CONTROL SYSTEMS

GATE CLASSES

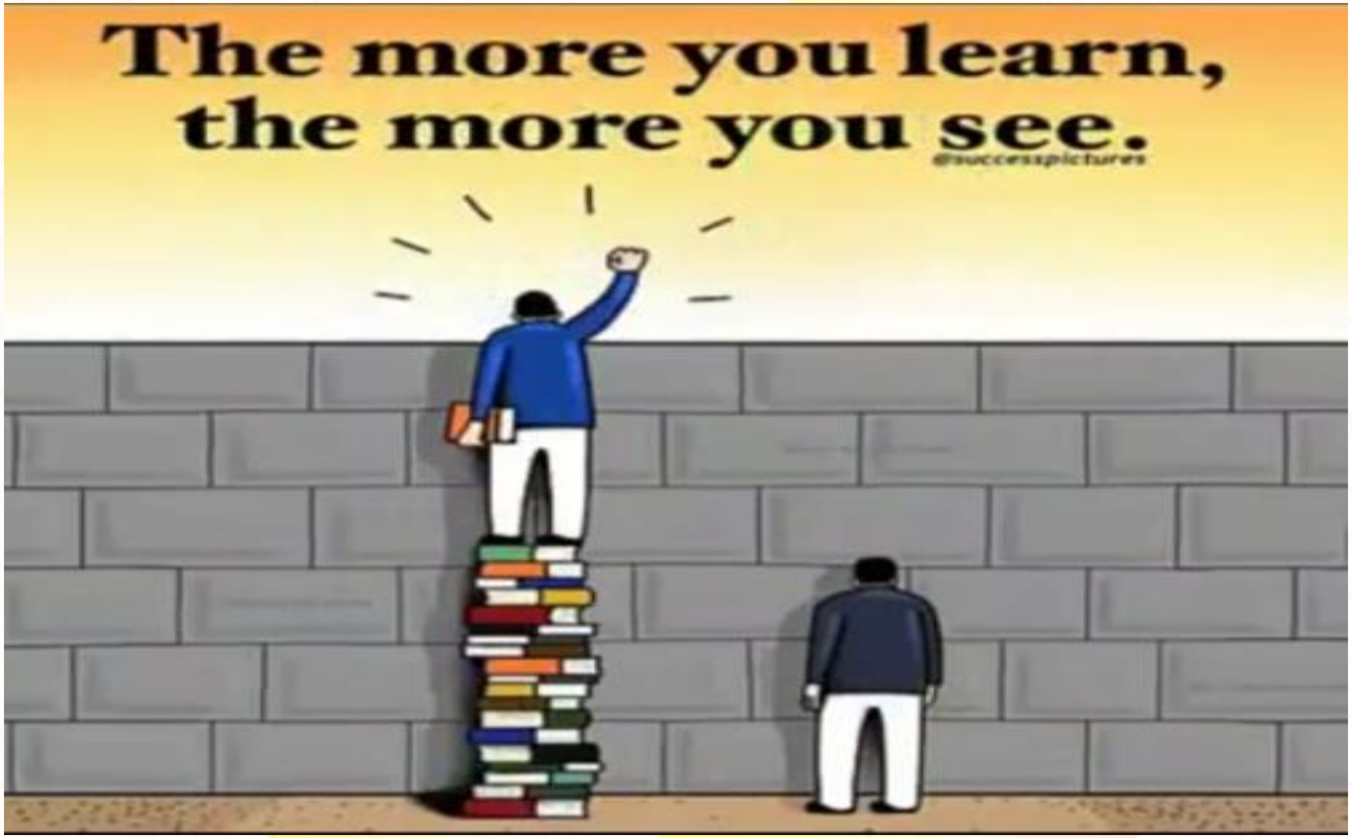
Dr. T.Devaraju
Professor of EEE
Sree Vidyanikethan Engineering College

LESSON PLAN

- Introduction to Control systems
- Modeling of Physical systems
- Transfer function - Block diagram reduction Techniques
- Transfer function through Signal flow graph
- Time response of second order systems
- Steady state and Transient analysis
- Time-domain specifications and Static error coefficients.
- Routh-Hurwitz stability, Finding the range of K for stability
- Concepts of state, state variables and state model.
- Derivation of state model from transfer function
- State transition matrix, Properties, determination of STM
- Conversion from SS to TF

DAY-1

- **Introduction to Control systems**
- **Modeling of Physical systems**
- **Transfer function from Block diagram**


The more you learn,
the more you see.

CONTROL SYSTEM

When a number of elements are combined together to form a system to produce desired output then the system is referred to as **control system**

The main **feature of a control system** is that there should be a clear mathematical relationship between input and output of the system.

When the relation between input and output of the system can be represented by a linear proportionality, the system is called a **linear control system**

The system used for controlling the position, velocity, acceleration, temperature, pressure, voltage and current etc. are examples of control systems

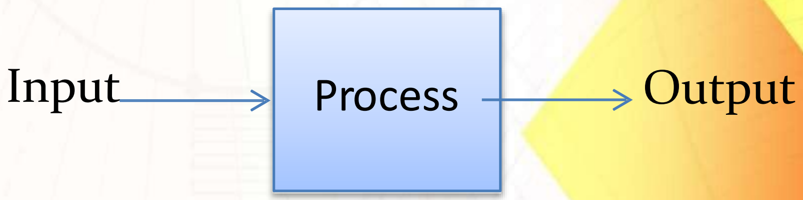
Types of control system.

Open loop control system

Closed loop control system

Definitions

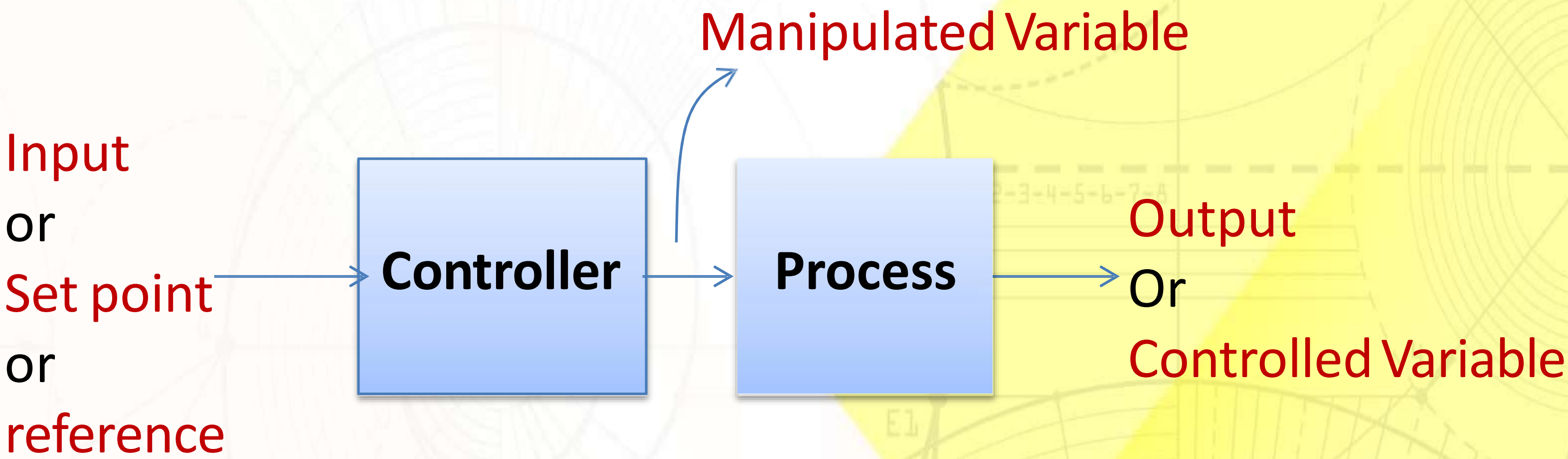
- **System** – An interconnection of elements and devices for a desired purpose.
- **Control System** – An interconnection of components forming a system configuration that will provide a desired response.
- **Process** – The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.



Definitions (Contd..)

- **Controlled Variable**– It is the quantity or condition that is measured and Controlled. Normally controlled variable is the output of the control system.
- **Manipulated Variable**– It is the quantity of the condition that is varied by the controller so as to affect the value of controlled variable.
- **Control** – Control means measuring the value of controlled variable of the system and applying the manipulated variable to the system to correct or limit the deviation of the measured value from a desired value.

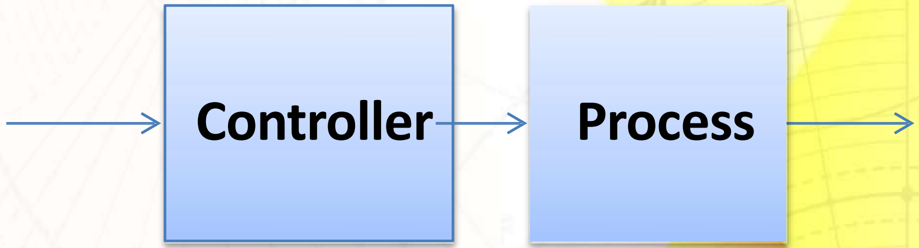
Definitions



- **Disturbances**– A disturbance is a signal that tends to adversely affect the value of the system. It is an unwanted input of the system.
- If a disturbance is generated within the system, it is called internal disturbance. While an external disturbance is generated outside the system.

Types of Control System

- **Open-Loop Control Systems** utilize a controller or control actuator to obtain the desired response.
- Output has no effect on the control action. No feedback – no correction of disturbance



Open-loop control system (without feedback).

Examples:- Washing Machine, Toaster, Electric Fan

- In other words output is neither measured nor fed back.

