GATE – CIVIL ENGINEERING

CONCRETE STRUCTURES (RCC & PSC)
Construction Materials and Management

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REINFORCED CEMENT CONCRETE

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Partial factors of safety:

In limit method of design partial factor of safety are used to account for uncertainty in material strength and uncertainty in loading.

a. For materials

$$f_d = \frac{f}{\gamma_m}$$

f_d : Design strength of material

f : Characteristic strength of material

γ_m : Partial safety factor appropriate to the material

b. For loads

$$F_d = \gamma_f . F$$

F_d : Design load

γ_f : Partial safety factor appropriate to the nature of loading

F : Characteristic load

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The value of partial safety factor for material strength should account for

- Possibility of deviation of the strength of material
- Accuracy of the calculation procedure
- Risk to life and economic consequences

The value of partial safety factor for loads should account for

- Unusual increase in loads beyond that used for deriving characteristic values.
- Unforeseen stress distribution
- Inaccurate assessment of the effects of loading
- Importance of the limit state considered

Load	IS specification
Dead loads	IS:875 (Part 1): 1987
Live loads	IS:875(Part 2):1987
Wind load	IS:875(Part 3): 1987
Seismic loads	IS:1893:1984

In general for LSD:

For loads, $\gamma_f = 1.5$

For materials

For concrete, $\gamma_m = 1.5$

For steel, $\gamma_m = 1.15$

Partial safety factors for various load combinations:

Load combination	Limit state of collapse			Limit state of serviceability		
	DL	LL	WL/EL	DL	LL	WL/EL
DL + LL	1.5	1.5	-	1.0	1.0	-
DL + WL	1.5 * or (0.9)	-	1.5	1.0	-	1.0
DL+LL+WL	1.2	1.2	1.2	1.0	0.8	0.8

- The value of 0.9 is considered when stability against overturning or reversal of stresses takes place
- For limit state of collapse, design loads are considered
- For limit state of serviceability, actual loads are considered.

Wind load, Earthquake load both may not occur simultaneously, maximum value of WL or EL is considered.

Increase in permissible stresses:

Where stress due to wind load or earthquake load, temperature and shrinkage effects are considered those due to live load, dead load and impact load stress specified may be exceeded upto $33\frac{1}{3}\%$

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eg. The member in a structure is subjected to a dead load = 100 kNm, live load = 160 kN/m, wind load = 80 kN/m and earthquake load = 120 kN/m.

a. The design load for limit state of collapse is …….

Design load for collapse is the maximum of the following combinations.

i. DL + LL combination = 1.5 (DL + LL) = 1.5 (100+160) = 390 kN/m

ii. DL + WL combination = 1.5 (DL + WL or EL)

= 1.5 (100 + 120) = 330 kN/m

iii. (DL + LL + WL) combination = 1.2 (DL + LL + EL)

= 1.2 (100 + 160 + 120) = 456 kN/m

Collapse load = 456 kN/m

b. The design load for serviceability is ………

Design load for serviceability is the maximum of the following combinations.

i. (DL + LL) combination = 1.0 (DL + LL) = 1.0 (100 + 160) = 260 kN/m

ii. (DL + WL) combination = 1.0 (DL + WL or EL)

