_n) is CE2 2019

a. $\frac{\gamma H}{cF_c}$ b. $\frac{cF_c}{\gamma H}$ **c.** $\frac{c}{F_c \gamma H}$ d. $\frac{F_c \gamma H}{c}$

03.c

H :Height of earth dam

c : Cohesion of soil

γ : Unit weight of soil

F_C :Factor of safety against cohesion

S_n : Taylor's stability number, $S_n = \frac{c}{F_c \gamma H}$

04.A granular soil has a saturated unit weight of 20 kN/m^3 and an effective angle of shearing resistance of 30°. The unit weight of water is 9.81 kN/m^3. A slope is to be made on this soil deposit in which the seepage occurs parallel to the slope up to the free surface. Under this seepage condition for a factor of safety of 1.5, the safe slope angle (in degree, round off to 1 decimal place) would be….. CE1 2019

04. 11.1

Saturated unit weight of soil, γ_{sat} =20kN/m^3

Effective angle of shearing resistance, ϕ =30^0

Unit weight of water, γ_w = 9.81kN/m^3

Seepage occurs parallel to the slope up to the free surface.

$$\gamma' = \gamma_{sat} - \gamma_w = 20-9.81=10.19\text{kN/m}^3$$

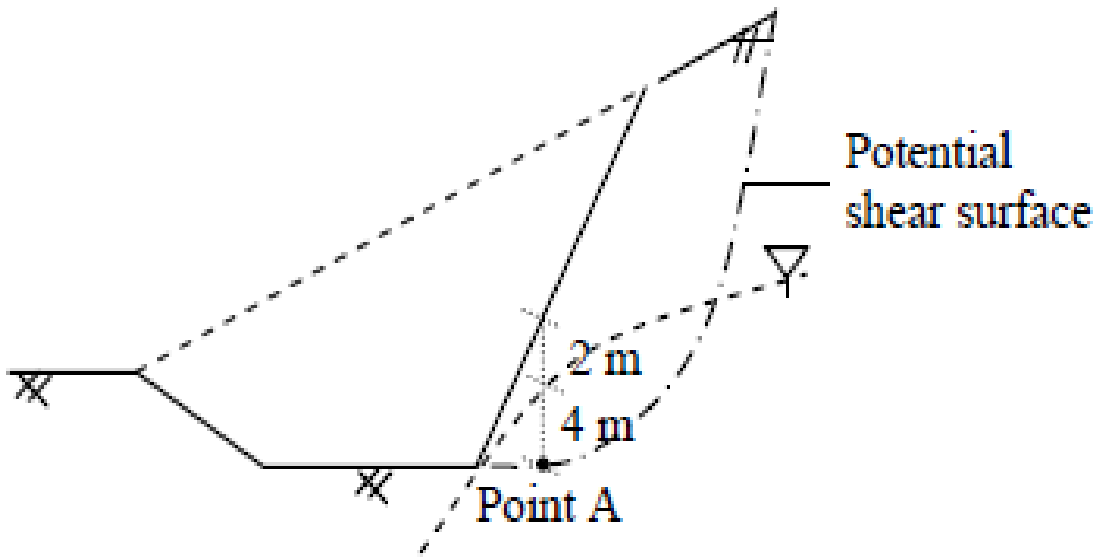
Factor of safety, F =1.5

Safe slope angle, $\beta = ?$

For an infinite slope, $F = \frac{\gamma'}{\gamma_{sat}} . \frac{\tan\phi}{\tan\beta}$

$$1.5 = \frac{10.19}{20} . \frac{\tan 30^0}{\tan\beta} \Rightarrow \tan\beta = 0.1961 \Rightarrow \beta = 11.090 = 11.10$$

05. For the construction of a highway, a cut is to be made as shown in the figure.



The soil exhibit $c' = 20\text{kPa}, \phi' = 18^0$, and the undrained shear strength = 80 kPa. The unit weight of water is 9.81 kN/m^3. The unit weights of the soil above and below the ground water table are 18 and 20 kN/m^3, respectively. If the shear stress at Point A is 50 kPa, the factors of safety against the shear failure at this point, considering the undrained and drained conditions, respectively. Would be