OPEN LOOP SYSTEM

Practical Examples

Electric Hand Drier – Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.

Automatic Washing Machine – This machine runs according to the pre-set time irrespective of washing is completed or not.

Bread Toaster – This machine runs as per adjusted time irrespective of toasting is completed or not.

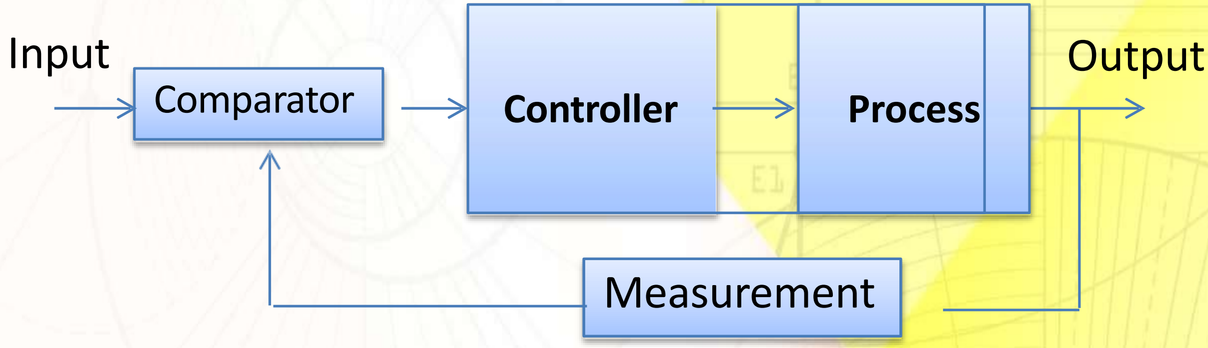
Timer Based Clothes Drier – This machine dries wet clothes for pre-adjusted time, it does not matter how much the clothes are dried.

Volume on Stereo System – Volume is adjusted manually irrespective of output volume level.

Open loop Control System (Contd..)

- Since in open loop control systems reference input is not compared with measured output, for each reference input there is fixed operating condition.
- Therefore, the accuracy of the system depends on calibration.
- The performance of open loop system is severely affected by the presence of disturbances, or variation in operating/ environmental conditions.

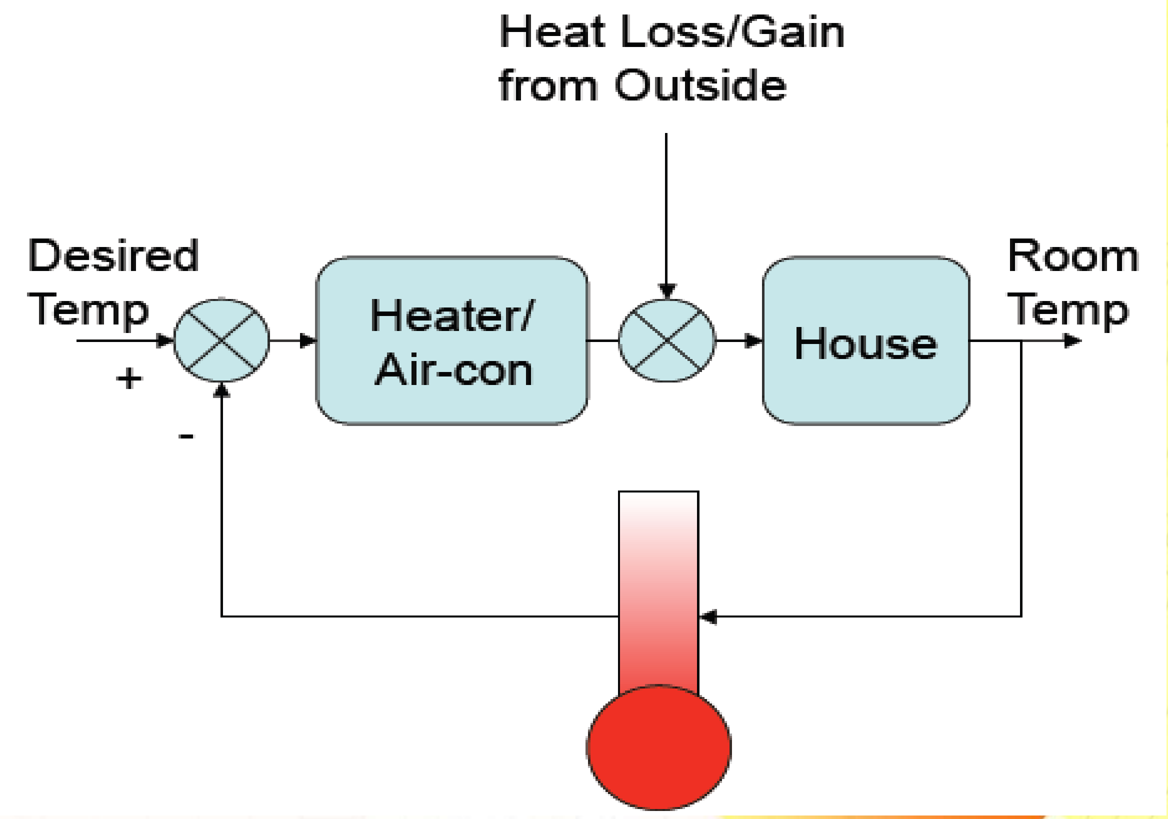
- **Closed-Loop Control Systems** utilizes feedback to compare the actual output to the desired output response.



Closed-loop feedback control system (with feedback).

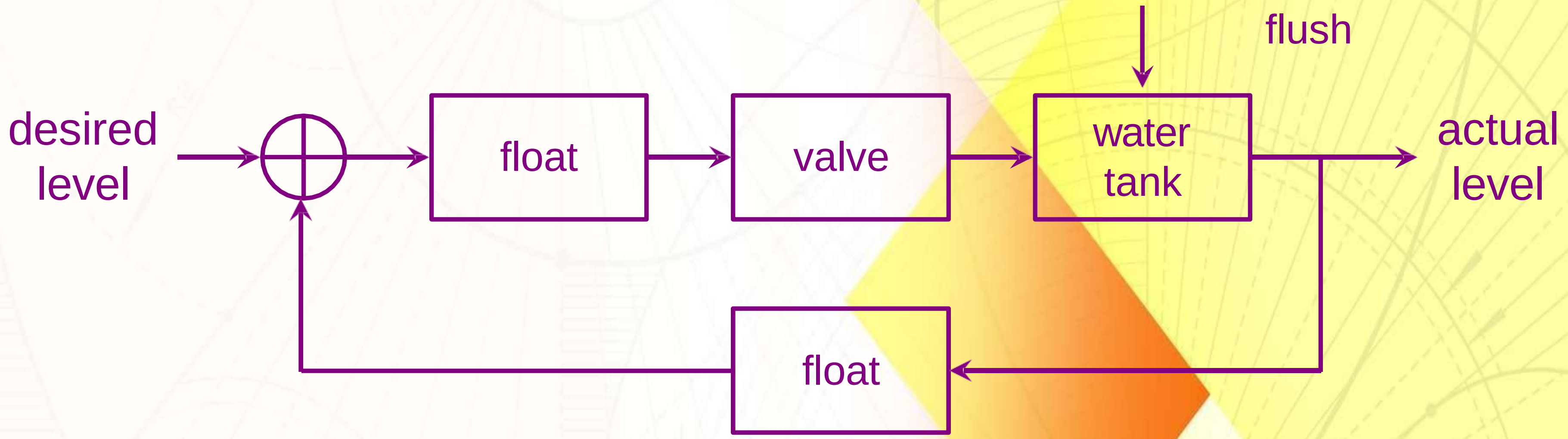
Examples of Control Systems

➢ **Room temperature control**



Float and valve example

- Float height determines desired water level
- Flush empties tank, float is lowered and valve opens
- Open valve allows water to enter tank
- Float returns to desired level and valve closes



Modeling of physical systems

The control systems can be represented with a set of mathematical equations known as **mathematical model**.

These models are useful for analysis and design of control systems. Analysis of control system means finding the output when we know the input and mathematical model.

Design of control system means finding the mathematical model when we know the input and the output.

The following mathematical models are mostly used.

- Differential equation model
- Transfer function model
- State space model

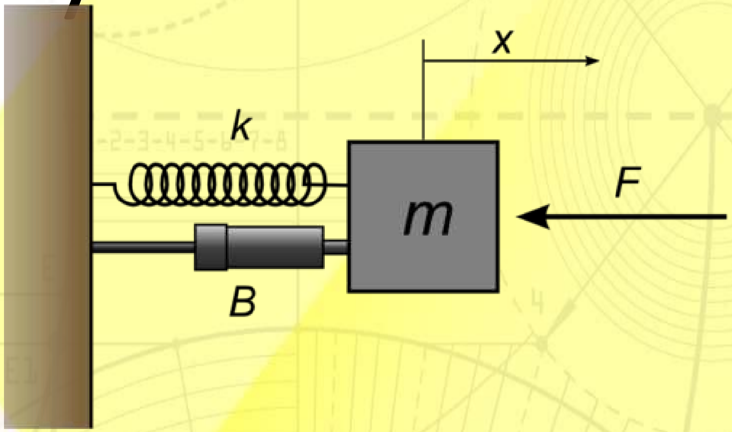
Various types of physical systems are
Mechanical systems,
Electrical systems
Thermal systems
Hydraulic systems
Chemical system etc.,