= 1.0 (100 + 120) = 220 kN/m

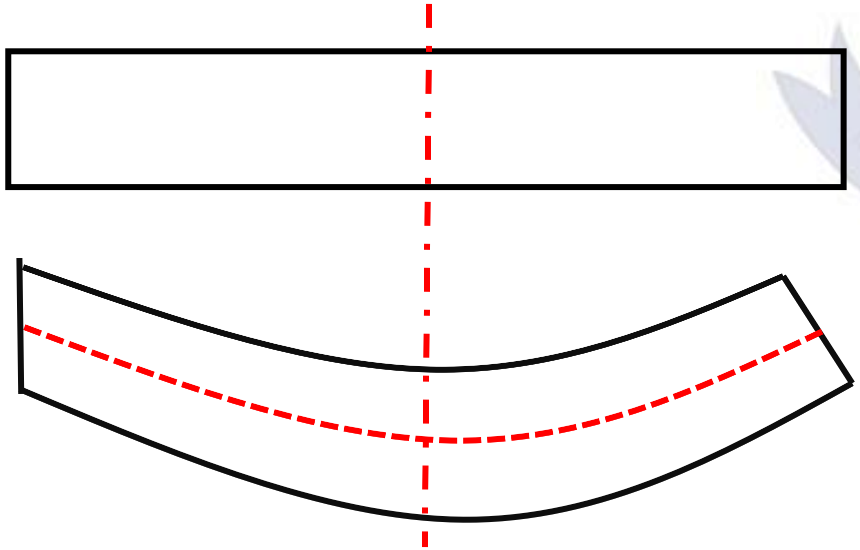
iii. (DL + LL + WL) combination = 1.0 × DL + 0.8 LL + 0.8 EL or WL

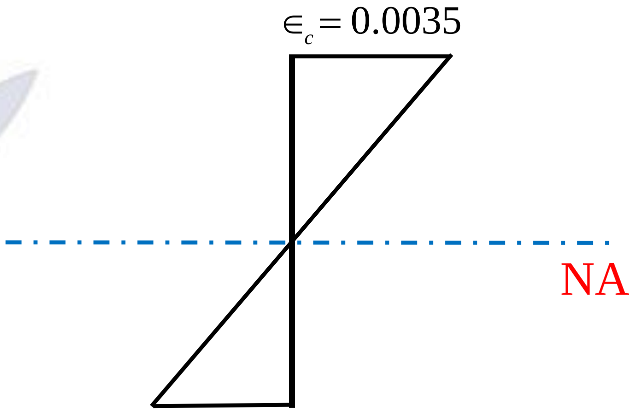
= 1 × 100 + 0.8 × 160 + 0.8 × 120 = 324 kN/m

Design load for serviceability = 324 kN/m

Assumptions in limit state of collapse (flexure)

1. Plane sections normal to axis remain plane after bending
   - It means that the strain at any point on the cross section is directly proportional to its distance from the neutral axis. The strain diagram across the section is linear.
2. Maximum strain in concrete at the outermost compression fiber is taken as 0.0035 in bending.
   - Concrete is a brittle material and gets crushed in compression at low strain of 0.0035.





Variation of strain across the section

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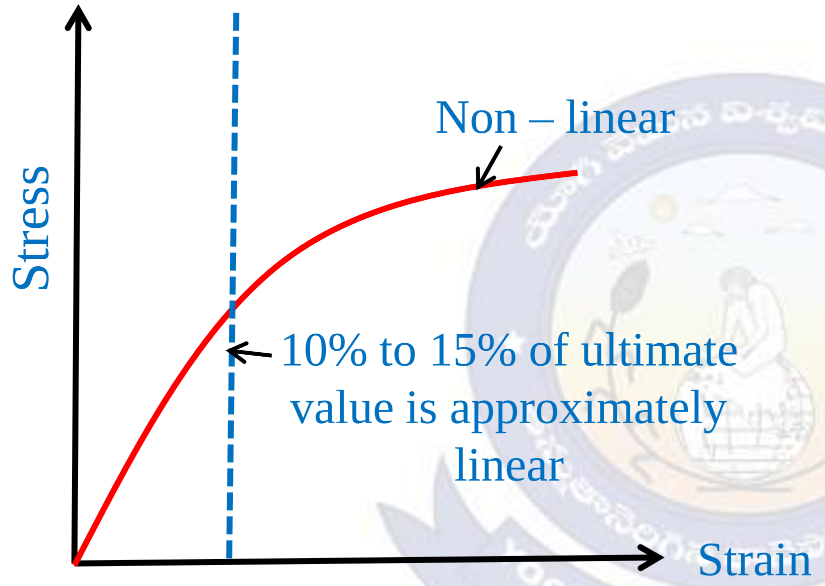
3. The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoidal, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test.