a. 1.6 and 0.9 b. 0.9 and 1.6 c. 0.6 and 1.2 d. 1.2 and 0.6 CE2 2017

05. a

Soil properties, $c' = 20\text{kPa}$, $\phi' = 18^0$

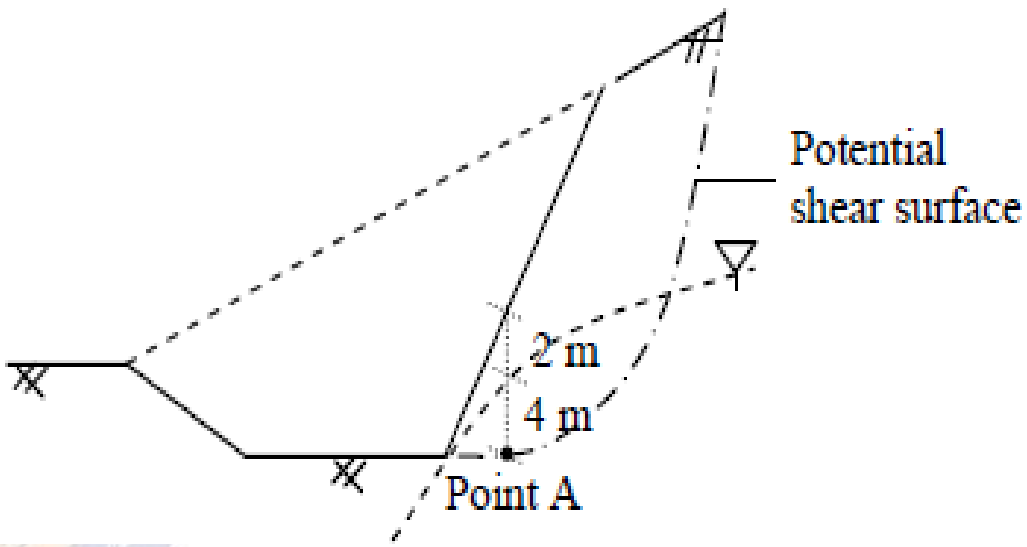
Undrained shear strength = 80 kPa

Unit weight of water, γ_w =9.81 kN/m^3

Unit weight of soil = 18 kN/m^3

Unit weight of saturated soil, γ_{sat} =20 kN/m^3

Shear stress at point A, τ_A = 50 kPa



Case 1: Undrained condition

$$F = \frac{\text{Resisting shear strength}}{\text{Actual shear stress}} = \frac{80}{50} = 1.6$$

Case 2: Drained condition

$$F = \frac{c' + \bar{\sigma}\tan\phi'}{\text{Actual shear stress}}$$

$\bar{\sigma}$: Stress at point A = 2 × 18 + 4 (20-9.81) = 76.76 kN/m^2.

$$F = \frac{20 + 76.76\tan 18^0}{50} = 0.899 \approx 0.9$$

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06. The infinite sand slope shown in the figure is on the verge of sliding failure. The ground water table coincides with the ground surface. Unit weight of water $\gamma_w = 9.81$ kN/m^3. The value of the effective angle of internal friction (in degrees, up to one decimal place) of the sand is …… CE1 2017

06. 34.34^0

For the infinite slope subjected to steady seepage parallel to the slope.

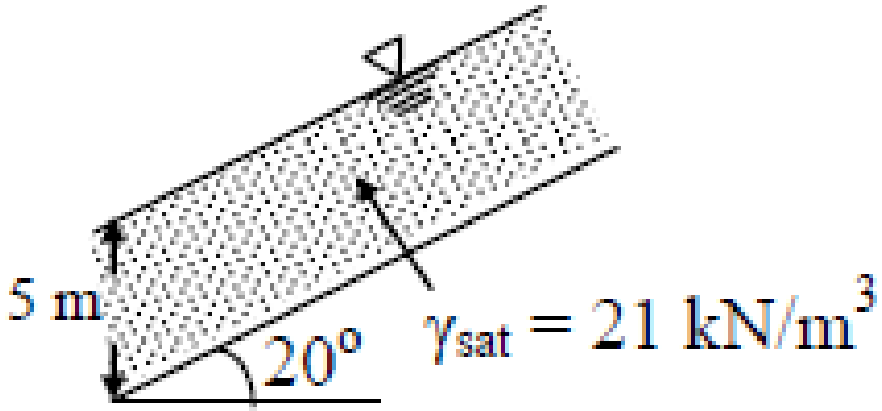
Saturated unit weight of soil, $\gamma_{sat} = 21$ kN/m^3

Unit weight of water, $\gamma_w = 9.81$ kN/m^3

Angle of the infinite slope with horizontal, $\beta = 20^0$

$$F = \frac{\gamma'}{\gamma_{sat}} \cdot \frac{\tan\phi}{\tan\beta}$$

$$1 = \frac{21 - 9.81}{21} \cdot \frac{\tan\phi}{\tan 20^0} \Rightarrow \tan\phi = 0.683 \Rightarrow \phi = 34.34^0$$



07. In friction circle method of slope stability analysis, if r defines the radius of the slip circle, the radius of friction circle is CE2 2015

a. $r\sin\phi$ b. r c. $r\cos\phi$ d. $r\tan\phi$

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08. An infinitely long slope is made up of a $c-\phi$ soil having the properties: cohesion (C) = 20 kPa, and dry unit weight (γ_d) = 16 kN/m^3. The angle of inclination and critical length of the slope are 40° and 5 m respectively. To maintain the limiting equilibrium, the angle of internal friction of the soil (in degrees) is ___ CE2 2014