Mathematical Model

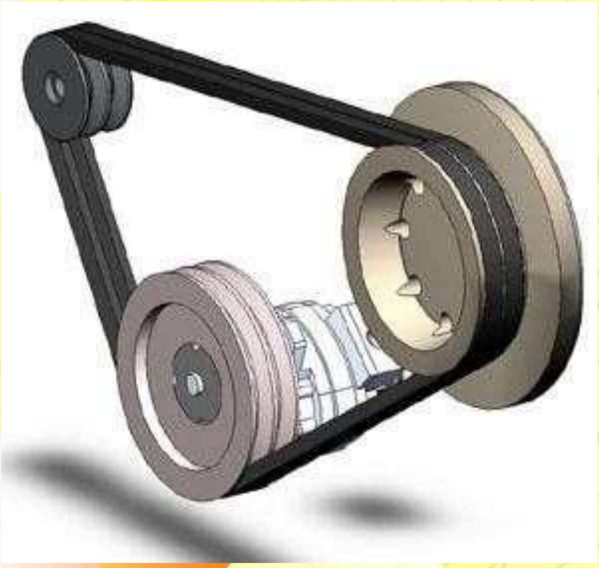
- A mathematical model is a set of equations (usually differential equations) that represents the dynamics of systems.
- In practice, the complexity of the system requires some assumptions in the determination model.
- How do we obtain the equations?
  - Physical law of the process
  - Examples:
    - Mechanical system (Newton's laws)
    - Electrical system (Kirchhoff's laws)

Basic Types of Mechanical Systems

- Translational System

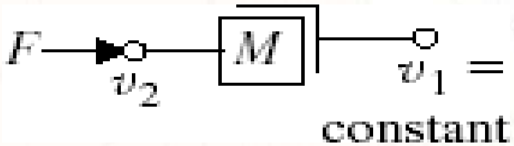


- Rotational System

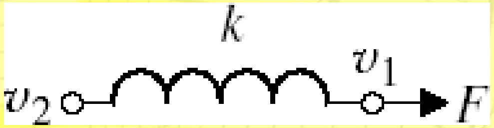


Translational Mechanical Systems

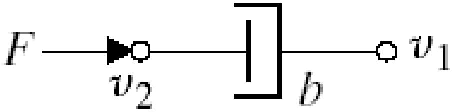
These systems mainly consist of three basic elements.
Mass, spring and dashpot or damper.



Translational Mass



Translational Spring



Translational Damper

Translational Mechanical Systems

- A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.

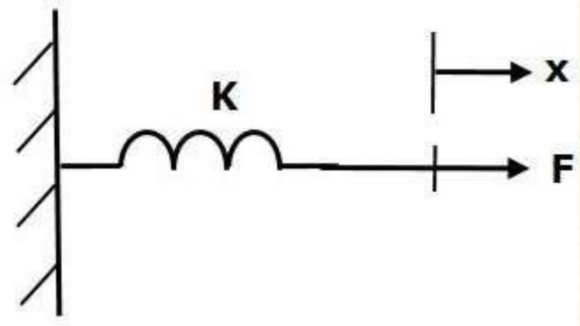
Translational Spring

Circuit Symbols

Translational Spring

Translational Mechanical Systems

➢Spring is an element, which stores **potential energy**.



F α x => $F_k = K\,x$

=> $F = F_k = K\,x$

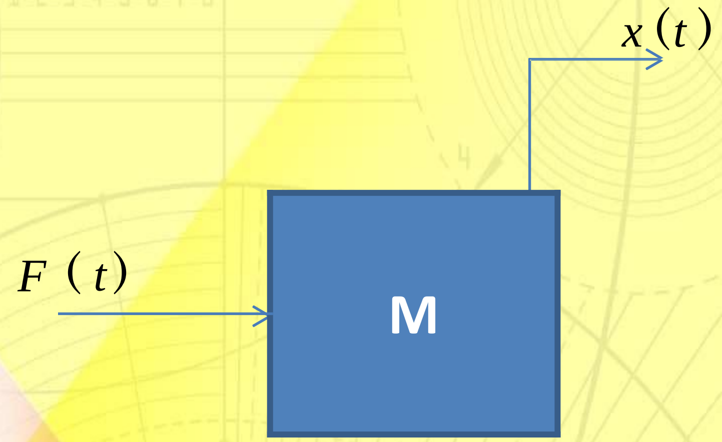
➢Where,

- **F** is the applied force
- $\mathbf{F_k}$ is the opposing force due to elasticity of spring
- **K** is spring constant
- **x** is displacement

Translational Mechanical Systems

- Translational Mass is an inertia element.
- A mechanical system without mass does not exist.
- If a force F is applied to a mass and it is displaced to x meters then the relation b/w force and displacements is given by Newton's law.

Translational Mass

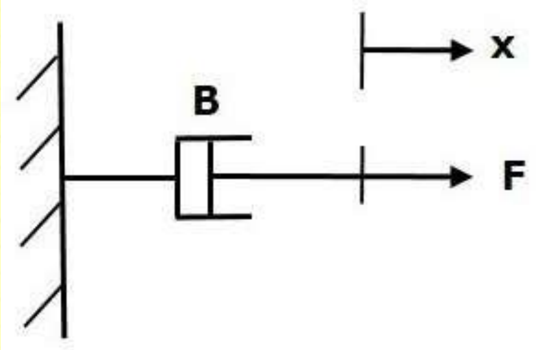


$$F_m \propto a \Rightarrow F_m = Ma$$

$$\Rightarrow F = F_m = M\frac{d^2x}{dt^2}$$

Translational Mechanical Systems

➢**Dash Pot:** If a force is applied on dashpot **B**, then it is opposed by an opposing force due to **friction** of the dashpot. This opposing force is proportional to the velocity of the body. Assume mass and elasticity are negligible.



$$F_b \alpha \text{ v} \quad => F_b = Bv = B\frac{dx}{dt}$$

$$=> \quad F = F_b = B\frac{dx}{dt}$$

Transfer function of Translational Mechanical Systems

- **First,** draw a free-body diagram, placing on the body all forces that
act on the body either in the direction of motion or opposite to it.

- **Second,** use Newton's law to form a differential equation of motion by summing the forces and setting the sum equal to zero.

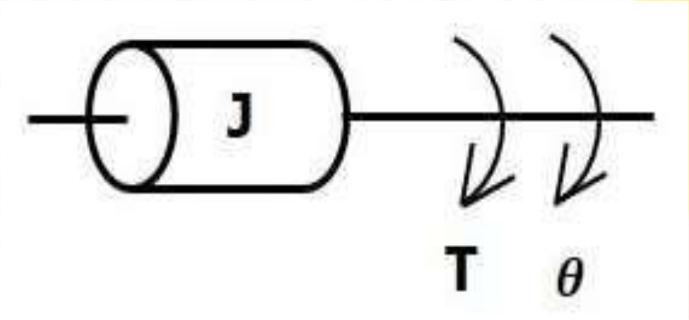
- **Finally,** assuming zero initial conditions, we take the Laplace transform of the differential equation, separate the variables, and arrive at the transfer function.

Rotational Mechanical Systems

➢These systems mainly consist of three basic elements. Those are **moment of inertia, torsional spring** and **dashpot**.

➢**Moment of Inertia**

In translational mechanical system, mass stores kinetic energy. Similarly, in rotational mechanical system, moment of inertia stores **kinetic energy**.



Rotational Mechanical Systems

$T_j \alpha \alpha$ => $J\alpha = T_j = J\,\dfrac{d^2\theta}{dt^2}$

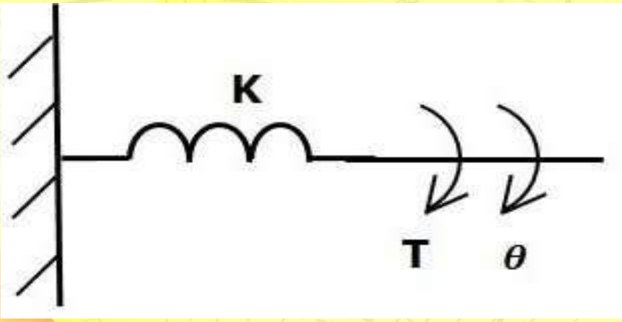
=> $T = T_j = J\dfrac{d^2\theta}{dt^2}$

- Where,
  - **T** is the applied torque
  - $\mathbf{T_j}$ is the opposing torque due to moment of inertia
  - **J** is moment of inertia
  - **α** is angular acceleration
  - **θ** is angular displacement

Rotational Mechanical Systems

- **Torsional Spring:**

In translational mechanical system, spring stores potential energy. Similarly, in rotational mechanical system, torsional spring stores **potential energy**.



$$T_k \alpha \theta \quad => \quad T_k = K\theta$$

$$=> \quad T = T_k = K\theta$$

- Where,
  - **T** is the applied torque
  - **T_k** is the opposing torque due to elasticity of torsional spring
  - **K** is the torsional spring constant
  - **θ** is angular displacement

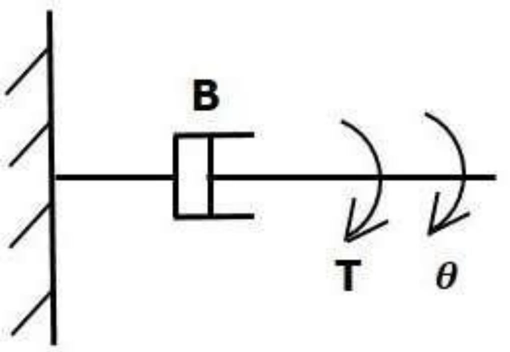
Rotational Mechanical Systems

➢ Dashpot

If a torque is applied on dashpot **B**, then it is opposed by an opposing torque due to the **rotational friction** of the dashpot.