- Due to size effect, the characteristic strength of concrete f_{ck} reduced to $\frac{2}{3} f_{ck} = 0.67 f_{ck}$
- Design compressive stress in concrete, $f_d = \frac{0.67 f_{ck}}{\gamma_m} = \frac{0.67 f_{ck}}{1.5} = 0.45 f_{ck}$

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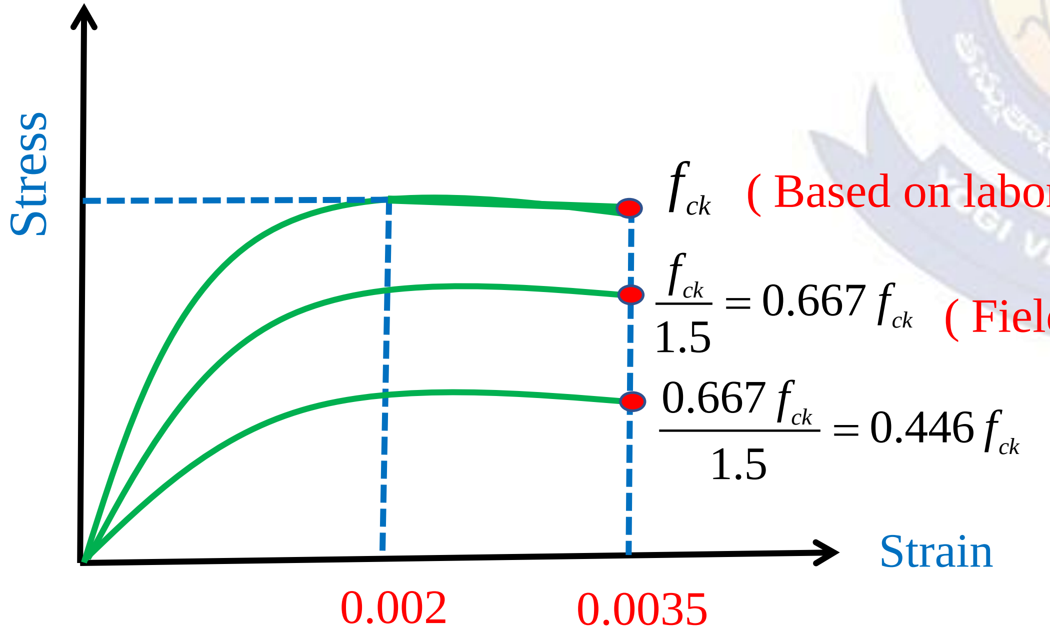
- As the size of concrete increases beyond 150 mm size of cube, compressive strength of concrete decreases upto a size of 450 mm and thereafter variation is very less. The actual size of structural concrete member may be more than the cube size tested in the laboratory. To account for size effect the design strength of concrete reduced to $0.67\, f_{ck}$
- The variation of stress-strain curve is parabolic upto a strain of 0.002 and thereafter stress remains constant upto maximum permissible strain of 0.0035. In Limit state method, it is considered as an idealized curve for concrete in compression. The curve is valid for all grades of concrete irrespective of percentage of tensile steel reinforcement.



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4. The tensile strength of concrete is ignored

- The cracked concrete does not contribute towards the enhancement of moment of resistance of the section. However, cracked concrete providing bond between the steel and concrete for developing tensile strain and thus stress in steel reinforcement

5. The stresses in reinforcement are derived from representative stress-strain curve for the type of steel used.

For design purposes the partial safety factor $\gamma_m = 1.15$ shall be applied.