08. 22.44

Cohesion of soil, $c = 20\,\text{kPa}$

Dry unit weight, $\gamma_d = 16\,\text{kN/m}^3$

Angle of inclination of slope, $\beta = 40^0$

Critical length of slope $= 5\,\text{m}$

Angle of internal friction of the soil, $\phi = ?$

For limiting equilibrium, factor of safety, $F = 1$

Critical Height, $H_c = \dfrac{c}{\gamma(\tan\beta - \tan\phi)\cos^2\beta}$

$$5 = \frac{20}{16(\tan 40^0 - \tan\phi)\cos^2 40^0} \qquad \tan 40^0 - \tan\phi = 0.426$$

$$\tan\phi = 0.413 \qquad \phi = 22.44^0$$

(OR)

Factor of safety, $F = \dfrac{\text{shear resistance}}{\text{shear stress}}$, $\dfrac{\tau_f}{\tau} = \dfrac{c + \sigma_n \tan\phi}{\tau}$

$$F = \frac{c + \gamma z \cos^2\beta . \tan\phi}{\gamma z \cos\beta \sin\beta} \Rightarrow 1 = \frac{20 + 16 \times 5 \cos^2 40^0 \tan\phi}{16 \times 5 \cos 40^0 \sin 40^0}$$

$$\tan\phi = 0.413 \Rightarrow \phi = 22.44^0$$

09. A long slope is formed in a soil with shear strength parameters: $c' = 0$ and $\phi' = 34^0$. A firm stratum lies below the slope and it is assumed that water table may occasionally rise to the surface, with seepage taking place parallel to the slope. Use $\gamma_{sat} = 18\,\text{kN/m}^3$ and $\gamma_w = 10\,\text{kN/m}^3$. The maximum slope angle (in degrees) to ensure a factor of safety of 1.5, assuming a potential failure surface parallel to the slope, would be

CE1 2014

a. 45.3 b. 44.7 c. 12.3 **d.** 11.3

09. d

Cohesion, $c' = 0$

Angle of internal friction, $\phi' = 34^0$

Saturated unit weight of soil, $\gamma_{sat} = 18\,\text{kN/m}^3$

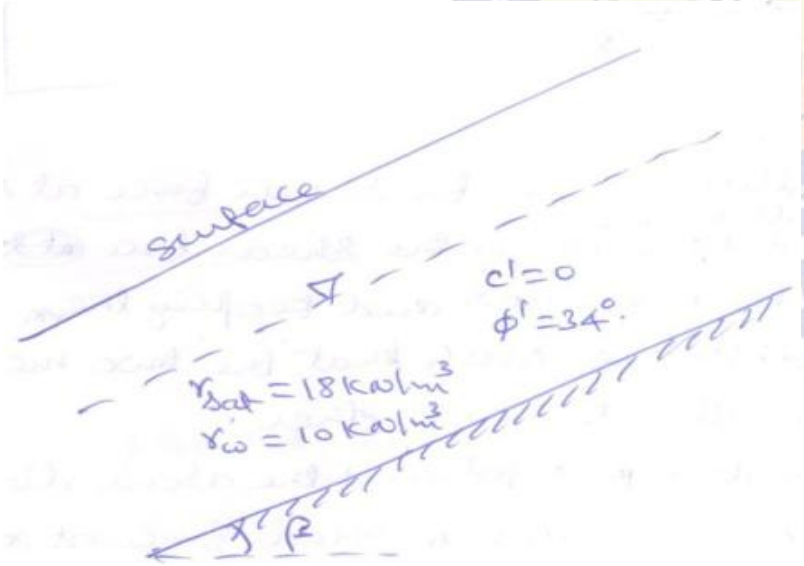
Unit weight of water, $\gamma_w = 10\,\text{kN/m}^3$

Maximum slope angle, $\beta = ?$

Factor of safety, $F = 1.5$

Factor of safety of failure surface when water table rises to the surface and parallel to the slope, $F = \frac{\gamma'}{\gamma_{sat}} . \frac{\tan\phi}{\tan\beta}$

