GATE – CIVIL ENGINEERING

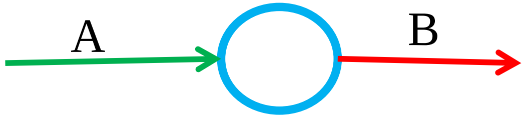
CONCRETE STRUCTURES (RCC & PSC)
Construction Materials and Management

Prof.B.Jayarami Reddy
Professor and Head
Department of Civil Engineering
Y.S.R. Engineering College of
Yogi Vemana University,
Proddatur, Y.S.R.(Dt.), A.P-516360.
E.mail : bjrcivilgate@gmail.com

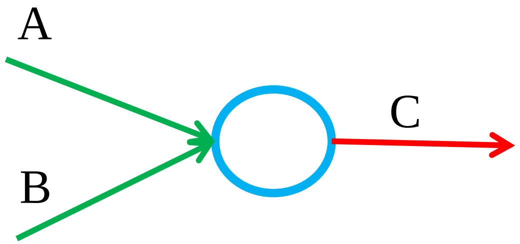
Common Partial situations in Network

Partial situation | **Representation**

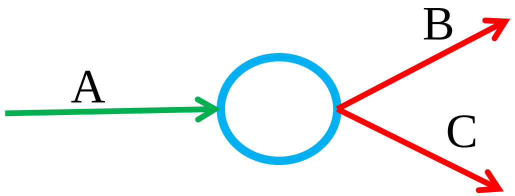
1. B is controlled by A.
   Operation B cannot begin until operation A is completed.



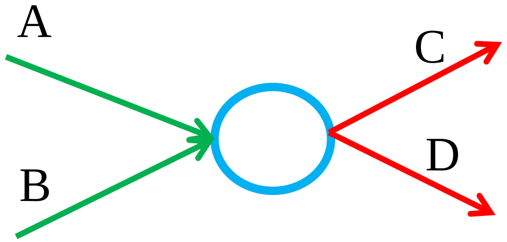
2. C is controlled by A and B. Operation C cannot begin
   until operations A and B are completed



3. Activities B and C are controlled by activity A.
   Neither of activities B and C can start unless A is completed.



4. Activities C and D are controlled by activities A and B.
   Neither of activities C and D can start until A and B
   are completed. However, C and D can be started
   independent of each other.

