Gear Trains

- A mechanism used to transmit power or motion from one shaft to another with the help of gear wheels.

- Like belt, rope and chain drives

- Widely used in modern world. Clocks, watches, lathes, ships, automobiles etc.

Train value

- The value of a gear train is defined as

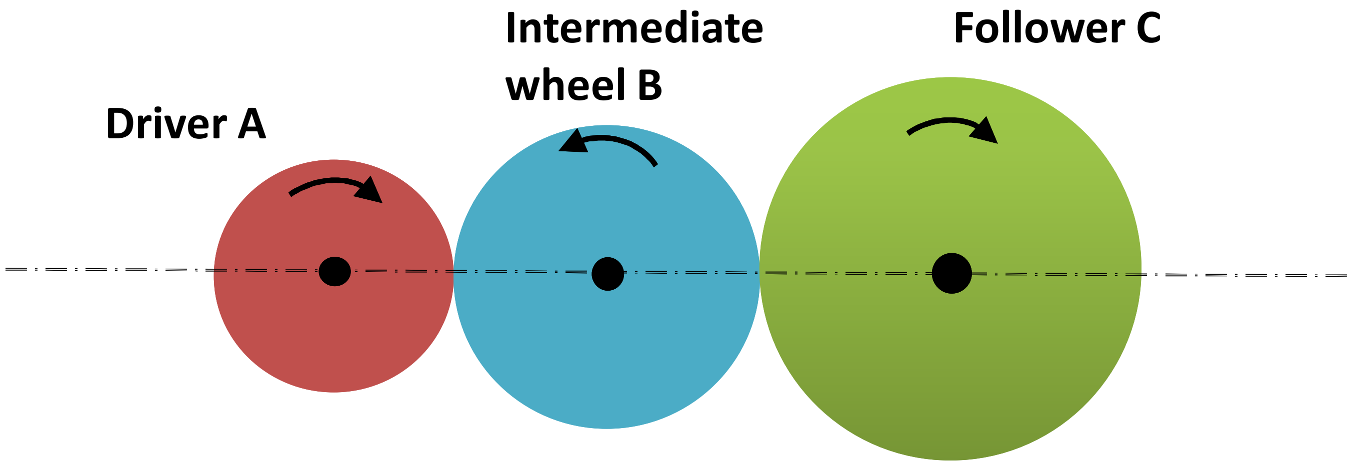
Train value= $\frac{speed\ of\ the\ follower\ gear}{speed\ of\ the\ driver\ gear}$

$$Train\ value = \frac{N_f}{N_d}$$

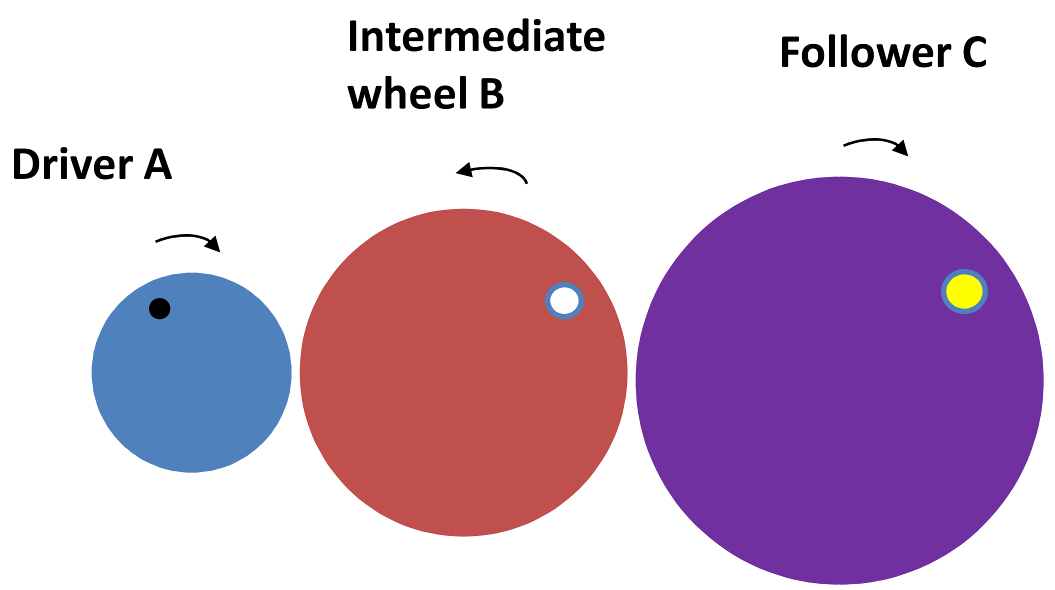
Types of gear trains

- Simple gear train
- Compound gear train
- Reverted gear train
- Epicyclic gear train