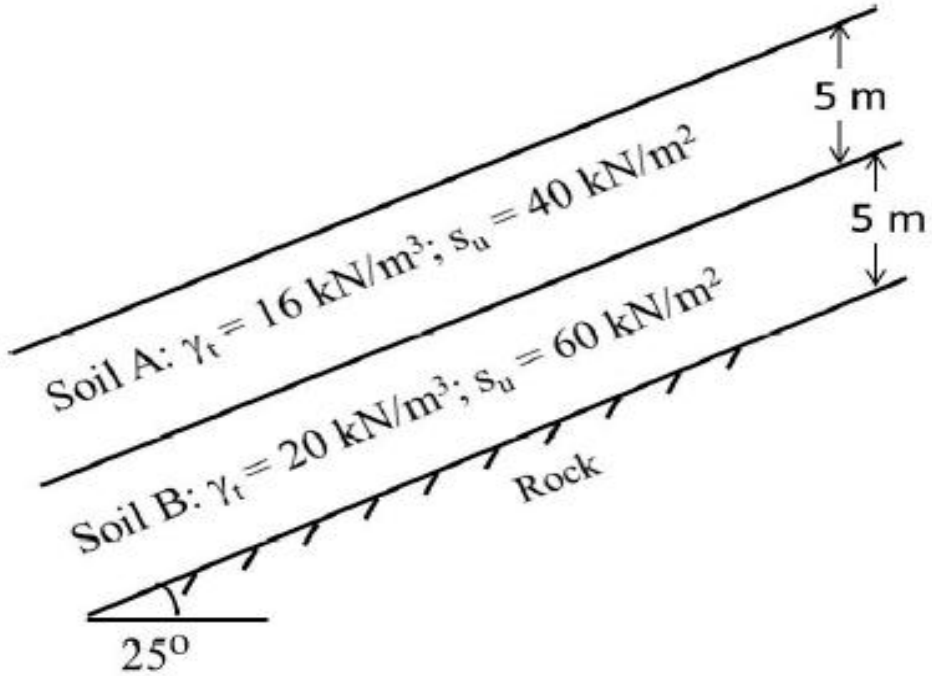
$$1.5 = \frac{(18-10)}{18} . \frac{\tan 34^0}{\tan\beta} \Rightarrow \tan\beta = 0.2 \Rightarrow \beta = 11.3^0$$



10. The soil profile above the rock surface for a 25° infinite slope is shown in the figure, where s_u is the undrained shear strength and γ_t is total unit weight. The slip will occur at a depth of 2013

a. 8.83 m b. 9.79 m c. 7.83 m d. 6.53 m

10. a



Slope of the rock surface, $\beta = 25^0$

For Soil A

Undrained shear stress, s_u = 40 kN/m^2

Total unit weight of soil, γ_{t1}= 16 kN/m

Height of soil, z_1 = 5m

For soil B

Undrained shear stress, s_u = 60 kN/m^2.

Total unit weight of soil, γ_{t2}=20 kN/m^2

Height of soil, z_2 = 5m

The slip will occur at a depth when the shear stress is greater than or equal to the shear strength of soil. $\tau \geq s_u$

$$(\gamma_1 z_1 + \gamma_2 z_2)\sin\beta.\cos\beta \geq s_u$$

$$(16\times 5 + 20\times z_2)\sin 25^0.\cos 25^0 \geq 60 \qquad z_2 \geq 3.83m$$

Therefore, depth of slip = 5 + 3.83 = 8.83m

11. The factor of safety of an infinite soil slope shown in the figure having the properties $c = 0$, $\phi = 35^0$, γ_{dry} = 16 KN/m^3 and γ_{sat} = 20 KN/m^3 is approximately equal to 2007