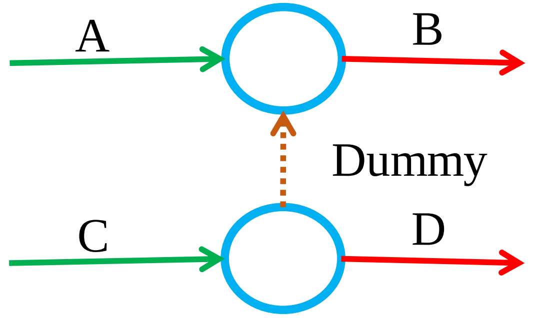


Prof. B. Jayarami Reddy

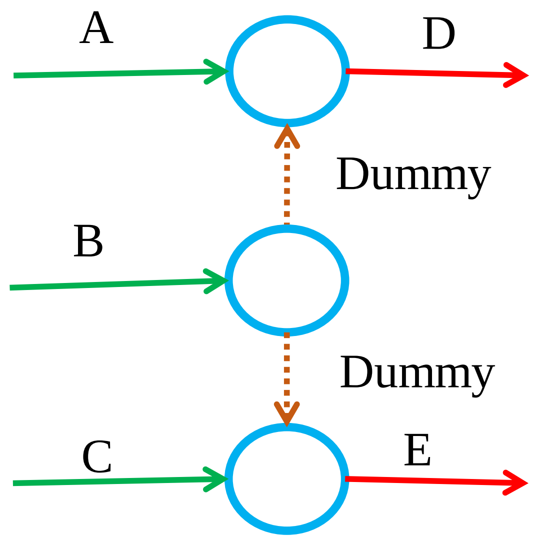
Common Partial situations in Network

Partial situation | Representation

5. Activity B is controlled by A and C, while Activity D is controlled by activity C only.

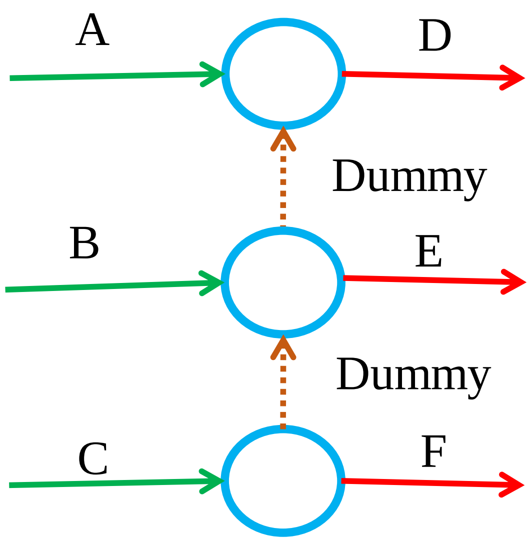


6. Activity D is controlled by A and B, while activity E is controlled by activity B and C.

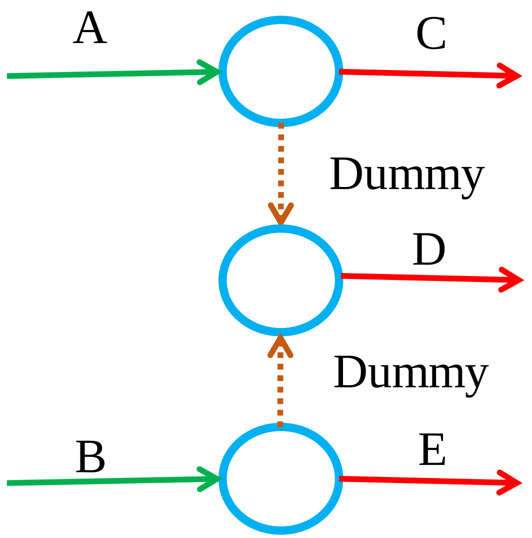


Prof. B. Jayarami Reddy

7. Activity D is controlled by A, B and C. However, activity E is controlled by B and C.

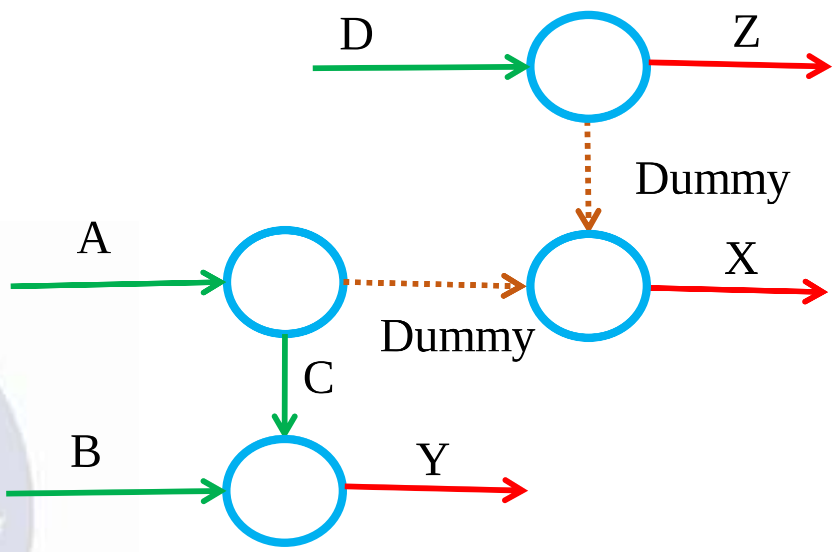


8. Activity A controls C and D, while activity B controls D and E. Thus, D is controlled by both A and B.



Prof. B. Jayarami Reddy

9. Activity X is controlled by D and A; activity Y is control led by A, B and C, while activity Z is controlled by D only.