Design stress for steel, $f_d = \dfrac{f_y}{1.15} = 0.87 f_y$

For Fe 250 grade steel, $f_d = 0.87 \times 250 = 217.5 \text{ N/mm}^2$

Strain at yield stress, $\varepsilon_y = \dfrac{f_y}{E_S} = \dfrac{250}{2 \times 10^5} = 0.00125$

Strain at design yield stress, $\varepsilon_{yd} = \dfrac{0.87 \times 250}{2 \times 10^5} = 0.00109$

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Since $\varepsilon_y > \varepsilon_{yd} \Rightarrow f_d = 217.5\,\text{N/mm}^2$

For Fe 415 grade steel

Strain at yield stress of $0.8\, f_{yd}$, $\varepsilon_{yd} = \dfrac{0.8 \times 0.87 \times 415}{2 \times 10^5} = 0.00144$

Strain at yield stress of $0.9\, f_{yd}$, $\varepsilon_{yd} = \dfrac{0.9 \times 0.87 \times 415}{2 \times 10^5} + 0.0003 = 0.00192$

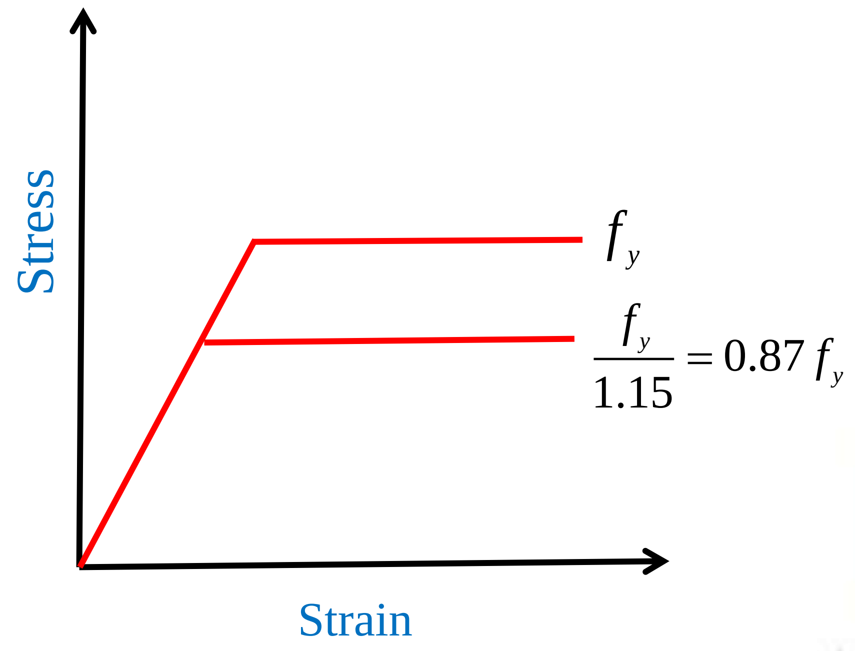
Strain at yield stress of f_{yd}, $\varepsilon_{yd} = \dfrac{0.87 \times 415}{2 \times 10^5} + 0.002 = 0.00380$