a. 0.70 b. 0.80

c. 1.00 d. 1.20

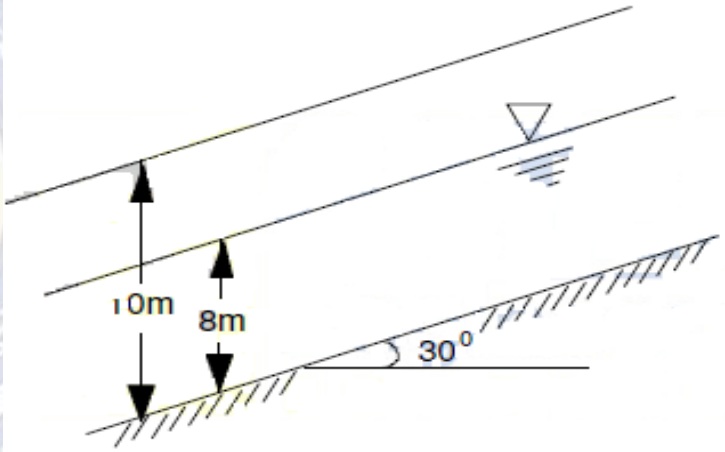
11. a

Cohesion, $c = 0$

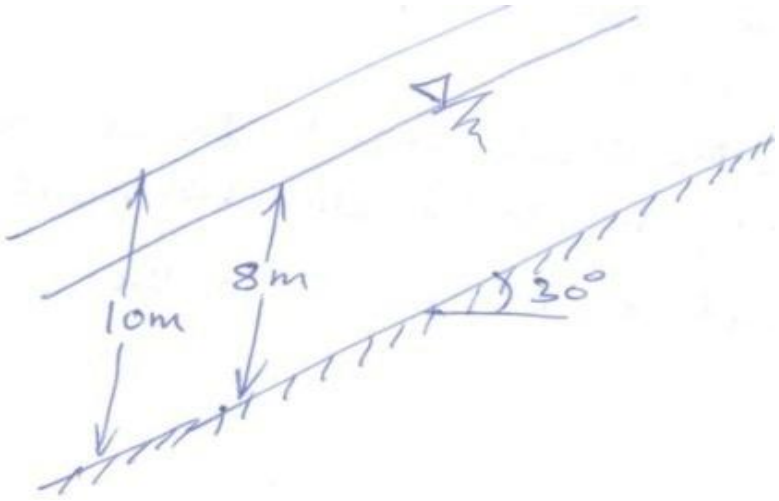
Angle internal friction, $\phi = 35^0$

Dry density of soil, γ_{dry} = 16 kN/m^3

Saturated density of soil, γ_{sat} = 20 kN/m^3



Factor of safety of an infinite slope, $F = \left(1 - \frac{\gamma_w h}{\gamma z}\right)\frac{\tan\phi'}{\tan\beta}$



Depth from the top of the slope, z = 10 m
Height of water above the failure plane, h = 8 m
Unit weight of water, γ_w = 10 kN/m^3
Angle of slope, β = 30°

$$F = \left(1 - \frac{10 \times 8}{20 \times 10}\right)\frac{\tan 35^o}{\tan 30^o} = 0.73$$

12. List-I below gives the possible types of failure for a finite soil slope and List-II gives the reasons for these different types of failure. Match the items in List-I with the items in List-II. 2006

List-I	**List-II**
P. Base failure	1. Soils above and below the toe have same strength
Q. Face Failure	2. Soil above the toe is comparatively weaker
R. Toe failure	3. Soil above the toe is comparatively stronger

a. P1 Q2 R3 b. P2 Q3 R1 c. P2 Q1 R3 **d.** P3 Q2 R1

12. d

Base failure can occur when the soil below the toe is relatively weak and soft and the slope is flat.

Face failure or slope failure can occur when the slope angle is very high and the soil close to the toe is quite strong or the soil in the upper part of slope is relatively weak.

Toe failure occurs in steep slopes when the soil mass above the base and below the base is homogeneous (same strength)

Type of failure	Soil condition
Base failure	Soil above the toe is comparatively stronger
Face failure	Soil above the toe is comparatively weaker
Toe failure	Soil above and below the toe have same strength

13. For two infinite slopes (one in dry condition and other in submerged condition) in a sand deposit having the angle of shearing resistance 30^0, factor of safety was determined as 1.5 (for both slopes). The slope angles would have been 2005

a. 21.05^0 for dry slope and 21.05^0 for submerged slope.

b. 19.47^0 for dry slope and 18.40^0 for submerged slope.

c. 18.4^0 for dry slope and 21.05^0 for submerged slope.

d. 22.6^0 for dry slope and 19.47^0 for submerged slope.

13. a

The factor of safety for the stability of an infinite slope in completely dry cohesionless soil is

$$F.S = \frac{\tan\phi}{\tan\beta} \qquad \tan\beta = \frac{\tan 30^0}{1.5} = 0.385 \Rightarrow \beta = 21.05^0$$

For submerged condition, $F.S = \dfrac{Tan\,\phi'}{Tan\,\beta}$ $\qquad \tan\beta = \dfrac{\tan\phi'}{1.5} = 0.385 \Rightarrow \beta = 21.05^0$

14. An infinite soil slope with an inclination of 35^0 is subjected to seepage parallel to its surface. The soil has $c' = 100$ KN/m^2 and $\phi' = 30^o$. Using the concept of mobilized cohesion and friction, at a factor of safety of 1.5 with respect to shear strength, the mobilized friction angle is 2004

a. 20.02^o **b.** 21.05^o c. 23.33^o d. 30.00^o

14. b

Angle of inclination of infinite soil slope, $\beta = 35^0$

Cohesion, $c' = 100\,\text{KN/m}^2$

Angle of internal friction, $\phi' = 30^0$

Factor of safety, $F_s = 1.5$

The factor of safety with respect to shear strength is given by

$$F_s = \frac{\tau_f}{\tau} = \frac{c' + \sigma_n' \tan\phi'}{\tau}$$

The shear strength mobilized at every point on the slip surface may be written as

$$\tau = \frac{c'}{F_s} + \sigma'_n \frac{\tan\phi'}{F_s} = c_m + \sigma'_n \tan\phi_m$$

$$c_m = \frac{c'}{F_s} \; ; \quad \tan\phi_m = \frac{\tan\phi'}{F_s}$$

$$\text{Tan}\,\phi_m = \frac{\tan 30^0}{1.5} = 0.3849 \Rightarrow \phi_m = 21.05^0$$

Data for Q.15-16 are given below. Solve the problems and choose correct answers.
A canal having side slopes 1:1 is proposed to be constructed in a cohesive soil to a depth of 10 m below the ground surface. The soil properties are : $\phi_u = 15^0$,c_u =12 kPa, e = 1.0, G = 2.65.