Prof. B. Jayarami Reddy

NUMBERING THE EVENTS

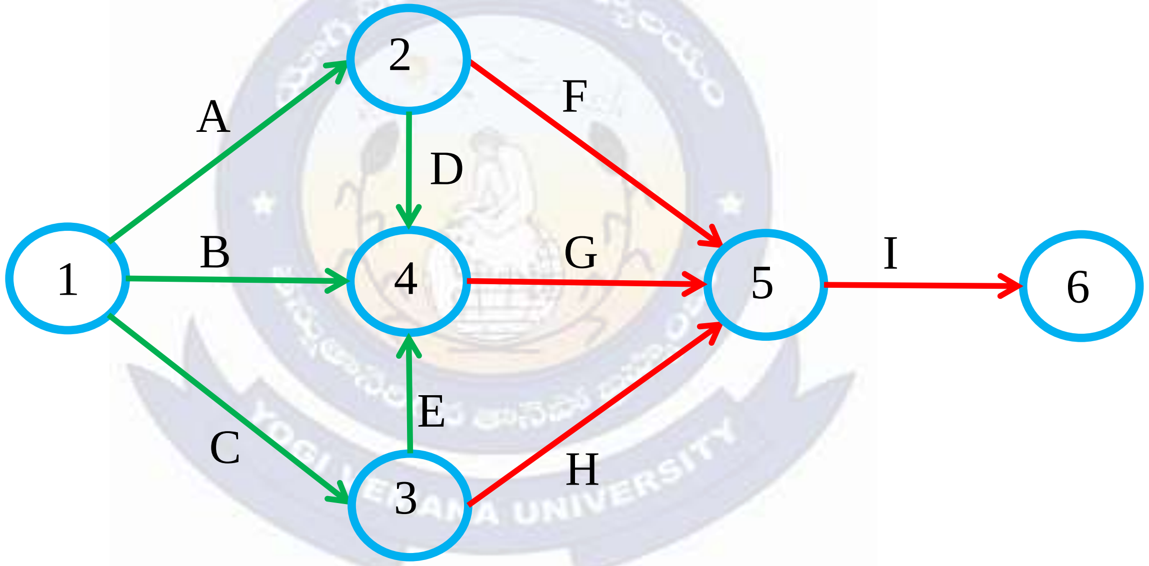
It is essential to number the events or node points.

The activities joining the nodes can better be identified on the network by the event numbers or node numbers at the tail and head of the activity.

In a big network, the problem of numbering can be simplified if the rules devised by D.R. Fulkerson are followed.

The sequential numbering to the events may be assigned in the following steps:

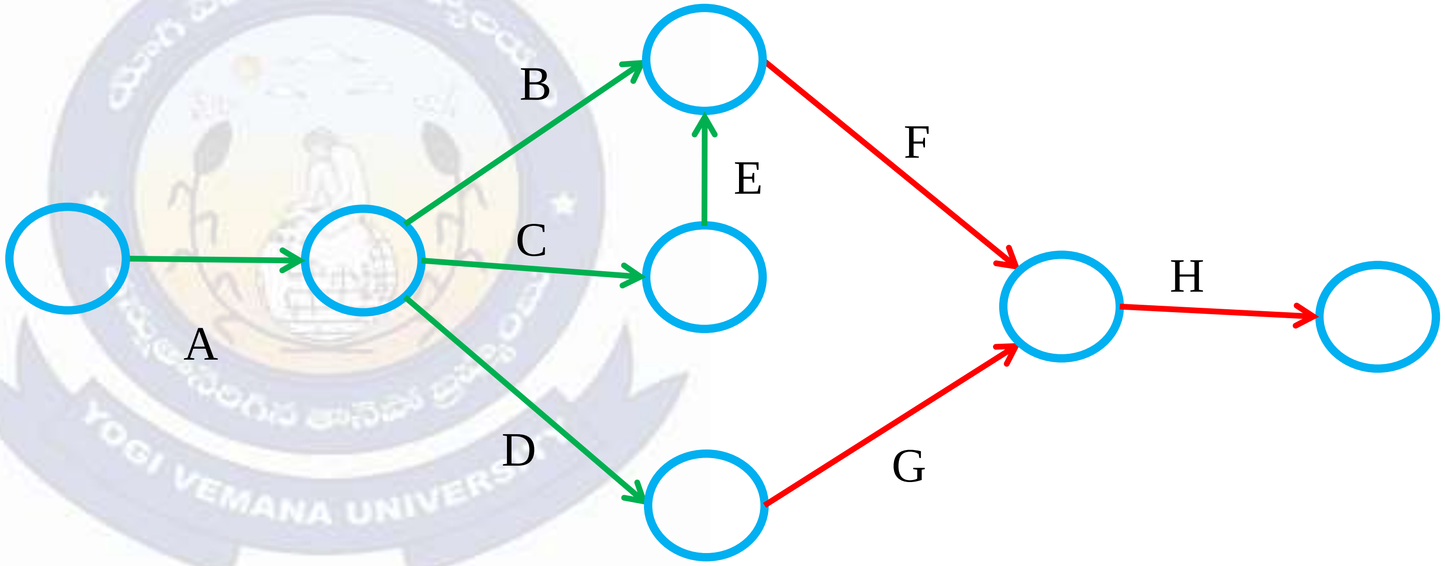
1. There is a single initial event in a network diagram. This initial event will have arrows coming out of it and non entering it. Number this initial event as 1.
2. Neglect all the arrows emerging out of the initial event numbered 1. Doing so will apparently provide one or more new initial events.
3. Number these apparently produced new initial events as 2, 3, 4 etc.
4. Again neglect all emerging arrows from these numbered events; this will create few more initial events.
5. Follow step 3.
6. Continue this operation until the last event, which has n emerging arrows, is numbered.



Prof. B. Jayarami Reddy

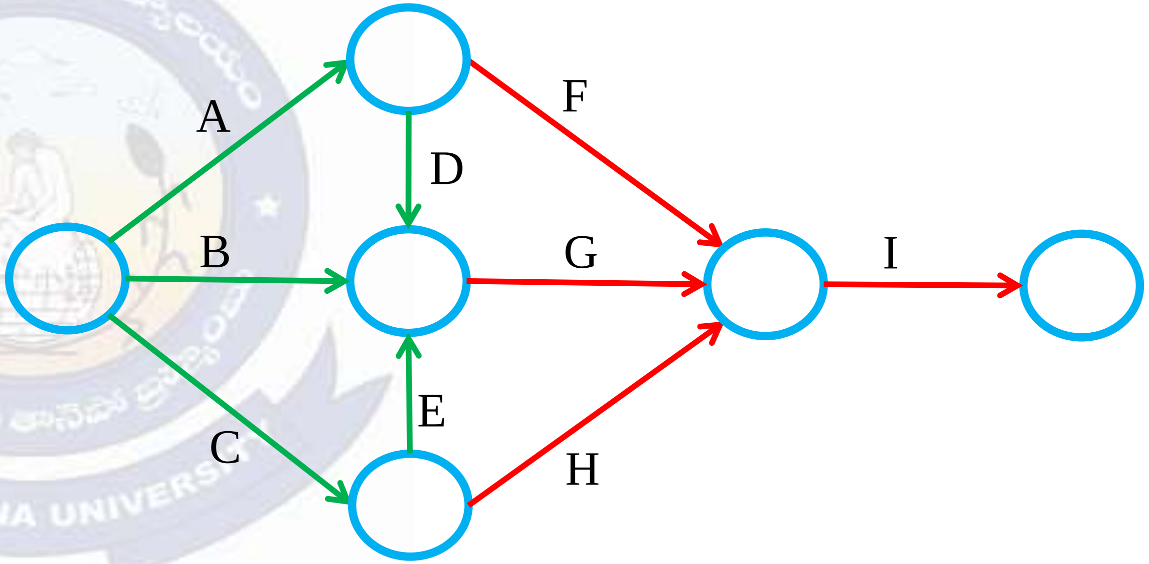
1. Draw an arrow diagram representation of the project having the activities as detailed below.