$$T_b \,\alpha\, \omega \quad \Rightarrow \quad T_b = B\omega = B\,\frac{d\theta}{dt}$$

$$\Rightarrow \quad T = T_b = B\,\frac{d\theta}{dt}$$

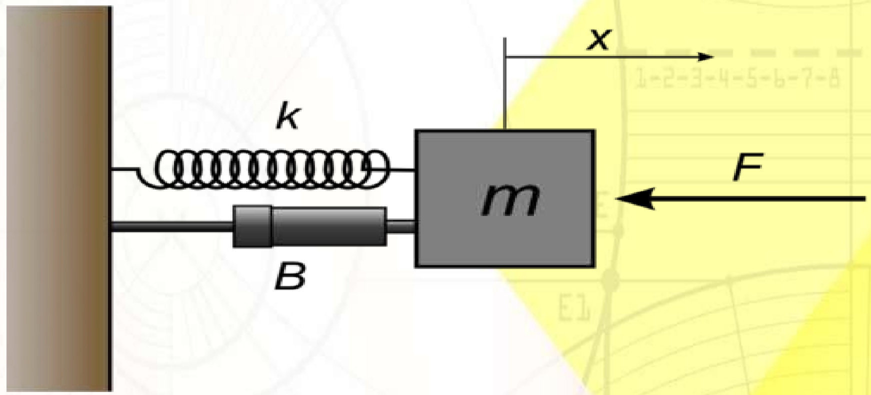


➢ Where,

- $\mathbf{T_b}$ is the opposing torque due to the rotational friction of the dashpot
- **B** is the rotational friction coefficient
- **ω** is the angular velocity
- **θ** is the angular displacement

Mechanical Translational System

- Consider the following system



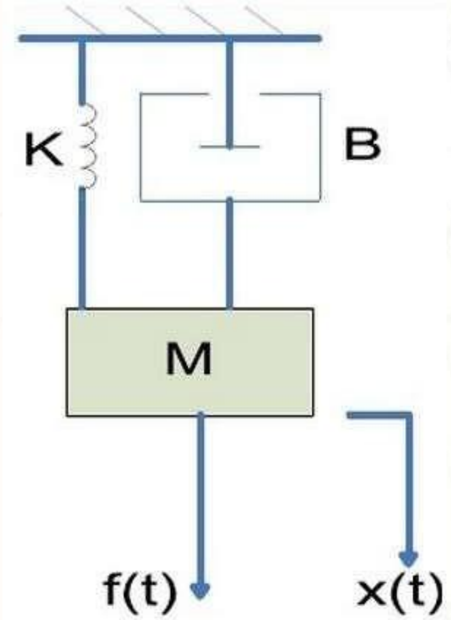
- Free Body Diagram

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

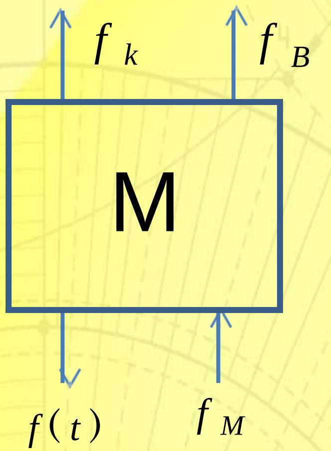
$$F = f_k + f_M + f_B$$

Find the transfer function MTS

Find the transfer function of the mechanicaltranslationalsystem given in Figure.



Free Body Diagram

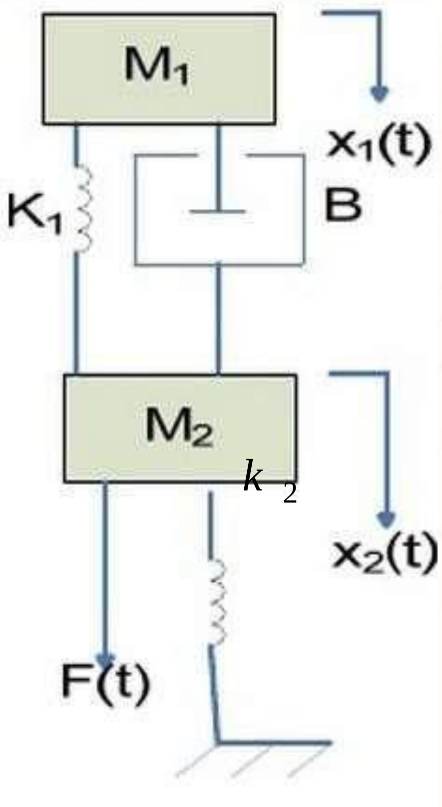


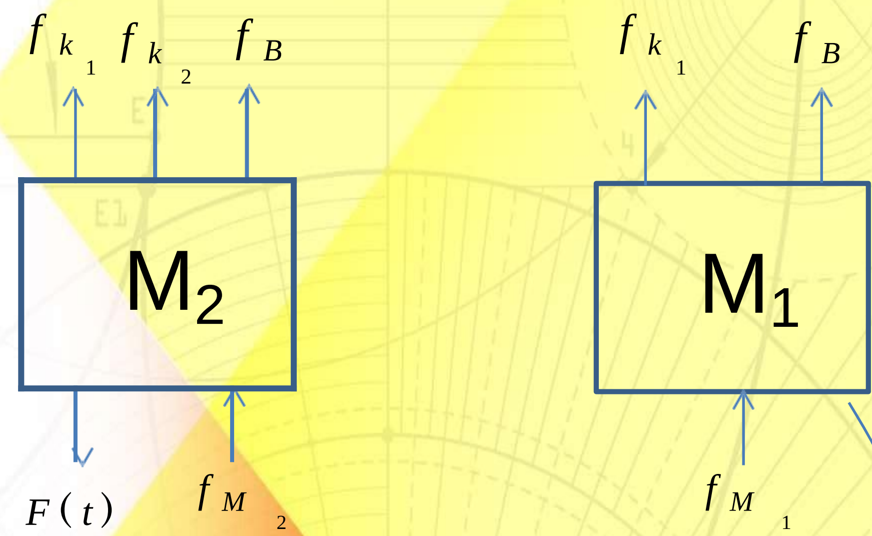
$$f(t) = f_k + f_M + f_B$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + k}$$

Modeling of a mechanical system

Draw the free body diagram for the mechanical system



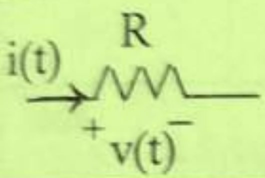
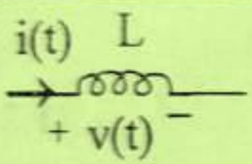
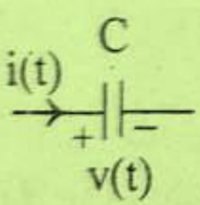


$$F(t) = f_{k_1} + f_{k_2} + f_{M_2} + f_B$$

$$0 = f_{k_1} + f_{M_1} + f_B$$

Mathematical Model of Electrical System

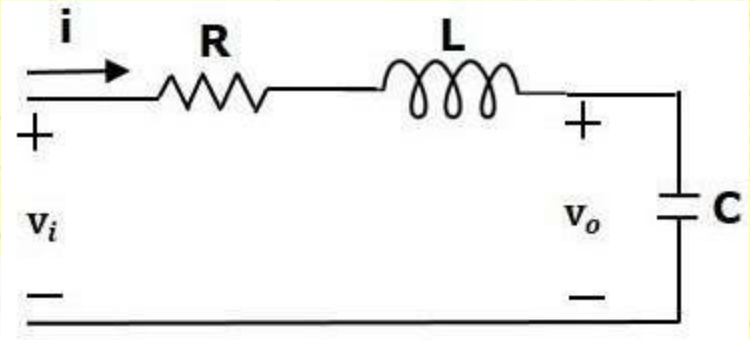
➤The mathematical model of electrical systems can be obtained by using resistor, capacitor and inductor

Element	Voltage across the element	Current through the elemen
	$v(t) = Ri(t)$	$i(t) = \frac{v(t)}{R}$
	$v(t) = L\frac{d}{dt}i(t)$	$i(t) = \frac{1}{L}\int v(t)\,dt$
	$v(t) = \frac{1}{C}\int i(t)\,dt$	$i(t) = C\frac{dv(t)}{dt}$

Mathematical Model of Electrical Systems:

The following mathematical models are mostly used.

- ➢Differential equation model
- ➢Transfer function model
- ➢State space model



Example: RLC Circuit

Mesh equation for this circuit is $v_i = Ri + L\frac{di}{dt} + v_o$

Where $i = C\frac{dv_o}{dt}$

$$\Rightarrow \frac{d^2v_o}{dt^2} + \left(\frac{R}{L}\right)\frac{dv_o}{dt} + \left(\frac{1}{LC}\right)v_o = \left(\frac{1}{LC}\right)v_i$$

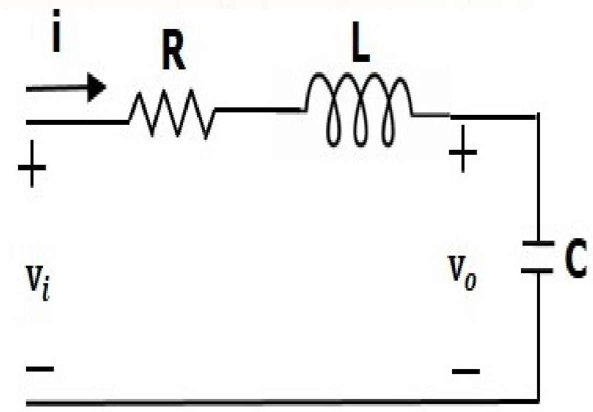
The above equation is a second order **differential equation**.