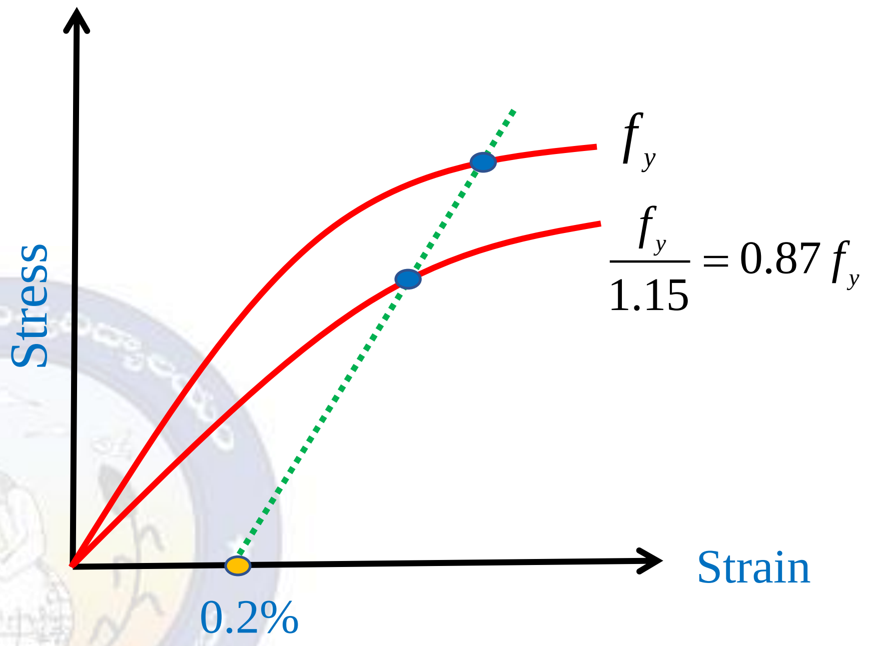
For Fe 500 grade steel,

Strain at yield stress of $0.9\, f_{yd}$, $\varepsilon_{yd} = \dfrac{0.9 \times 0.87 \times 500}{2 \times 10^5} + 0.0003 = 0.00226$

The design stress values are obtained based on the strain at yield for Fe415 and Fe50 grade steel. Typical points on the stress-strain curves are as shown in table.

Stress Level	Fe 415		Fe 500	
	Strain	Stress	Strain	Stress
$0.80f_{yd}$	0.00144	288.7	0.00174	347.8
$0.85f_{yd}$	0.00163	306.7	0.00195	369.6
$0.90f_{yd}$	0.00192	324.8	0.00226	391.3
$0.95f_{yd}$	0.00241	342.8	0.00277	413.0
$0.975f_{yd}$	0.00276	351.8	0.00312	423.9
$1.0f_{yd}$	0.00380	360.9	0.00417	434.8





6. The maximum strain in tension reinforcement in the section at failure shall not be less than

$$\varepsilon_s \nless \frac{f_y}{1.15 E_s} + 0.002$$

f_y : Characteristic strength of steel
E_s : Modulus of elasticity of steel

- Failure of member is always ductile in nature. i.e. considerable deformation gives before failure
- It restricts the depth of neutral axis

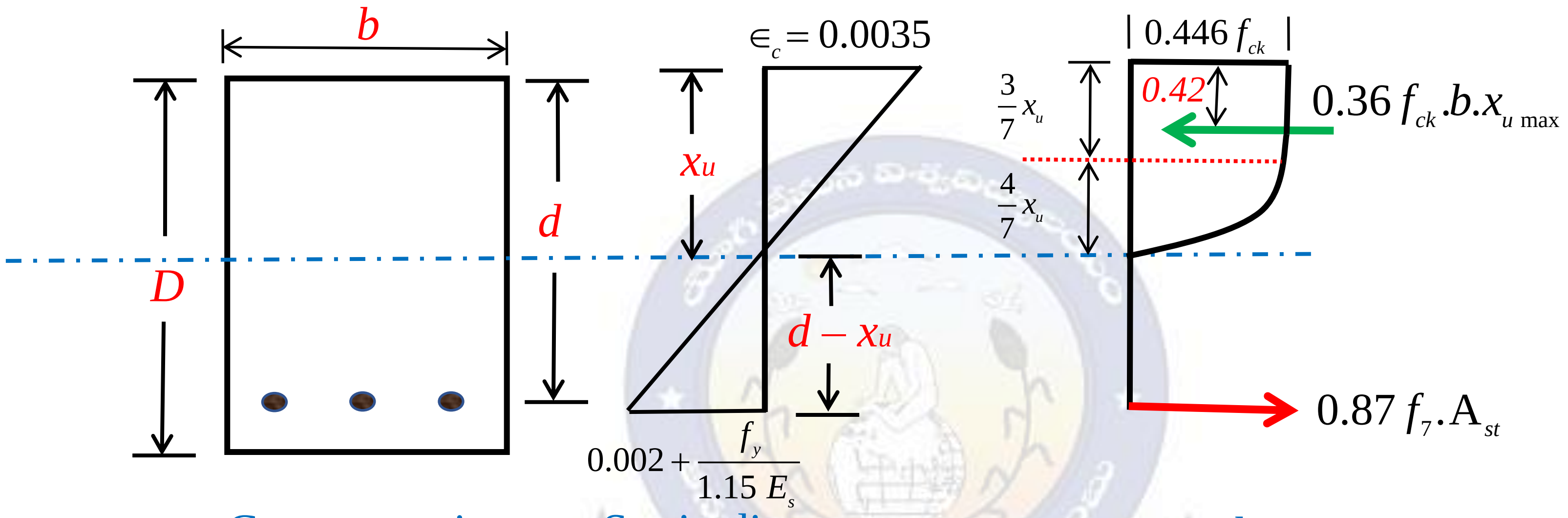
For Fe 250 grade steel, $\varepsilon_s \nless \frac{250}{1.15\times 2\times 10^5}+0.002 \Rightarrow \varepsilon_s \nless 0.00308$