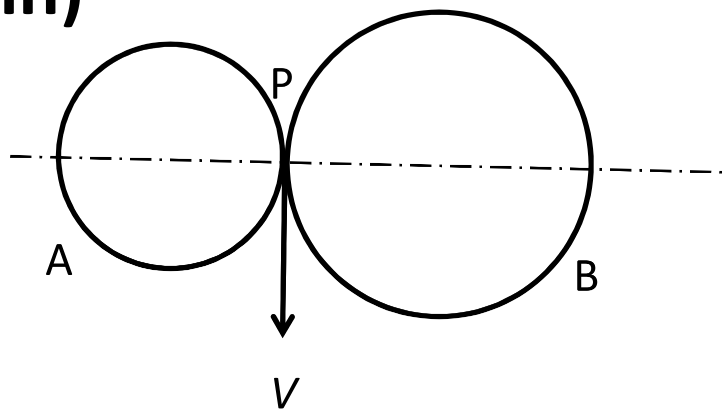
Simple gear train



Simple gear train



Train Value (Simple gear train)



V= Pitch line velocity of the gears A and B at the contact point P

$$V = \pi D_A N_A = \pi D_B N_B$$

$$\frac{N_B}{N_A} = \frac{D_A}{D_B}$$

But $$\frac{D_A}{D_B} = \frac{T_A}{T_B}$$

Therefore $$\frac{N_B}{N_A} = \frac{T_A}{T_B} \quad \ldots\ldots\ldots..(1)$$

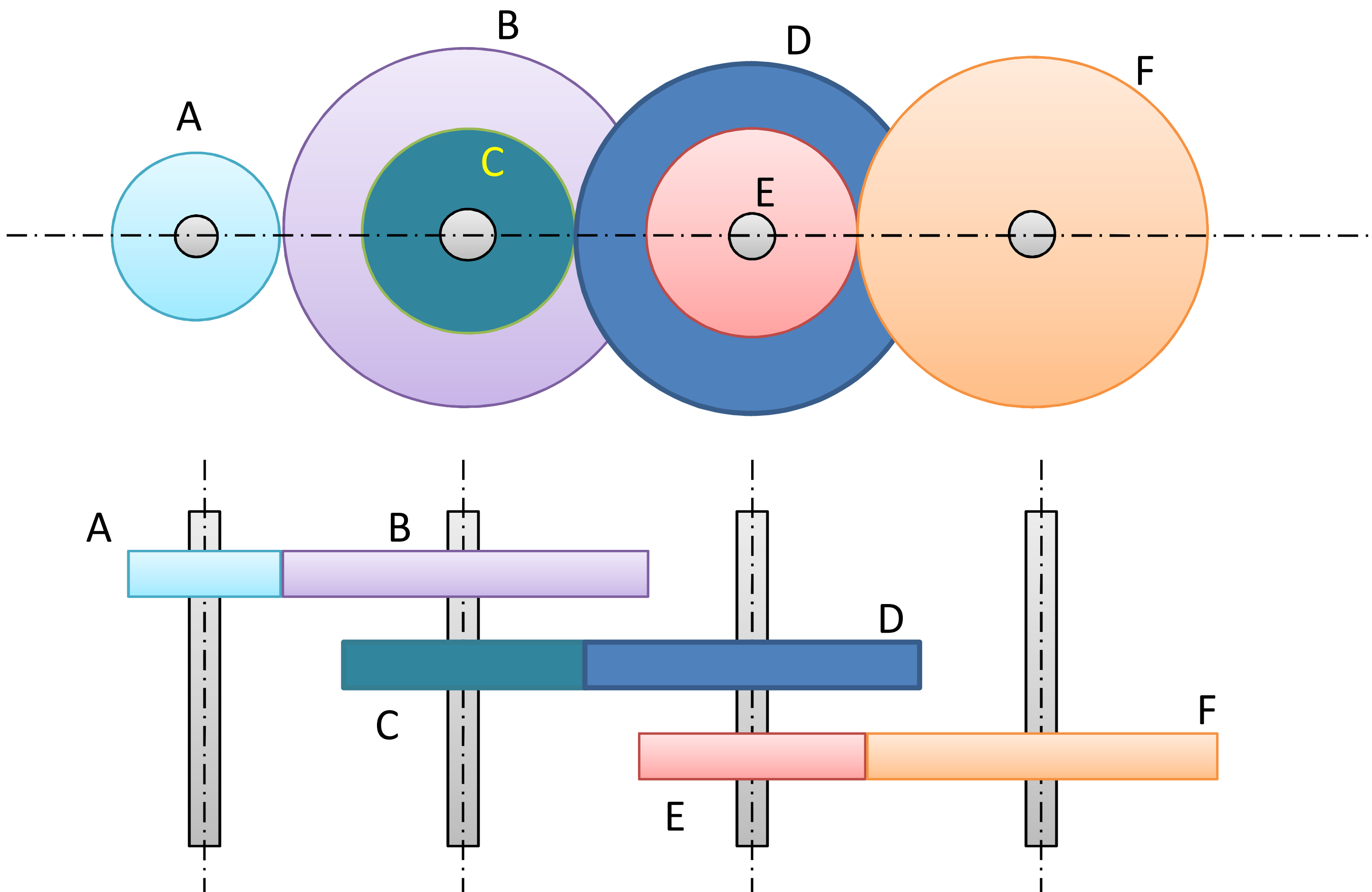
Similarly, for gears B and C $$\frac{N_C}{N_B} = \frac{T_B}{T_C} \quad \ldots\ldots\ldots..(2)$$

From (1) and (2), Train value is $$\frac{N_C}{N_A} = \frac{T_A}{T_C}$$

Thus the train value of the simple gear train does not depend on the intermediate wheel. But, direction of rotation of the follower will be influenced by the intermediate wheel in the simple gear train.