Job Activity	Immediate Predecessors
A	-
B	A
C	A
D	A
E	C
F	B,E
G	D
H	F,G



2. Draw the network diagram for the project having the following activities

Job Activity	Immediate Predecessors
A	-
B	-
C	-
D	A
E	C
F	A
G	B,D,E
H	C
I	F,G,H



Prof. B. Jayarami Reddy

3. Draw the network for the project having activities with the interrelationship given.

A and B start at the same time.

C follows A but preceeds E

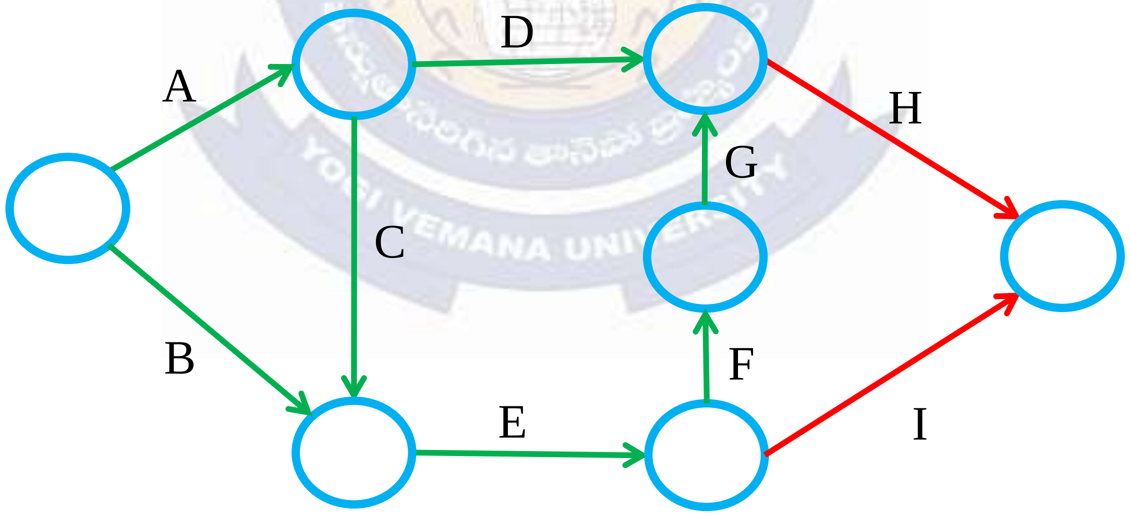
D follows A but preceeds H

E follows B and C but preceeds F and I

F follows E but preceeds G

G follows F but preceeds H

H and I are terminate at the same time.



Prof. B. Jayarami Reddy

Programme Evaluation Review Technique

INTRODUCTION

PERT was developed by the U.S Navy to accelerate the development of the Polaris Fleet Ballistic Missile.

PERT: Programme Evaluation and Review Technique

- used for planning, scheduling and monitoring the project.

TIME ESTIMATES

- A project is composed of many diversified activities which contribute to its completion.
- An important characteristic of any project is its duration.
- Its effectiveness and economy is often dependent on the project duration
- Total project duration depends on the time taken by each activity.
- Estimation of time required for any activity requires wide experience of similar activities.
- Estimation of these time estimates can be done in two ways

Prof. B. Jayarami Reddy

1. Deterministic approach: Planner has enough knowledge about the activity and gives a single estimate of the duration which is almost accurate.

2. Probabilistic Approach: Planner does not have much idea about the activity as there is little or no past history about it. The limits with in which the duration will lie, is estimated.

- PERT follows the probabilistic approach and absorbs the uncertainties into the time estimates for activity and project durations.
- PERT is well suited for those projects where there is insufficient or no background information for estimation of time duration.
- PERT is used in R & D type projects such as space industry, defence industry etc.
- PERT analysis is event oriented i.e. in this analysis interest is more focussed on the events (start or completion of activity) rather than the activities.

In order to take into account the uncertainties involved in the activity times three kinds of time estimates are made for each activity in PERT.