15. If there is a sudden drawdown of water in the canal and if Taylor's stability number for the reduced value of ϕ_v is 0.126, the factor of safety with respect to cohesion against the failure of bank slopes will be 2003

a. 1.85 b. 1.18 c. 0.84 **d.** 0.5

15. d

If there is sudden drawdown of water in the canal, the soil remains saturated for sometime. Therefore saturated unit weight will be considered for computations.

$\gamma_{sat} = \gamma_{sub} + \gamma_w$ =8.09 + 9.81= 17.9 kN/m^3

Taylor's stability number, S_n = 0.126

Taylor's stability number, $S_n = \dfrac{c_u}{F_c.\gamma_{sat}.H} = \dfrac{c_u}{S_n.\gamma_{sat}.H} = \dfrac{12}{0.126 \times 17.9 \times 10} = 0.53$

16. If Taylor's Stability number S_n is 0.08 and if the canal flows full, the factor of safety with respect to cohesion against failure of the canal bank slopes will be

a. 3.7 **b.** 1.85 c. 1.0 d. None of these 2003

16. b

Side slope of canal = 1:1

Depth of canal, = 10 m

$\phi_u = 15^0$, $c_u = 12$ kPa, $G = 2.65$, $e = 1$

Taylors stability number, $S_n = 0.08$

When the canal is full, the canal bank slopes will be submerged under water.

$$\gamma_{sub} = \left(\frac{G-1}{1+e}\right)\gamma_w = \left(\frac{2.65-1}{1+1}\right)9.81 = 8.09\,\text{kN/m}^3$$

Taylors stability number, $S_n = \dfrac{c_u}{F_c.\gamma_{sub}.H} = \dfrac{c_u}{S_n.\gamma_{sub}.H} = \dfrac{12}{0.08\times 8.09\times 10} = 1.85$

17. A granular soil possesses saturated density of 20 kN/m^3. Its effective angle friction is 35 degrees. If the desired factor of safety is 1.5, the safe angle of slope for this soil, when seepage occurs at and parallel to the slope surface, will be

a. 25° b. 23° c. 20° **d.** 13° 2003

17. d

Saturated density of soil, γ_{sat} = 20 kN/m^3

Effective angle of internal friction, $\phi' = 35^0$

Factor of safety, F = 1.5

Angle of slope of the soil = β

$$F = \frac{\gamma' \tan\phi'}{\gamma_{sat}.\tan\beta}$$

$$1.5 = \frac{(20-10)\tan 35^0}{20\tan\beta} \Rightarrow \tan\beta = 0.2334 \Rightarrow \beta = 13.140 \;\; ; \;\; 13^0$$

18. An infinite slope is to be constructed in a soil. The effective stress strength parameters of the soil are $c' = 0$ and $\phi' = 30^0$. The saturated unit weight of the slope is 20 KN/m^3 and the unit weight of water is 10 KN/m^3. Assuming that seepage is occurring parallel to the slope, the maximum slope angle for a factor of safety of 1.5 would be 2002

a. 10.89^0 b. 11.30^0 c. 12.48^0 d. 14.73^0

18. a

Effective cohesion, $c' = 0$

Effective angle of friction, $\phi' = 30^0$

Saturated unit weight of soil, γ_{sat}=20 KN/m^2

Factor of safety, $F = 1.5$

Slope angle, $\beta = ?$

Seepage occurring parallel to the slope

For cohesionless soils, $F = \dfrac{\gamma' \tan\phi'}{\gamma_{sat}.\tan\beta} \Rightarrow 1.5 = \dfrac{10}{20}.\dfrac{\tan 30^0}{\tan\beta} \Rightarrow \tan\beta = 0.1924$

;

$\beta = 10.89^0$

19. The critical slip circle for a slope is shown below along with the soil properties. The length of the arc of the slip circle is 15.6 m and area of soil within the slip circle is 82 m^2. The radius of the slip circles is 10.3 m. The factor of safety against the slip circle failure is nearly equal to 2001