For Fe 415 grade steel, $\varepsilon_s \nless \frac{415}{1.15\times 2\times 10^5}+0.002 \Rightarrow \varepsilon_s \nless 0.00380$

For Fe 500 grade steel, $\varepsilon_s \nless \frac{500}{1.15\times 2\times 10^5}+0.002 \Rightarrow \varepsilon_s \nless 0.00417$

- Minimum strains at collapse ensures that the design stress in tensile steel at collapse is always equal to f_{yd}.
- For mild steel, strain corresponding to $0.87\ f_y,\ \varepsilon_s = \frac{0.87\times 250}{2\times 10^5} = 0.00109$
- For Fe 250, the strain of 0.00308 is much larger than that required for the design stress . It ensures sufficient ductility in mild steel besides causing the design stress.

Singly Reinforced Beams:



Cross – section Strain diagram Stress diagram

C_1: Compressive resistance due to rectangular portion of rectangular stress block

$$= 0.446\, f_{ck} \times \frac{3}{7} x_u \times b = 0.191\, f_{ck}.b.x_u$$

C_2: Compressive resistance due to parabolic portion of stress block

$$= \frac{2}{3} \times 0.446\, f_{ck} . \frac{4}{7} x_u.b = 0.169\, f_{ck} b.x_u$$

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C = Total compressive resistance of concrete

$$C = C_1 + C_2$$

$$= 0.191 f_{ck} b\, x_u + 0.169\, f_{ck} b.x_u$$

$$C = 0.36 f_{ck}.b.x_u$$

T= Tensile force due to tensile steel reinforcement

$$T = 0.87\, f_y A_{st}$$

a : Distance of compressive force from the extreme compression fibre

$$= \frac{C_1 a_1 + C_2 a_2}{C_1 + C_2}$$

$$= \frac{0.191\, f_{ck}.b.x_u \times \frac{1}{2}.\frac{3}{7} x_u + 0.169\, f_{ck}.b.x_u \left(\frac{3}{7} x_u + \frac{3}{8}.\frac{4}{7} x_u \right)}{0.36\, f_{ck}.b.x_u} = 0.42\, x_u$$

$$a = 0.42\, x_u$$

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Z : lever arm

= Distance between the compressive force C and tensile force T

$$z = d - 0.42\, x_u$$

If $M_u < M_{u,\lim} \Rightarrow$ Under reinforced section

If $M_u = M_{u,\lim} \Rightarrow$ Balanced section

If $M_u > M_{u,\lim} \Rightarrow$ Redesign the section either by increasing the dimension or design as doubly reinforced section.

Limiting depth of neutral axis ($x_{u,\max}$):

From the strain diagram,

$$\frac{x_u}{d - x_u} = \frac{0.0035}{0.002 + \dfrac{f_y}{1.15\,E_s}}$$

$$\frac{x_u}{d} = \frac{0.0035}{0.0055 + \dfrac{f_y}{1.15\,E_s}} \Rightarrow x_u = \left(\frac{0.0035}{0.0055 + \dfrac{f_y}{1.15\,E_s}}\right) d$$

When the strain in concrete and steel reaches its maximum values, x_u becomes $x_{u,\max}$.