Modeling of physical systems

Differential Equation Model

Differential equation model is a time domain mathematical model of control systems. Apply basic laws to the given control system to find the differential equation model in terms of input and output

Consider the following electrical system



$$v_i = Ri + L\frac{di}{dt} + v_o$$

$$i = c\frac{dv_o}{dt}$$

$$v_i = RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + v_o$$

$$\frac{d^2v_o}{dt^2} + \left(\frac{R}{L}\right)\frac{dv_o}{dt} + \left(\frac{1}{LC}\right)v_o = \left(\frac{1}{LC}\right)v_i$$

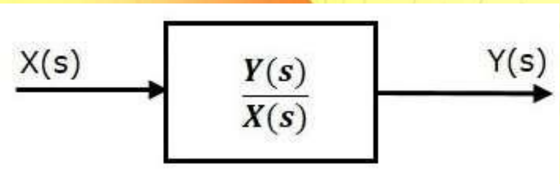
Transfer Function Model

- The **Transfer function** of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output and Laplace transform of input by assuming all the initial conditions are zero.

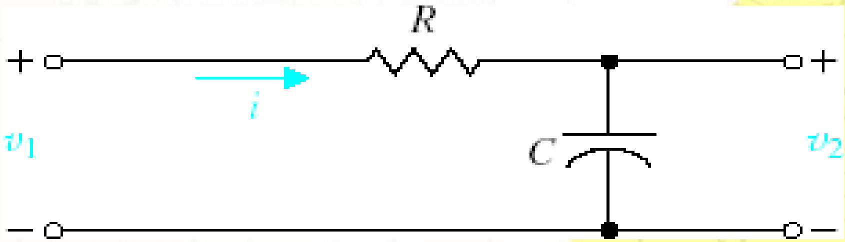
- If x(t) and y(t) are the input and output of an LTI system, then the corresponding Laplace transforms are X(s) and Y(s).

$$i.e.,\ Transfer\ Function = \frac{Y(s)}{X(s)}$$

- The transfer function model of an LTI system is shown in the following figure.



Transfer Function of Linear System



$$V_1(s) = \left(R + \frac{1}{Cs} \right) I(s)$$

$$V_2(s) = \left(\frac{1}{Cs} \right) I(s)$$

Transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{\left(\frac{1}{Cs} \right)}{\left(R + \frac{1}{Cs} \right)} = \frac{1}{1 + sRC}$$

Transfer function model

Transfer function model is an s-domain mathematical model of control systems.

The **Transfer function** of a Linear Time Invariant (LTI) system is defined as the ratio of Laplace transform of output variable to the Laplace transform of input variable by assuming all the initial conditions are zero.

If x(t) and y(t) are the input and output of an LTI system, then the corresponding Laplace transforms are X(s) and Y(s).

Transfer Function=Y(s)/X(s)

$$\frac{d^2 v_o}{dt^2} + \left(\frac{R}{L}\right)\frac{dv_o}{dt} + \left(\frac{1}{LC}\right) v_o = \left(\frac{1}{LC}\right) v_i$$

$$s^2 V_o(s) + \left(\frac{sR}{L}\right) V_o(s) + \left(\frac{1}{LC}\right) V_o(s) = \left(\frac{1}{LC}\right) V_i(s)$$

$$\Rightarrow \left\{s^2 + \left(\frac{R}{L}\right) s + \frac{1}{LC}\right\} V_o(s) = \left(\frac{1}{LC}\right) V_i(s)$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \left(\frac{R}{L}\right) s + \frac{1}{LC}}$$

Analogous Systems

Electrical Analogous of mechanical Translational System:

As the electrical systems has two types of inputs either voltage or current source. There are two types of analogies .

- Force- Voltage analogy
- Force- Current analogy

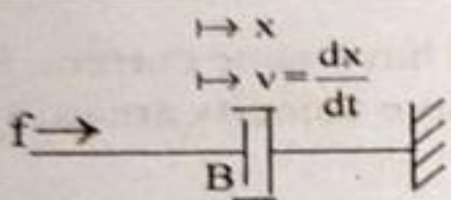
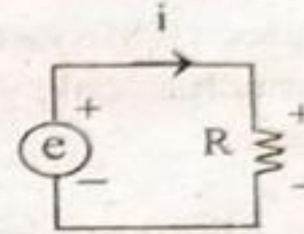
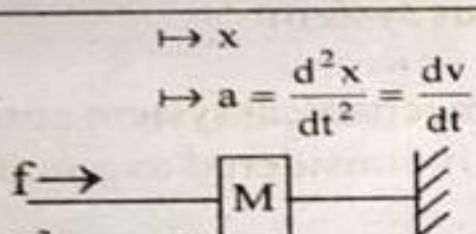
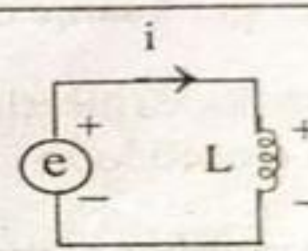
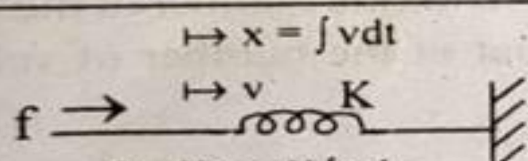
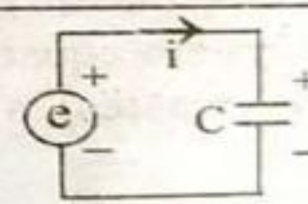
Force- Voltage Analogy:

TABLE 3

Item	Mechanical system	Electrical system (mesh basis system)
Independent variable (input)	Force, f	Voltage, e
Dependent variable (output)	Velocity, v	Current, i
	Displacement, x	Charge, q
Dissipative element	Frictional coefficient of dashpot, B	Resistance, R
Storage element	Mass, M	Inductance, L
	Stiffness of spring, K	Inverse of capacitance, 1/C
Physical law	Newton's second law $\Sigma F = 0$	Kirchoff's voltage law $\Sigma V = 0$
Changing the level of independent variable	lever $\frac{f_1}{f_2} = \frac{l_1}{l_2}$	Transformer $\frac{e_1}{e_2} = \frac{N_1}{N_2}$

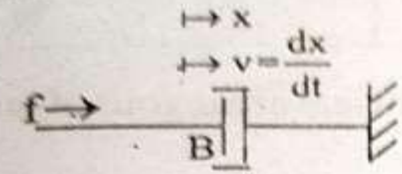
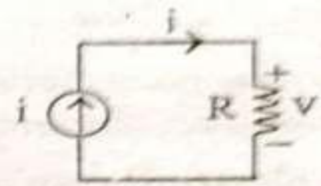
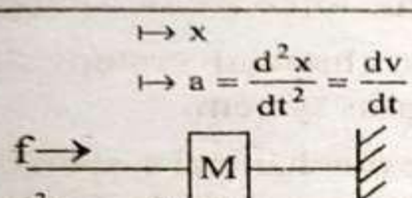
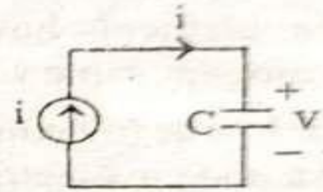
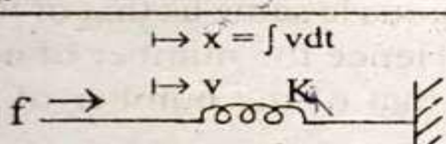
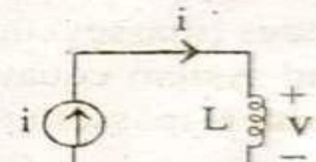
CS Scanned with CamScanner

Force- Voltage Analogy:

Mechanical system	Electrical system
Input : Force Output : Velocity	Input : Voltage source Output : Current through the element
 $f = B\frac{dx}{dt} = Bv$	 $e = Ri$
 $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	 $e = L\frac{di}{dt}$
 $f = Kx = K\int v\,dt$	 $e = \frac{1}{C}\int i\,dt$

Dr.T.Devaraju,Professor of
EEE Sree Vidyanikethan

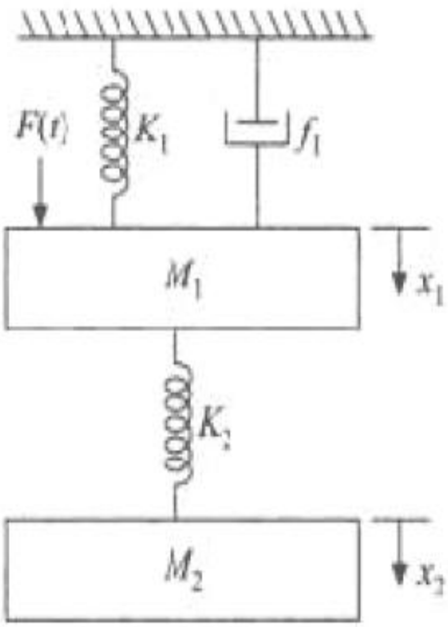
Force- Current Analogy:

Mechanical system	Electrical system
Input : Force Output : Velocity	Input : Current source Output : Voltage across the element
 $f = B\frac{dx}{dt} = Bv$	 $i = \frac{1}{R}v$
 $f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$	 $i = C\frac{dv}{dt}$
 $f = Kx = K\int v dt$	 $i = \frac{1}{L}\int v dt$

Modeling of physical systems

Electrical system		Mechanical system			
		Translational		Rotational	
Voltage	V	Force	f	Torque	T
Current	i	Velocity	u	angular velocity	ω
Charge	q	Displacement	x	angular displacement	θ
Inductance	L	Mass	M	Moment of Inertia	J
Capacitance	C	Compliance	$\frac{1}{K}$	Compliance	$\frac{1}{K}$
Resistance	R	Damping coefficient	B	Damping coefficient	B