a. 1.05 b. 1.22 c. 0.78 **d.** 1.28

19. d

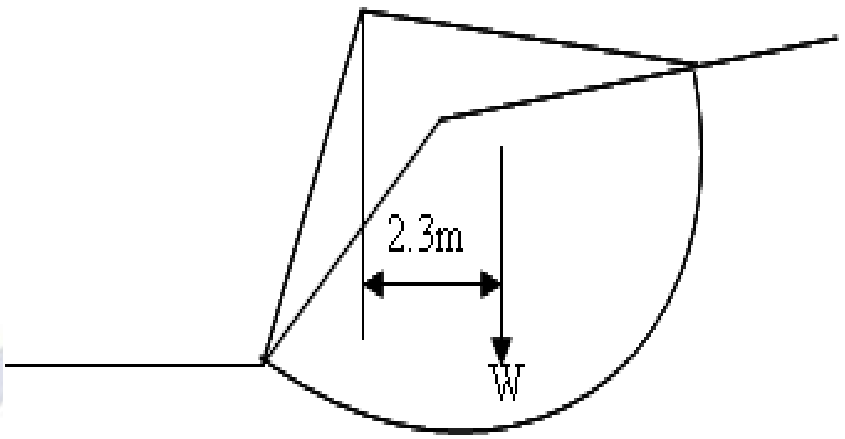
Cohesion, $c_u = 30\ \text{kN/m}^2$

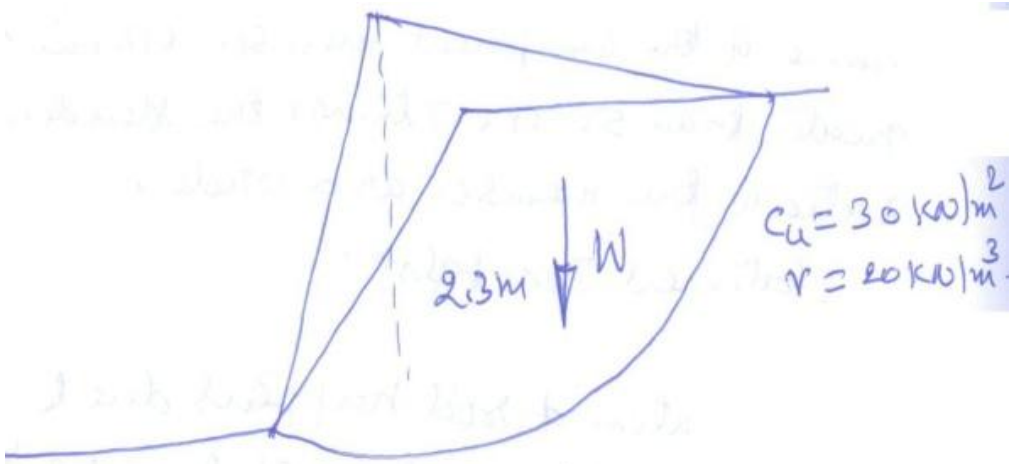
Density of soil, $\gamma = 20\ \text{kN/m}^3$

Length of the arc of the slip circle, $L = r.\theta = 15.6\ \text{m}$

Area of soil within the slip circle, $A = 82\ \text{m}^2$

Radius of the slip circle, $r = 10.3\ \text{m}$





slip circle for a slope.

Eccentricity from centre of rotation : e

Factor of safety against the slip circle failure, $F = \dfrac{\text{Restoring moment}}{\text{Sliding moment}}$

Sliding moment $= W.e$

Restoring moment $= c_u.L.r$

$$F = \frac{c_u.L.r}{W.e} = \frac{30 \times 15.6 \times 10.3}{82 \times 1 \times 20 \times 2.3} = 1.28$$

20. With respect to a $c-\phi$ soil in an infinite slope, identify if the following two statements are TRUE or FALSE. 2001

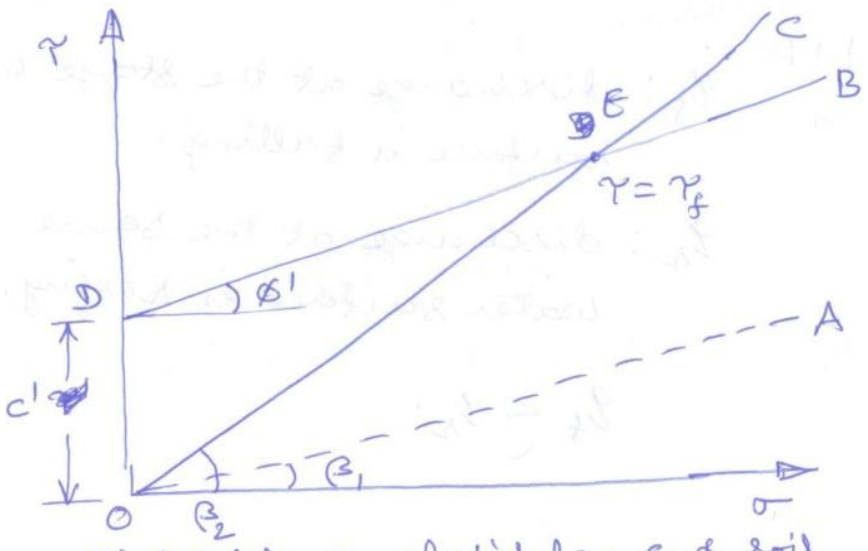
I. The stable slope angle can be greater than ϕ .

II. The factor of safety of the slope does not depend on the height of soil in the slope.

a. Both statements are FALSE. **b.** I is TRUE but II is FALSE.

c. I is FALSE but II is TRUE d. Both statements are TRUE

21. b



Stability analysis for c-φ soil

For $c-\phi$ soil in an infinite slope,

If slope angle $\beta_1 < \phi'$, slope is stable whatever be its depth.