Compound gear train

Each intermediate gear shaft carries two gears which are fastened together rigidly.



Train value of Compound gear train

$$\frac{N_B}{N_A} = \frac{T_A}{T_B}$$ ---------------- (1)

$$\frac{N_D}{N_C} = \frac{T_C}{T_D}$$ ---------------- (2)

$$\frac{N_F}{N_E} = \frac{T_E}{T_F}$$ ---------------- (3)

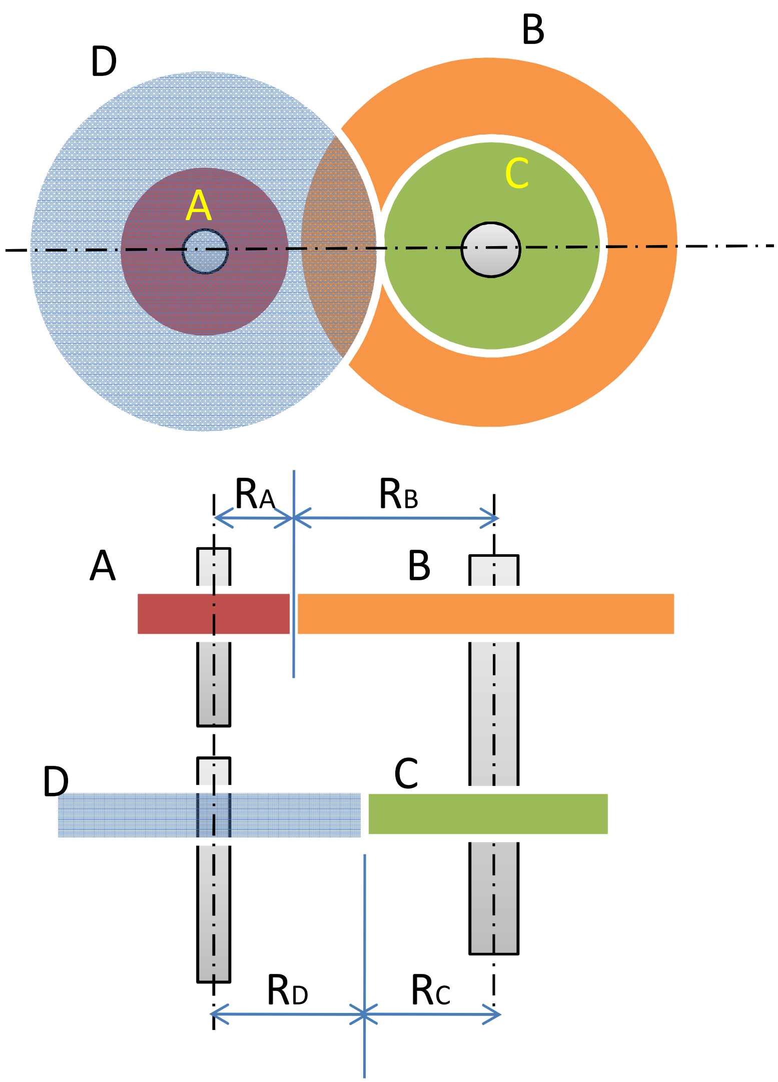
From (1), (2) and (3) $$\frac{N_B}{N_A} \text{x} \frac{N_D}{N_C} \text{x} \frac{N_F}{N_E} = \frac{T_A}{T_B} \text{x} \frac{T_C}{T_D} \text{x} \frac{T_E}{T_F}$$

$$\text{Train value} = \frac{N_F}{N_A} = \frac{T_A}{T_B} \text{x} \frac{T_C}{T_D} \text{x} \frac{T_E}{T_F}$$

Intermediate gears affect the train value and direction. Smaller size of the gears.

Compound gear trains are compact i.e. provide larger velocity ratio in limited space.

Reverted gear train



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Special type of compound gear train where the first and the last gear have the same axis.

Eg. Clock wherein minute and hour hands are attached to gears on the same axis, back gear of a lathe.

$$R_A + R_B = R_C + R_D$$

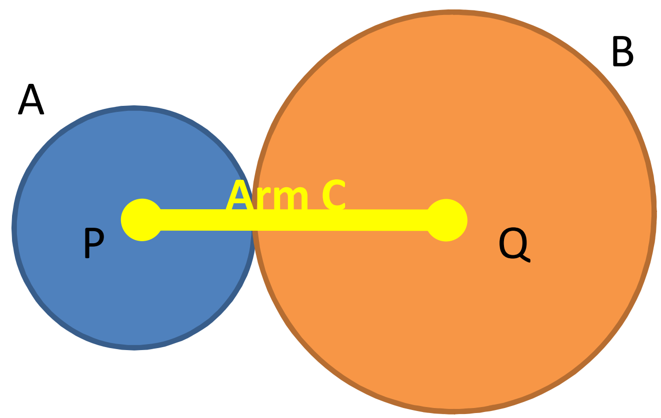
(m=D/T)

(D=mT)

$$T_A + T_B = T_C + T_D$$

Epicyclic Gear Train

- Axes are fixed in simple and compound gear trains
- In Epicyclic gear trains, the axes of some of the gears revolve about one fixed axis.