Modeling of physical systems



$$F(t) = M_1 \frac{d^2 x_1}{dt^2} + f_1 \frac{dx_1}{dt} + K_1 x_1 + K_2(x_1 - x_2)$$

$$M_2 \frac{d^2 x_2}{dt^2} + K_2(x_2 - x_1) = 0$$

$$F(t) = M_1 \frac{dv_1}{dt} + f_1 v_1 + K_1 \int v_1 dt + K_2 \int (v_1 - v_2) dt$$

$$M_2 \frac{dv_2}{dt} + K_2 \int (v_2 - v_1)\, dt = 0$$

$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_2} \int (i_1 - i_2)\, dt$$

$$i(t) = C_1 \frac{de_1}{dt} + \frac{e_1}{R_1} + \frac{1}{L_1} \int e_1\, dt + \frac{1}{L_2} \int (e_1 - e_2)\, dt$$

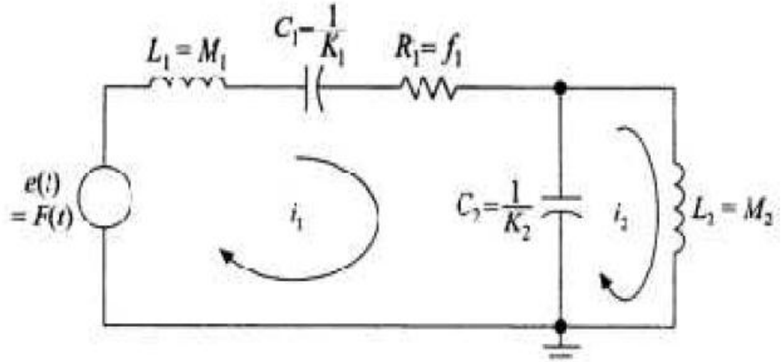
$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_2} \int (i_2 - i_1)\, dt$$

$$C_2 \frac{de_2}{dt} + \frac{1}{L_2} \int (e_2 - e_1)\, dt = 0$$

F-V and F-I analogous circuits

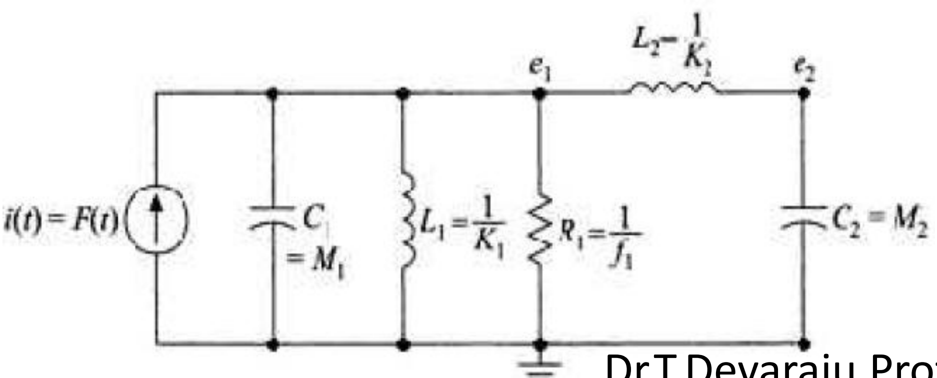
$$e(t) = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int (i_1 - i_2)\, dt$$

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1)\, dt$$

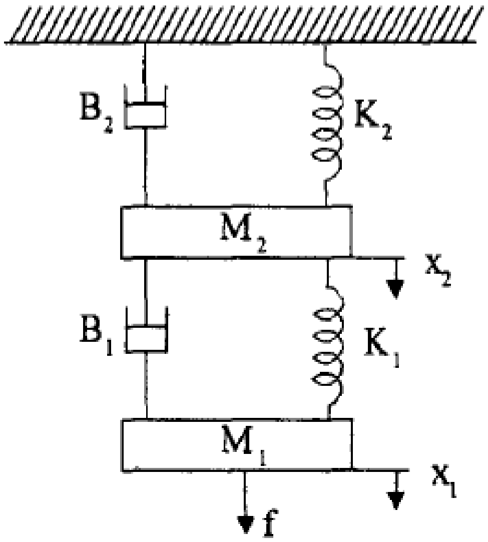


$$i(t) = C_1 \frac{de_1}{dt} + \frac{e_1}{R_1} + \frac{1}{L_1}\int e_1\, dt + \frac{1}{L_2}\int (e_1 - e_2)\, dt$$

$$C_2 \frac{de_2}{dt} + \frac{1}{L_2}\int (e_2 - e_1)\, dt = 0$$

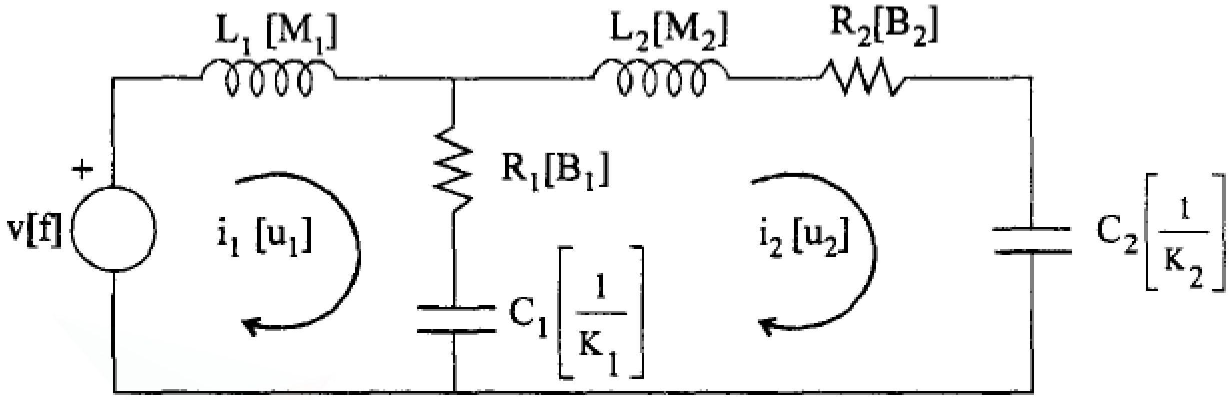


Modeling of physical systems

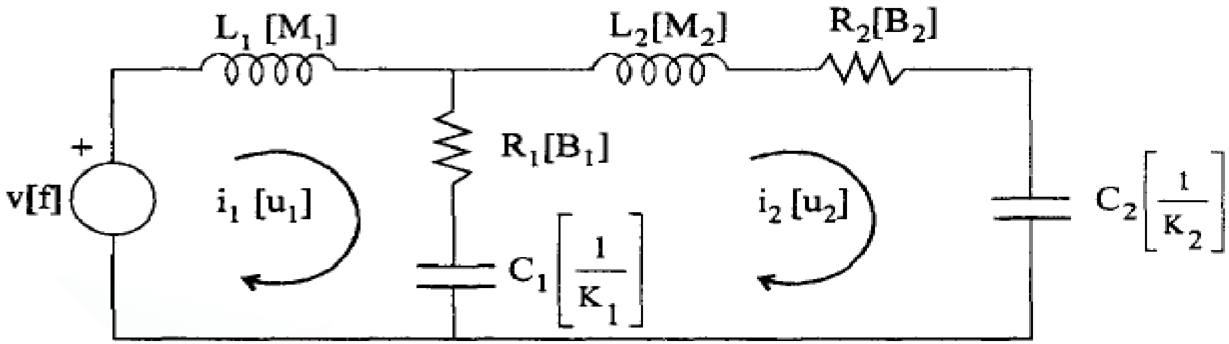


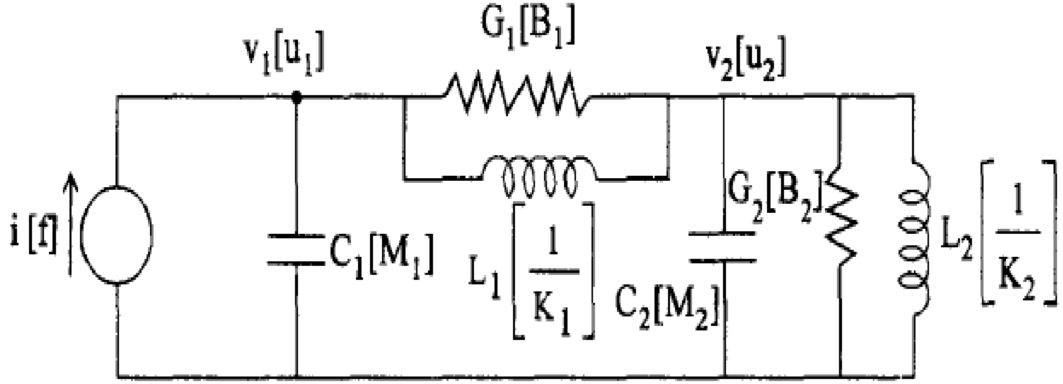
$$M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} + K_1 (x_1 - x_2) = f$$

$$M_2 \frac{d^2 x_2}{dt} + B_2 \frac{dx_2}{dt} + B_1 \frac{d(x_2 - x_1)}{dt} + K_2 x_2 + K_1(x_2 - x_1) = 0$$