If slope angle $\beta_2 > \phi'$, slope is stable up to a certain depth where shear stress (τ) is equal to the shear strength $(\tau)_f$. ie. up to E, the slope is stable.

E is the point of intersection of the slope line and shear strength line.

Statement I is true.

Factor of safety, $F_c = \dfrac{c'}{S_n.\gamma.H}$

Factor of safety depends on the height of soil in the slope.

Statement II is false.

22. A deep cut of 7 m has to be made in a clay with unit weight 16 KN/m^3 and a cohesion of 25 KN/m^2. What will be the factor of safety if one has to have a slope angle of ? Stability number is given to be 0.178 (from Taylor's chart) for a depth factor of 3. 2000

a. 0.80 b. 1.1 **c.** 1.25 d. 1.0

22. c

Depth of cut $= 7\,\text{m}$

Unit weight of soil, $\gamma = 16\ \text{kN/m}^3$

Cohesion, $c = 25\ \text{kN/m}^2$

Slope angle, $\beta = 30^0$

Factor of safety, $F_c = ?$

Taylors stability number, $S_n = 0.178$

$$S_n = \frac{c}{F_c.\gamma.H} \qquad F_c = \frac{25}{0.178 \times 16 \times 7} = 1.254$$

23. A 40^0 slope is excavated to a depth of 10 m is a deep layer of saturated clay of unit weight 20 KN/m^3; the relevant shear strength parameters are c_u=72 kN/m^2 and $\phi_u = 0$ .The rock ledge is at a great depth. The Taylor's stability coefficient for $\phi_u = 0$ and 40^0 slope angle is 0.18. The factor of safety of the load is 1998

a. 2.0 b. 2.1 c. 2.2 d. 2.3

23. a

Slope angle, $\beta = 40^0$

Depth of excavation =10 m

Saturated unit weight of soil, γ = 20 kN/m^3

Cohesion, c = 72 kN/m^2

Angle of shearing resistance, $\phi = 0$

Taylors stability number, S_n = 0.18

Factor of safety = F_c

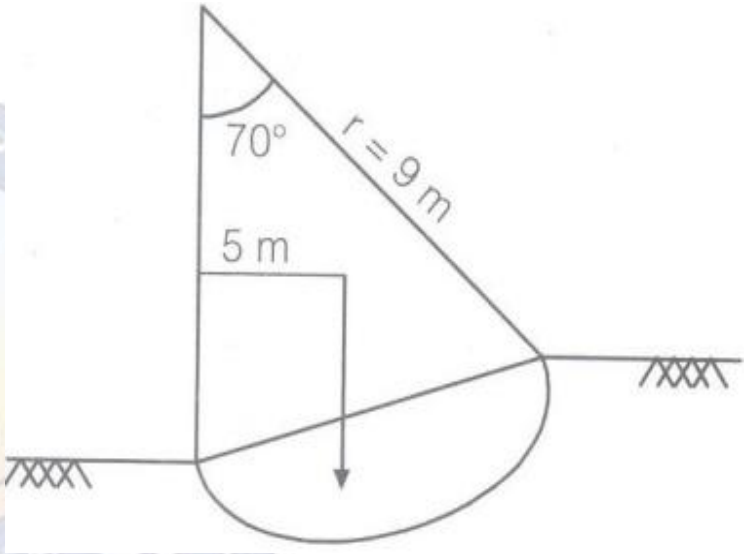
$$S_n = \frac{c}{F_c.\gamma.H} \Rightarrow 0.18 = \frac{72}{F_c \times 20 \times 10} \Rightarrow F_c = 2$$

24. The problem of slope stability analysis assuming circular slip surfaces and considering equilibrium of the free body as a whole is statically determinate. True / False 1994

24. True

The problem of slope stability analysis assuming circular slip surfaces and considering equilibrium of the free body as a whole is statically determinate.

25. For the trial slip circle shown in figure, calculate the factor of safety. [(Given: weight of the soil = 346 kN/m, unit weight of soil=19 kN/m^3, c_u =20 kN/m^2] 1993



25. 1.144

Weight of the soil, $W = 346$ KN/m

Unit weight of soil, $\gamma = 19$ KN/m^3

Cohesion, $c_u = 20$ KN/m^2

Radius of the slip circle, $r = 9$ m

Eccentricity, $e = 5$ m

Factor of safety, $F = \dfrac{\text{Restraining moment}}{\text{Sliding moment}}$ $\quad F = \dfrac{c_u.L.r}{W.e}$

Length of slip circle, $L = \dfrac{2\pi r.\theta}{360^0} = \dfrac{2\pi \times 9 \times 70^0}{360} = 10.995$ m

$$F = \frac{20 \times 10.995 \times 9}{346 \times 5} = 1.144$$