An Epicyclic gear train can provide larger velocity ratio for a given number of gears.

Applications: differential gear of automobile, wrist watches, hoists, pulley blocks

Train value of epicyclic gear train

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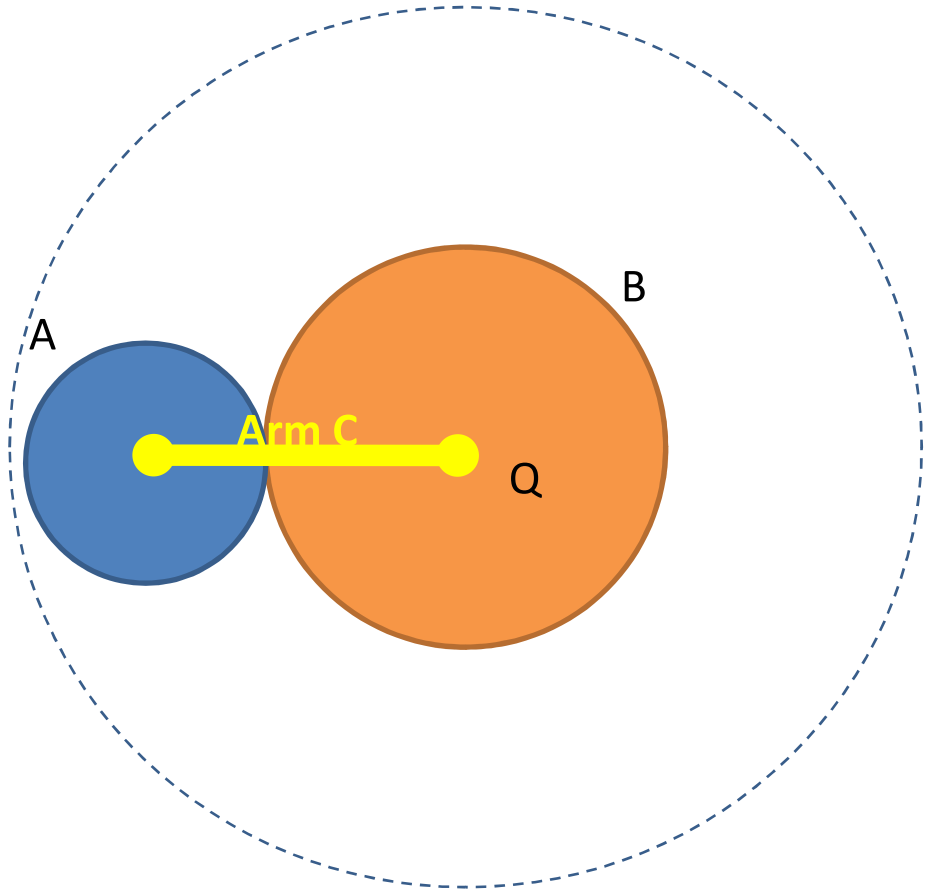


Train value of epicyclic gear train

- Various methods are employed to determine the velocity ratio of an epicyclic gear train.
- Relative velocity method
- Tabular or Algebraic method

Tabular or Algebraic method

Operation	Revolution of the Arm C	Revolution of the gear A	Revolution of the gear B
Arm C fixed	0	+1	$-(T_A/T_B)$
Multiply by x	0	$+x$	$-x \cdot (T_A/T_B)$
Add y	$+y$	$x+y$	$y - x \cdot (T_A/T_B)$

Torques in epicyclic gear train

- Assuming that different gears constituting the epicyclic gear are moving with uniform speed,

  Algebraic sum of the torques applied externally must be equal to zero.

- In three torques are applied externally,
- T_d - driving torque
- T_r - resisting torque
- T_h - holding down or braking torque