Modeling of physical systems



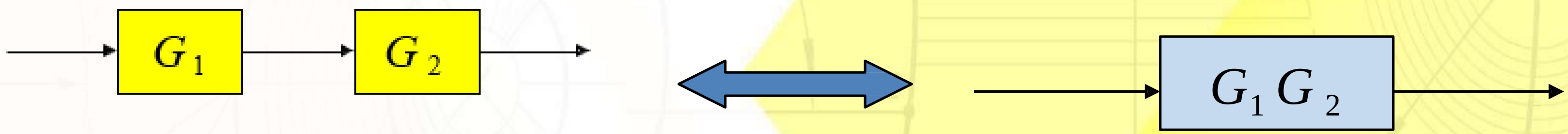


Block Diagram Algebra

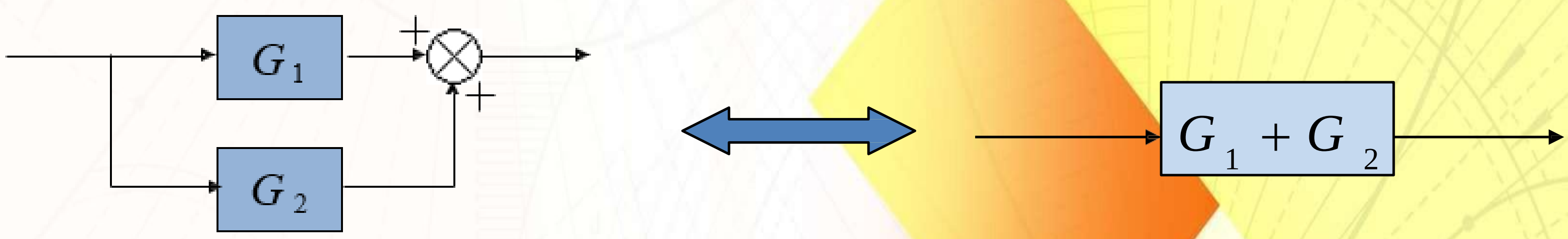
- We often represent control systems using block diagrams. A block diagram consists of blocks that represent transfer functions of the different variables of interest.
- If a block diagram has many blocks, not all of which are in cascade, then it is useful to have rules for rearranging the diagram such that you end up with only one block.

Reduction techniques

1. Combining blocks in cascade

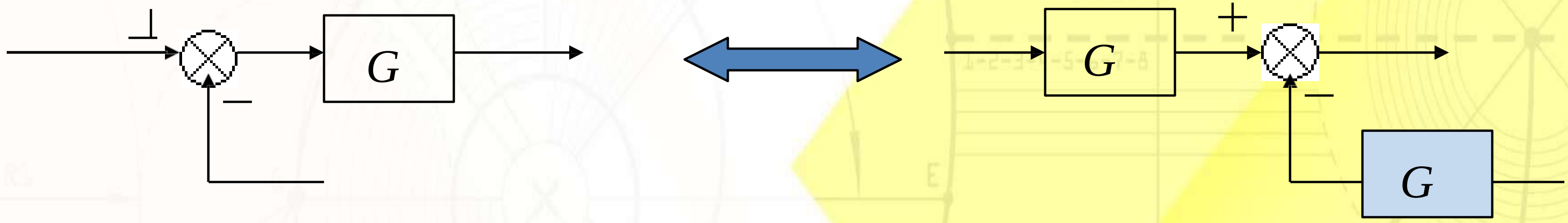


2. Combining blocks in parallel

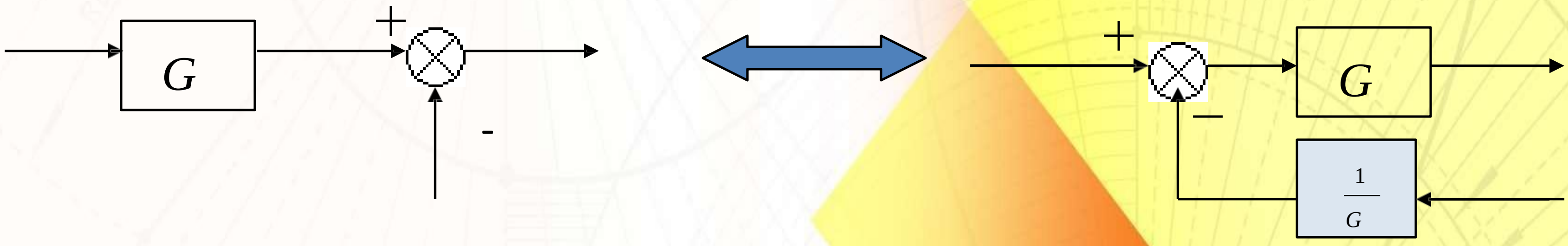


Reduction techniques

3. Moving a summing point behind a block



4. Moving a summing point ahead of a block



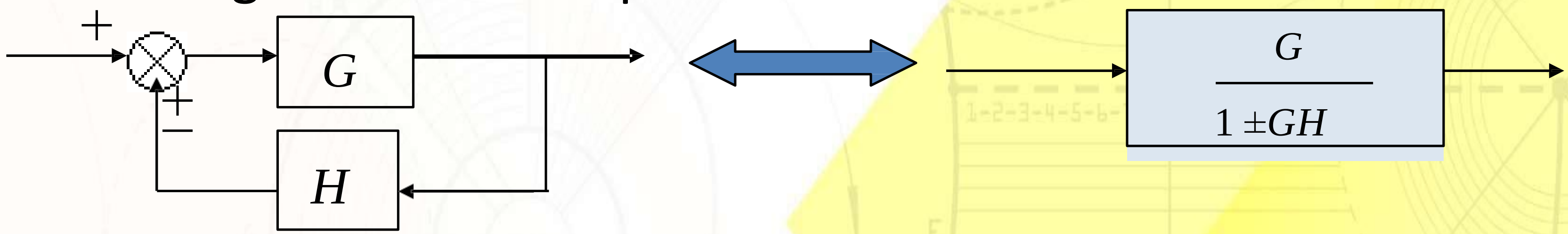
Reduction techniques

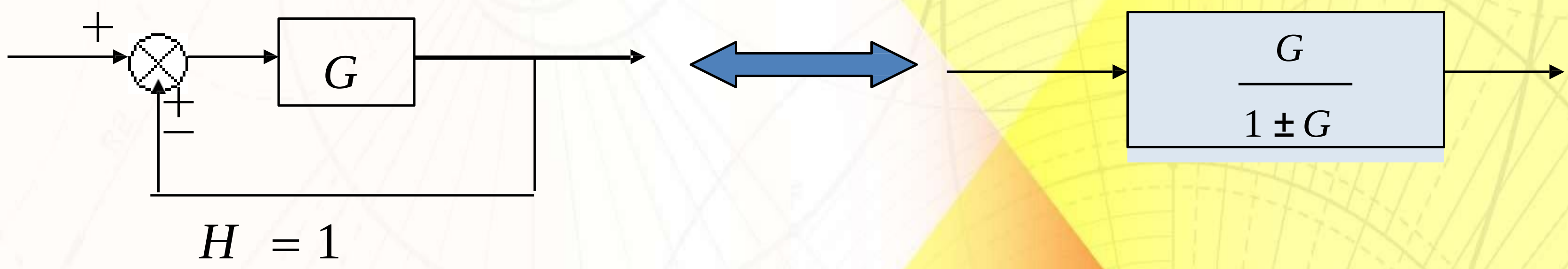
4. Moving a pickoff point behind a block

5. Moving a pickoff point ahead of a block

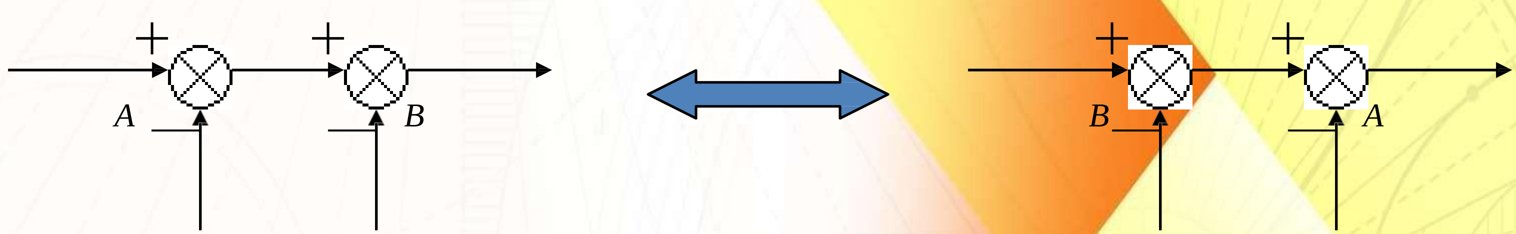
Reduction techniques

6. Eliminating a feedback loop

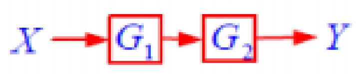
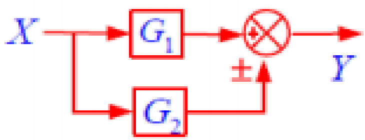
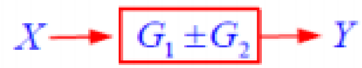
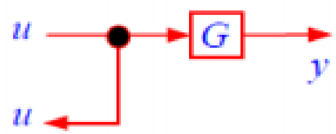
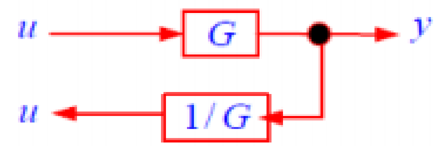
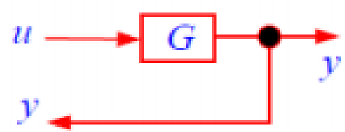
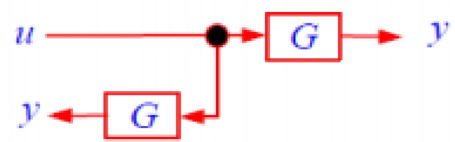
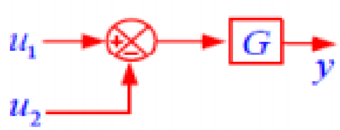
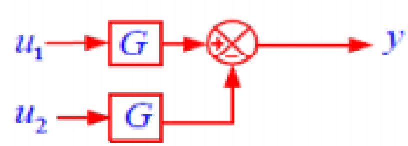
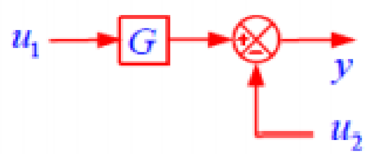
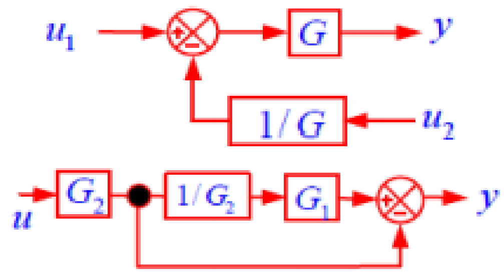