For Fe415 grade steel, $x_{u,\max} = \left(\dfrac{0.0035}{0.0055 + \dfrac{415}{1.15 \times 2 \times 10^5}} \right) d = 0.48$ d

Grade of steel	$x_{u,\max}$
Fe 250	$0.53d$
Fe 415	$0.48d$
Fe 500	$0.46d$

- Limiting depth of neutral axis corresponds to balanced section or balanced failure in limit state of flexure.

If $x_u < x_{u,\max} \Rightarrow$ Under reinforced section Tension failure occurs

If $x_u > x_{u,\max} \Rightarrow$ Over reinforced section compression failure occurs.

Tension failure:

- Tension steel yields before ultimate strength is reached
- Beam undergoes considerable deformation and develops extensive crack before failure
- Ductile failure occurs and it gives advance warning before failure
- Large strains develop in tension steel and hence extensive cracks developed in concrete.

Compression failure:

- Concrete fails by compression
- Tension steel does not yield before ultimate strength is reached.
- The failure is sudden since the concrete is a brittle material.

Limiting Moment of Resistance ($M_{u,\lim}$):

Limiting moment of resistance occurs at the balanced section. i.e. $x_u = x_{u,\max}$

$$M_{u,\lim} = 0.36 f_{ck}.b.x_{u,\max}(d - 0.42 x_{u,\max})$$

For Fe415 grade steel,

$$M_{u,\lim} = 0.36 f_{ck}.b \times 0.48d\left(d - 0.42 \times 0.48d\right) = 0.138 f_{ck}.bd^2$$

$$M_{u\lim} = 0.148 f_{ck} b.d^2 \qquad \text{(Fe250)}$$

$$= 0.138 f_{ck} bd^2 \qquad \text{(Fe415)}$$

$$= 0.133 f_{ck} bd^2 \qquad \text{(Fe500)}$$

Limiting percentage of tensile reinforcement ($p_{t,\lim}$):

$p_{t,\lim}$: Percentage of tensile steel reinforcement at balanced section

$$C = T$$

$$0.36 f_{ck}.b.x_{u,\max} = 0.87 f_y A_{st} \qquad \frac{A_{st}}{b} = \frac{0.36 f_{ck}}{0.87 f_y}.x_{u,\max}$$

Limiting % of tensile reinforcement, $p_{t,\lim} = \frac{A_{st}}{bd} \times 100$

$$= \frac{0.36 f_{ck}}{0.87 f_y}.\frac{x_{u,\max}}{d} \times 100$$

$$p_{t,\lim} = 41.38.\frac{f_{ck}}{f_y}.\frac{x_{u,\max}}{d}$$

If $p_t < p_{t,\lim} \Rightarrow$ the section is Under reinforced section

If $p_t > p_{t,\lim} \Rightarrow$ the section is Over reinforced section

If $p_t = p_{t,\lim} \Rightarrow$ the section is Balanced section

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Moment of Resistance increases as the % of steel reinforcement increases upto the balanced steel reinforcement and thereafter remains constant.

eg. For M20 grade concrete and Fe415 grade steel,

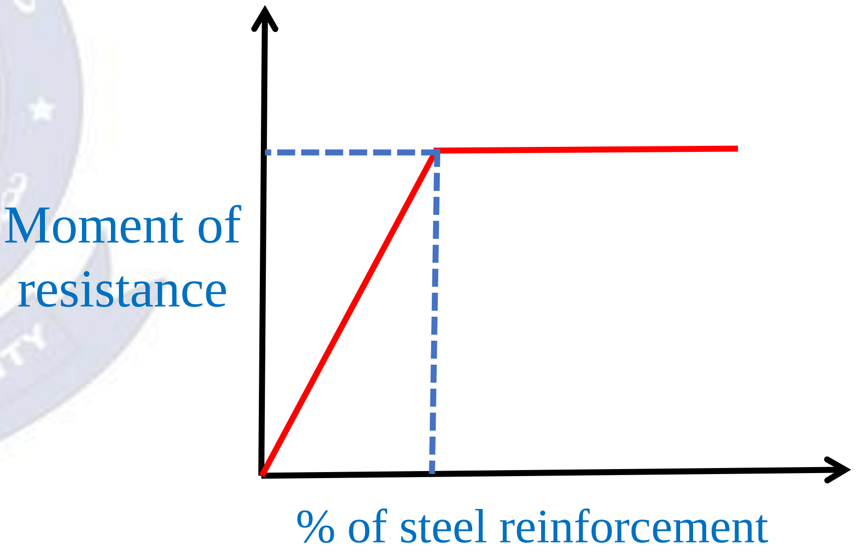
$$p = 41.38 \times \frac{20}{415} \times 0.48$$