therefore, $T_d + T_r + T_h = 0$

Torques in epicyclic gear train…

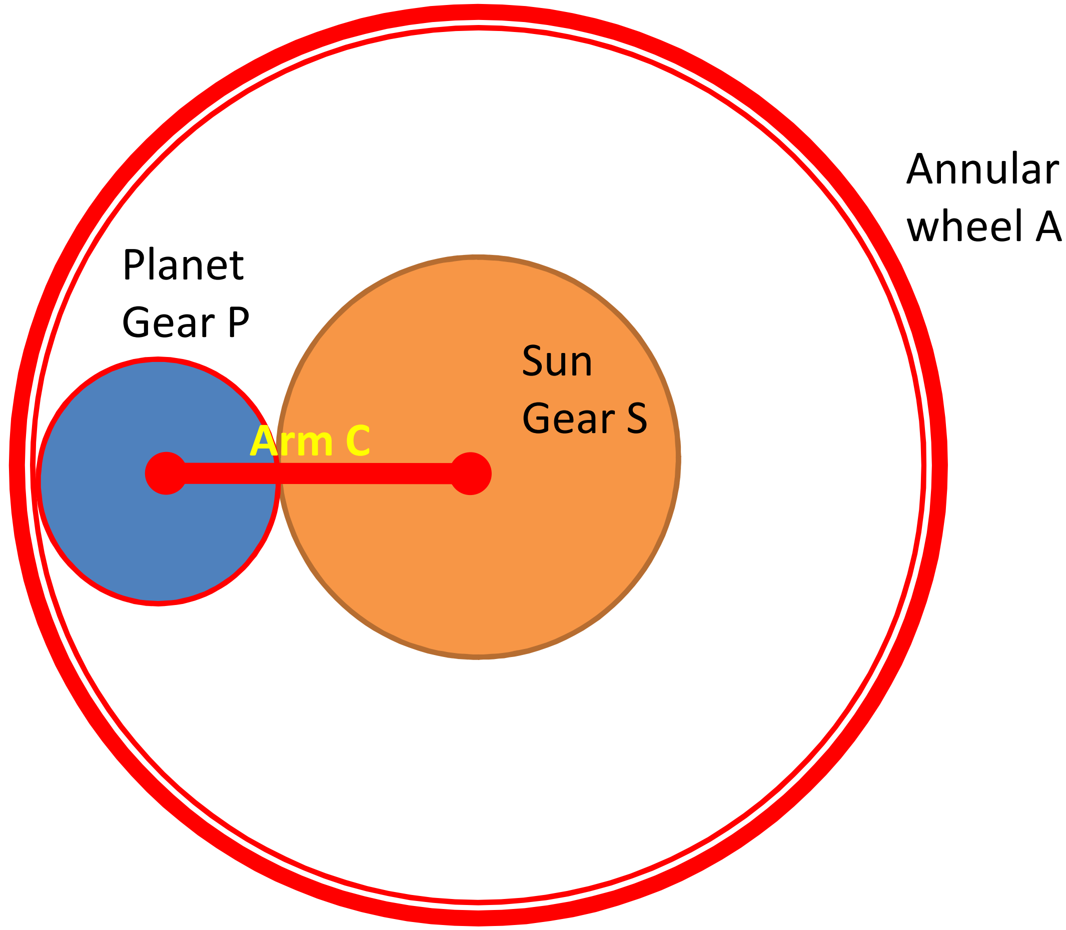
- Neglecting friction losses of teeth during mesh and at bearings, the total energy must be equal to zero.