7. Swap with two neighboring summing points



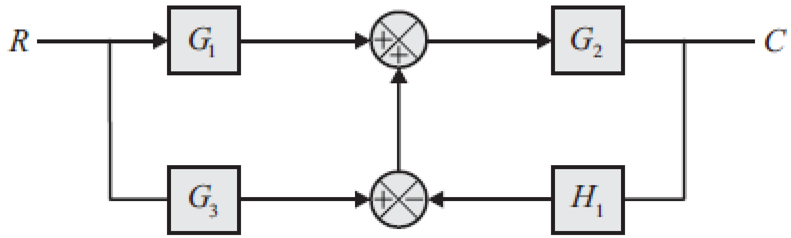
Rules in block diagram reduction

<table>
<tr><th></th><th>Manipulation</th><th>Original Block Diagram</th><th>Equivalent Block Diagram</th><th>Equation</th></tr>
<tr><td>1</td><td>Combining Blocks in Cascade</td><td></td><td></td><td>$Y=(G_1G_2)X$</td></tr>
<tr><td>2</td><td>Combining Blocks in Parallel; or Eliminating a Forward Loop</td><td></td><td></td><td>$Y=(G_1 \pm G_2)X$</td></tr>
<tr><td>3</td><td>Moving a pickoff point behind a block</td><td></td><td></td><td>$y = G u$
$u = \frac{1}{G} y$</td></tr>
<tr><td>4</td><td>Moving a pickoff point ahead of a block</td><td></td><td></td><td>$y = Gu$</td></tr>
<tr><td>5</td><td>Moving a summing point behind a block</td><td></td><td></td><td>$e_2 = G(u_1 - u_2)$</td></tr>
<tr><td>6</td><td>Moving a summing point ahead of a block</td><td></td><td></td><td>$y = Gu_1 - u_2$

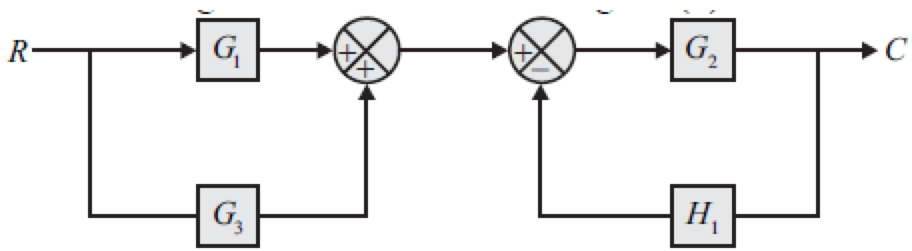
$y = (G_1 - G_2)u$</td></tr>
</table>

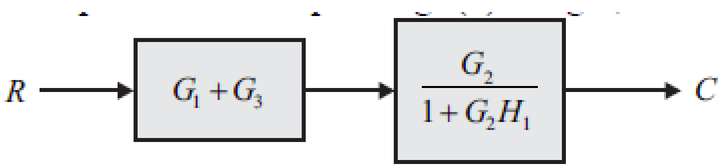
Block diagram reduction technique

What is the overall transfer function of the block diagram

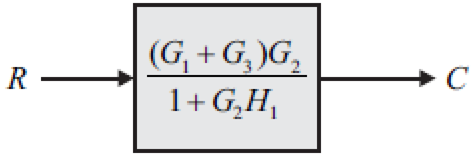


Rearrange the above block

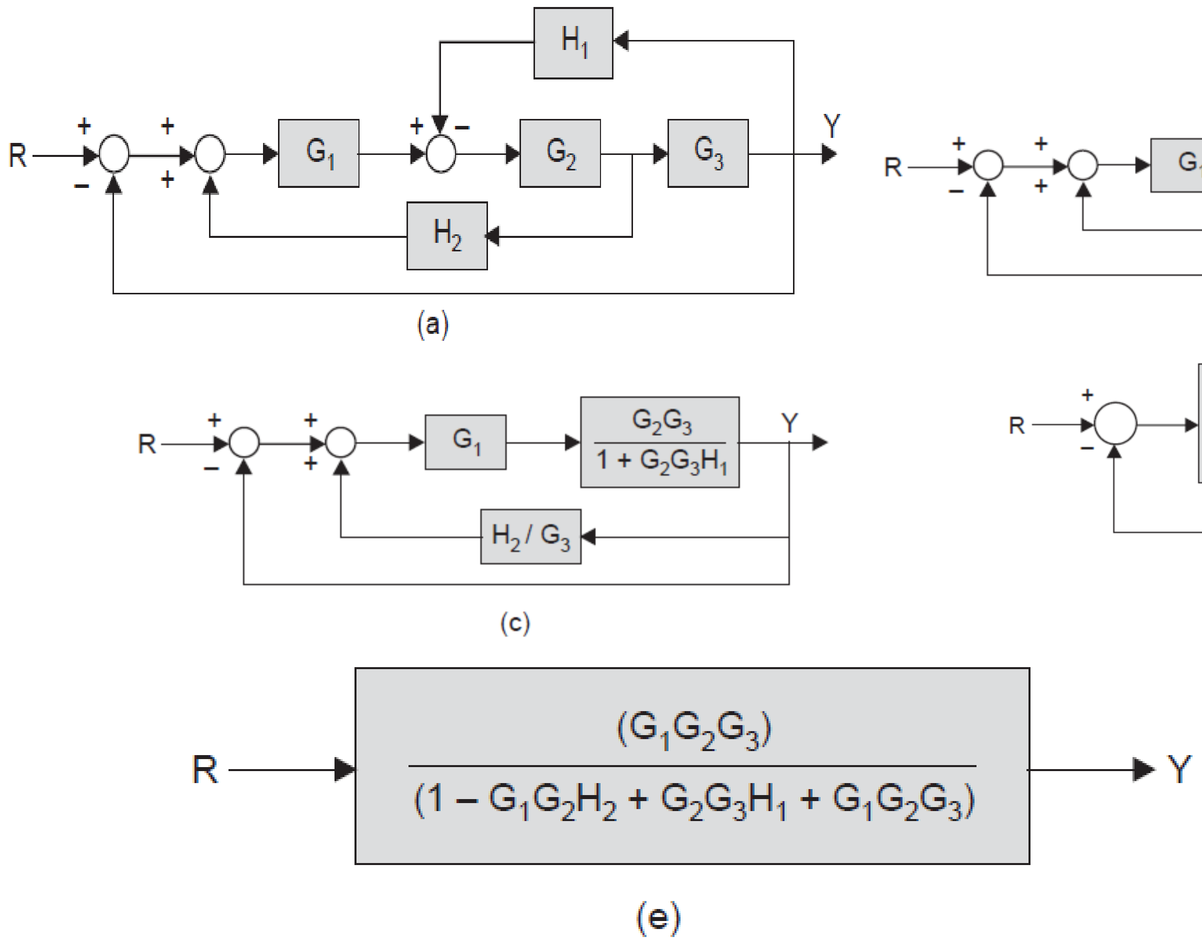




cascade combination

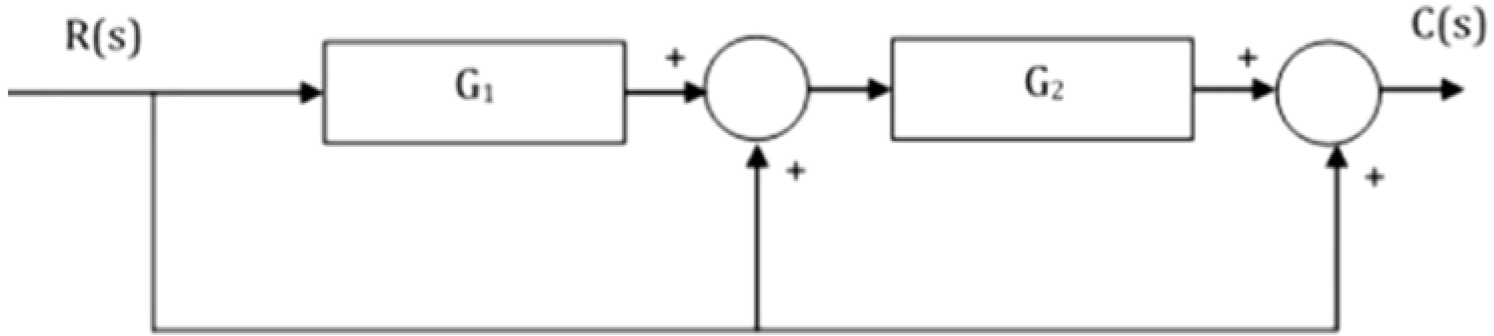


Block diagram reduction technique



Block diagram reduction technique

Find the transfer function $\frac{C}{R}$



We will discuss