(or)

$$0.36 \times 20 \times b \times 0.48d = 0.87 \times 4.15 A_{st}$$

$$\frac{A_{st}}{bd} = 0.0096$$

$$p = \frac{A_{st}}{bd} \times 100 = 0.96\%$$



Types of sections:

x_u : Actual depth of neutral axis

$$C = T$$

$$0.36\, f_{ck} b.x_u = 0.87\, f_y A_{st}$$

$$x_u = ?$$

a. If $x_u < x_{u,\max}$, the section is Under reinforced section

$$MR = T.z$$

$$MR = 0.87\, f_y A_{st} (d - 0.42\, x_u)$$

$$MR = 0.87\, f_y . A_{st} . d \left(1 - \frac{f_y . A_{st}}{f_{ck} . bd} \right)$$

b. If $x_u > x_{u,\max}$, the section is Over reinforced section

$$MR = C.z$$

x_u becomes equal to $x_{u,\max}$

$$MR = 0.36\, f_{ck}.b.x_{u\max}(d - 0.42\, x_{\max})$$

or

$$MR = M_{u,\lim} = 0.148\, f_{ck} b.d^2 \qquad \text{(Fe250)}$$

$$= 0.138\, f_{ck} bd^2 \qquad \text{(Fe415)}$$

$$= 0.133\, f_{ck} bd^2 \qquad (Fe500)$$

- Moment of resistance of the over reinforced section is equal to that of balanced section since the depth of neutral axis is limited.

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c. If $x_u = x_{u,\max}$, the section is balanced section

$$MR = C.z \quad or \quad MR = T.z$$

$$MR = 0.36\, f_{ck}.b.x_{u\max}(d - 0.42\, x_{\max})$$

$$MR = 0.87\, f_y A_{st}(d - 0.42\, x_{u,\max})$$

$$MR = M_{u,\lim} = 0.148\, f_{ck} b.d^2 \qquad \text{(Fe250)}$$

$$= 0.138\, f_{ck} bd^2 \qquad \text{(Fe415)}$$

$$= 0.133\, f_{ck} bd^2 \qquad (Fe500)$$

- For balanced section, steel may mitigate failure, but collapse of structure is due to crushing of concrete.
- Balanced sections are not advisable for design since it causes sudden collapse by crushing of concrete.
- $x_{u,\max}$ depends on grade of steel only and it is independent of grade of concrete.

Tension reinforcement:

Minimum reinforcement:

- The minimum area of tension reinforcement shall be not less than that given by

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

A_s : minimum area of tension reinforcement

b : Width of beam

d : Effective depth

f_y : Characteristic strength of reinforcement, N/mm^2

- If the steel provide is less than the minimum steel, sudden collapse occurs in beam due to breakage of bars.

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Maximum Reinforcement:

- The maximum area of tension reinforcement shall not exceed 0.04bD
- Maximum area of tension reinforcement = 4% of the gross cross sectional area.
- The limit of maximum steel reinforcement is based on ease in placing and compacting concrete in formwork.

Eg. For Fe500 grade steel reinforcement, minimum percentage of steel is ……….

$$\frac{A_s}{bd} = \frac{0.85}{f_y}$$

$$p = \frac{A_s}{bd} \times 100 = \frac{0.85}{f_y} \times 100 = \frac{0.85}{500} \times 100 = 0.17\%$$

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