Therefore, $T_d \,.\, \omega_d + T_r \,.\, \omega_r + T_h \,.\, \omega_h = 0$

Since $\omega_h = 0$,

$$T_d \,.\, \omega_d + T_r \,.\, \omega_r = 0$$

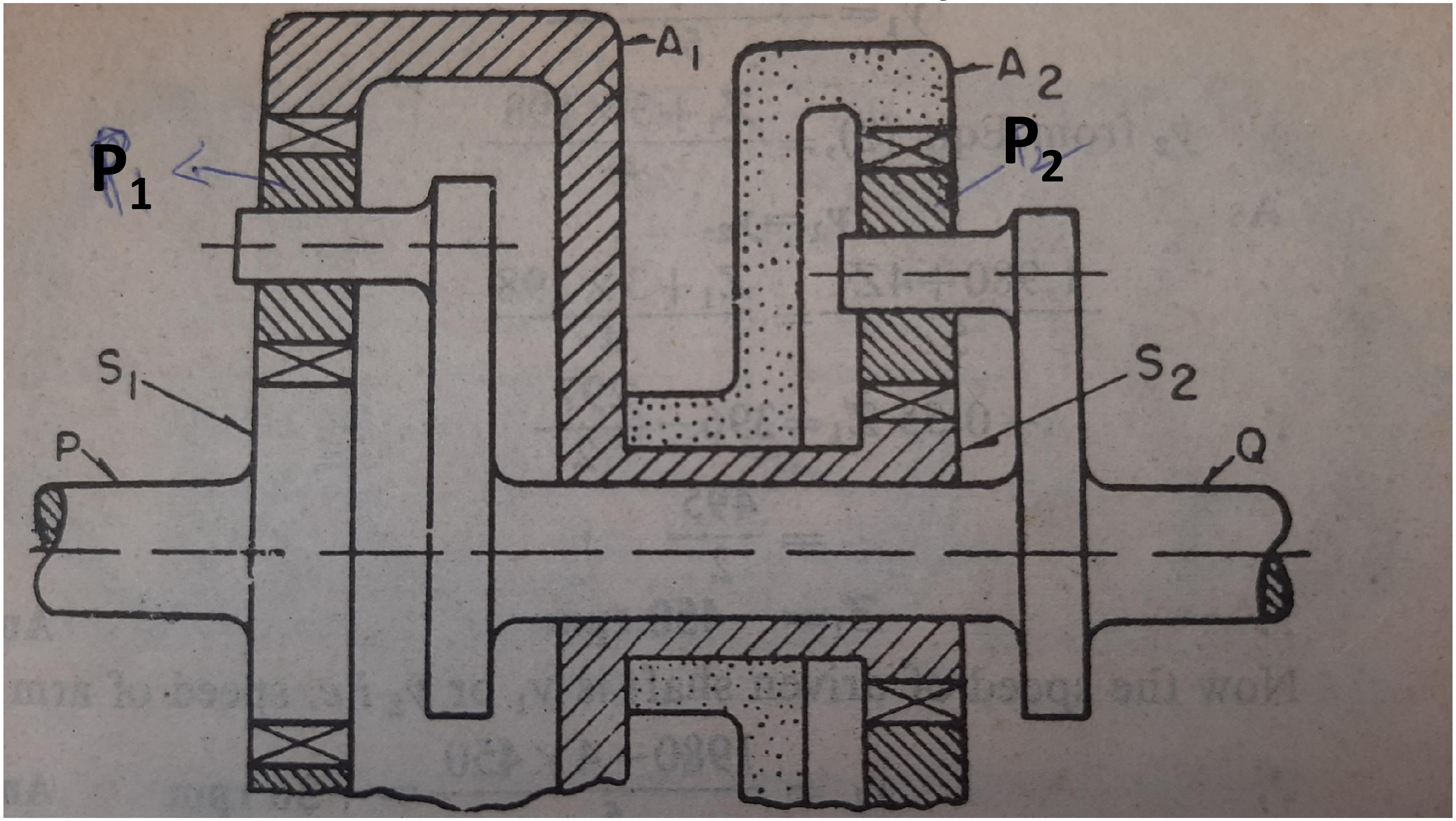
Sun and Planet Gear



Speeds in Sun and Planet gear Tabular or Algebraic method

Operation	Revolution of the Arm C	Revolution of the Sun gear S	Revolution of the Planer gear P	Revolution of the Annular wheel A
Arm C fixed	0	+1	$-(T_S/T_P)$	$-(T_S/T_P) . (T_P/T_A) = -(T_S/T_A)$
Multiply by x	0	$+x$	$-x . (T_S/T_P)$	$-x . (T_S/T_A)$
Add y	$+y$	$x+y$	$y-x . (T_S/T_P)$	$y-x . (T_S/T_A)$

Numerical example 1



The number of teeth on S1=24, A1=96, S2=30 ,A2=90.
If the shaft P rotates at 1980 rpm and annular wheel A2 is fixed, find the speed of the shaft Q. Also, find the speed of the shaft Q if the annular wheel A2 rotates at 198 rpm in the same direction as that of S1.

Numerical Example

Operation	Arm	S_1	P_1	A_1	S_2	P_2	A_2
Arm fixed	0	+1	$-(24/P_1)$	-(24/96) = - (1/4)	-(24/96) = -(1/4)	$+(24/96) \times (30/P_2)$	$+(24/96) \times (30/P_2) \times$ (P2/90)= + (1/12)
Arm fixed + x revolution to S1	0	$+ x$		$-(1/4)\ x$	$-(1/4)\ x$		$+(1/12).\ x$
Add $+ y$ to arm	$+ y$	$x+y$		$-(1/4)x + y$	$-(1/4)x + y$		$(1/12)x + y$

Numerical example
Case 1:

- Annular wheel A_2 is fixed.
- Therefore $(1/12)x + y = 0$
- $or\ x = -12y$
- Speed $S_1 = x + y = 1980\ rpm$
- $-12y + y = 1980$
- *Speed of arm i.e shaft Q*$= y = -1980/11$
- $= -180\ rpm$
- *Shaft Q rotates at 180 rpm in the direction opposite to that of the shaft P.*
- $x = 1980 - y = 1980 + 180 = 2160\ rpm$
- Speed of $S_2 = -(1/4)x + y = -2180/4 - 180$
- $= -545 - 180 = -725\ rpm$

Case 2

- Speed of Annular wheel A_2 =198 rpm
- Therefore (1/12) $x + y = 198$
- or $x = 12\,(198 - y)$
- Speed $S_1 = x + y = 1980$ rpm
- $12(198 - y) + y = 1980$
- $12 \times 198 - 11\,y = 1980$
- $y = (12 \times 198 - 1980)/11$
- $= 2 \times 198/11 = 36$ rpm
- *Speed of arm i.e shaft Q= y= 36 rpm*
- *Shaft Q rotates at 36 rpm in the same direction as that of the shaft P.*

Numerical example 2

- Refer to the previous numerical example.
- If the torque on the shaft P is 300 N-m, find the torque on the shaft Q and the holding down torque on A_2 when is A_2 fixed.
- $T_d . \omega_d + T_r . \omega_r + T_h . \omega_h = 0$
- $T_d . N_d + T_r . N_r + T_h . N_h = 0$
- $300 \times 1980 + T_r \times -180 + T_h \times 0 = 0$
- $T_r = 300 \times 1980/180 = 3300$ N-m

Numerical example 2

- Also, $T_d + T_r + T_h = 0$
- Therefore, $300 + 3300 + T_h = 0$
- or $T_h = -3600$ N-m

Questions on Gear trains

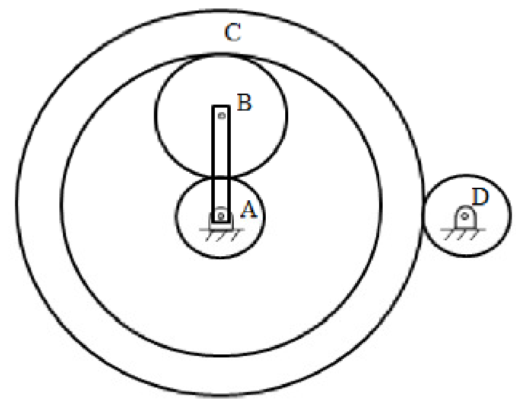
1. An epicyclic gear train is shown in the figure below. The number of teeth on the gears A, B and D are 20, 30 and 20, respectively. Gear C has 80 teeth on the inner surface and 100 teeth on the outer surface. If the carrier arm AB is fixed and the sun gear A rotates at 300 rpm in the clockwise direction, then the rpm of D in the clockwise direction is

(A) 240

(B) -240

(C) 375

(D) -375



Answer : (C) 375

Solution: when arm is fixed, it becomes simple train.

$$\frac{N_B}{N_A} = -\frac{T_A}{T_B} \qquad N_C = -\frac{T_A}{T_C}N_A = -\frac{20}{80}(-300) = 75\,rpm$$

$$\frac{N_C}{N_B} = \frac{T_B}{T_C} \qquad N_D = -\frac{T_C}{T_D}N_C = -\frac{100}{20}(75) = -375\,rpm = 375\,rpm\ (C.W)$$

$$\frac{N_C}{N_A} = -\frac{T_A}{T_C}$$

As epicyclic gear train

$$y = 0$$

$$N_A = x + y = -300\ rpm$$

$$N_C = y - x\,\frac{T_A}{T_C} = 0 - (-300)\,\frac{20}{80} = +75\ rpm$$

$$N_D = -\frac{T_C}{T_D}N_C = -\frac{100}{20}(75) = -375\ rpm = 375\ rpm\ (C.W)$$

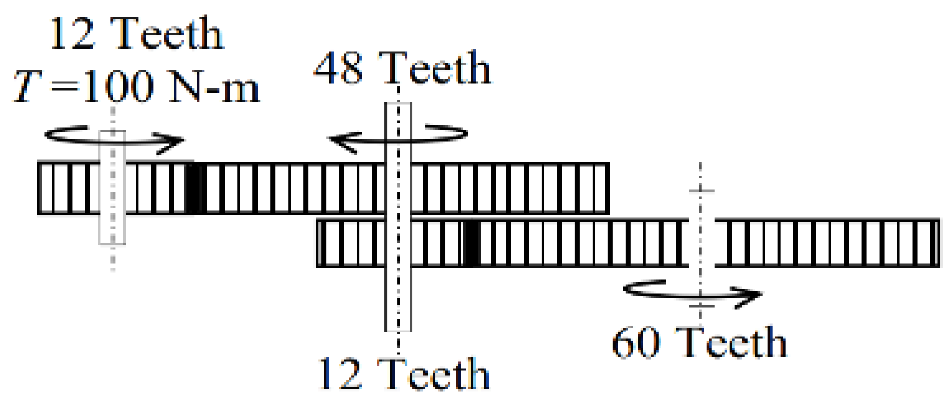
2. A frictionless gear train is shown in the figure. The leftmost 12-teeth gear is given a torque of 100 N-m. The output torque from the 60-teeth gear on the right in N-m is

(A) 5

(B) 20

(C) 500

(D) 2000



Answer : (D) 2000

Solution:

$$\frac{N_4}{N_1} = \frac{T_1 T_3}{T_2 T_4} = \frac{12 \text{ x } 12}{48 \text{ x } 60} = \frac{1}{20}$$

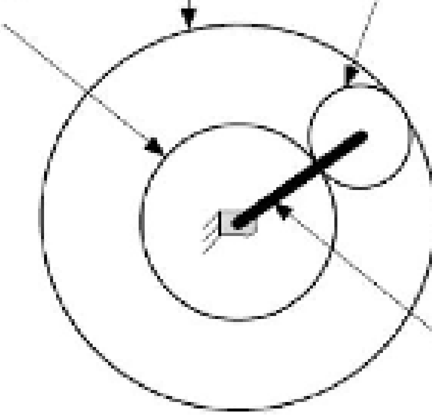
$$t_1 \text{x } N_1 = t_4 \text{x } N_4$$

$$t_1 \text{x } \omega_1 = t_4 \text{x } \omega_4$$

$$t_4 = \frac{N_1}{N_4} t_1 = 20\ (100) = 2000\ N - m$$

3. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the sun gear rotates counterclockwise at 100 rpm. Let the number of teeth on the sun, planet and outer gears to be 50, 25, and 100, respectively. The ratio of magnitudes of angular velocity of the planet gear to the angular velocity of the carrier arm is _____________.

- **Answer : 3**
- Sol: $N_S = x + y = +100\, rpm$



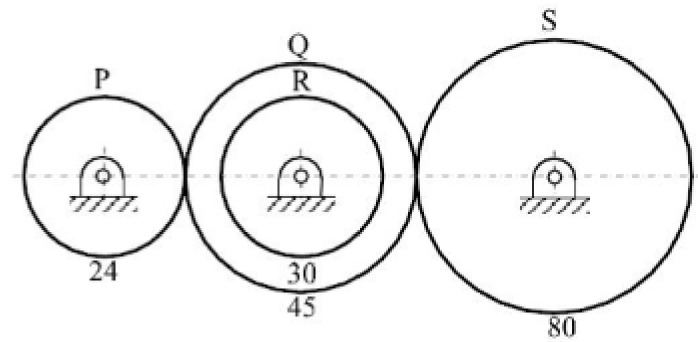
$$N_A = y - x\,\frac{T_S}{T_A}$$

$$N_A = y - x\,\frac{50}{100} = 0 \qquad 2y = x$$

$$N_P = y - x\,\frac{T_S}{T_P} \qquad \frac{N_P}{y} = 1 - \frac{x}{y}\,\frac{T_S}{T_P} = 1 - 2 \text{ x } \frac{50}{25} = -3$$

4. A gear train shown in the figure consist of gears P, Q, R and S. Gear Q and gear R are mounted on the same shaft. All the gears are mounted on parallel shaft and the number of teeth of P, Q, R and S are 24, 45, 30 and 80, respectively. Gear P is rotating at 400 rpm. The speed (in rpm) of the gear S is__________

- Answer : 120



$$\frac{N_S}{N_P} = \frac{T_P}{T_S}$$

$$N_S = \frac{24}{80} \text{ x } 400 = 120\ rpm$$

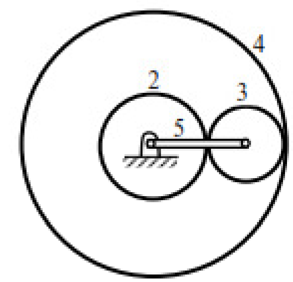
5. In the gear train shown, gear 3 is carried on arm 5. Gear 3 meshes with gear 2 and gear 4. The number of teeth on gear 2, 3, and 4 are 60, 20, and 100, respectively. If gear 2 is fixed and gear 4 rotates with an angular velocity of 100 rpm in the counterclockwise direction, the angular speed of arm 5 (in rpm) is

(A) 166.7 counterclockwise

(B) 166.7 clockwise

(C) 62.5 counterclockwise

(D) 62.5 clockwise



Answer : (C) 62.5 counterclockwise

$$N_{s=2} = x + y = 0$$

$$N_{A=4} = y - x\,\frac{T_S}{T_A}$$

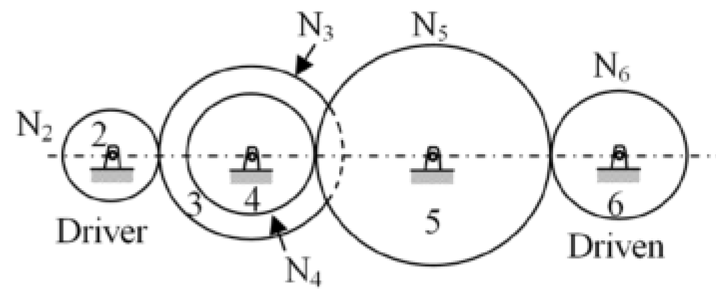
$$N_4 = y - x\,\frac{60}{100} = 100$$

$$y = -\,x$$

$$N_4 = y - x\,(0.6) = 100$$

$$y = \frac{100}{1.6} = 62.5\,rpm\;(C.C.W)$$

6. A gear train is made up of five spur gears as shown in the figure. Gear 2 is driver and gear 6 is driven member. N_2, N_3, N_4, N_5 and N_6 represent number of teeth on gears 2,3,4,5, and 6 respectively, The gear(s) which act(s) as idler(s) is/ are



(A) Only 3

(B) Only 4

(C) Only 5

(D) both 3 and 5

Answer : (C) Only 5

Solution:

$$\frac{N_3}{N_2} = -\frac{T_2}{T_3}$$

$$\frac{N_5}{N_4} = -\frac{T_4}{T_5}$$

$$\frac{N_6}{N_5} = -\frac{T_5}{T_6}$$

$$\frac{N_3}{N_2} X \frac{N_5}{N_4} X \frac{N_6}{N_5} = -\frac{T_2}{T_3} X - \frac{T_4}{T_5} X - \frac{T_5}{T_6} == -\frac{T_2 T_4}{T_3 T_6}$$

Clarifications

- Sir,
  In the concept of addendum circle cutting at the tangent to avoid interference. i the figure the addendum is passing into the base circle(tooth tip of one gear into other) which is not desired. what will we do to avoid it?.Is it possible for such type of gears to function?