1. Optimistic time estimate (t_0)
2. Pessimistic time estimate (t_p)
3. Most likely time estimate (t_m)

Prof. B. Jayarami Reddy

1. Optimistic Time Estimate (t_0)

It is the minimum time required for an activity if everything goes perfectly well without any problems or adverse conditions developed during the execution of the activity.

No provisions are made for delays or setbacks and better than normal conditions are assumed to prevail during the execution of the activity.

2. Pessimistic Time Estimate (t_p)

It is the maximum time required for an activity if everything goes wrong and abnormal situations prevail.

It does not include the possible effects of major catastrophes such as flood. earthquakes, fire, labour strikes, etc

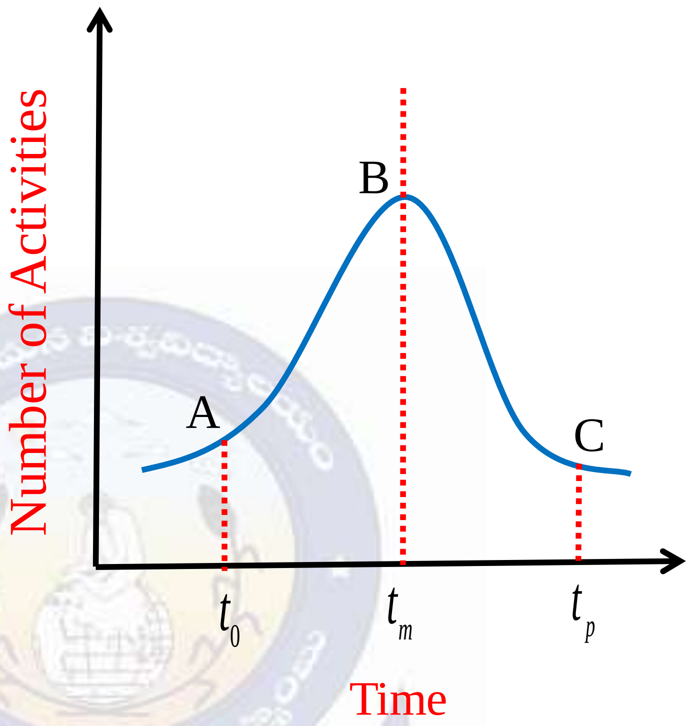
3. Most Likely time Estimate (t_m)

It is the time required to complete the activity if normal conditions prevail.

Most likely time estimate lies between pessimistic and optimistic time estimates.

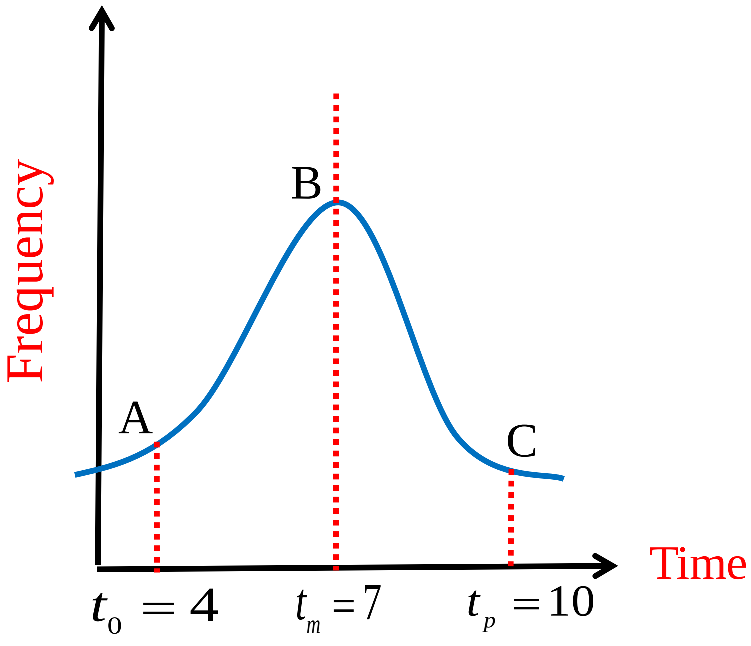
Prof. B. Jayarami Reddy

FREQUENCY DISTRIBUTION

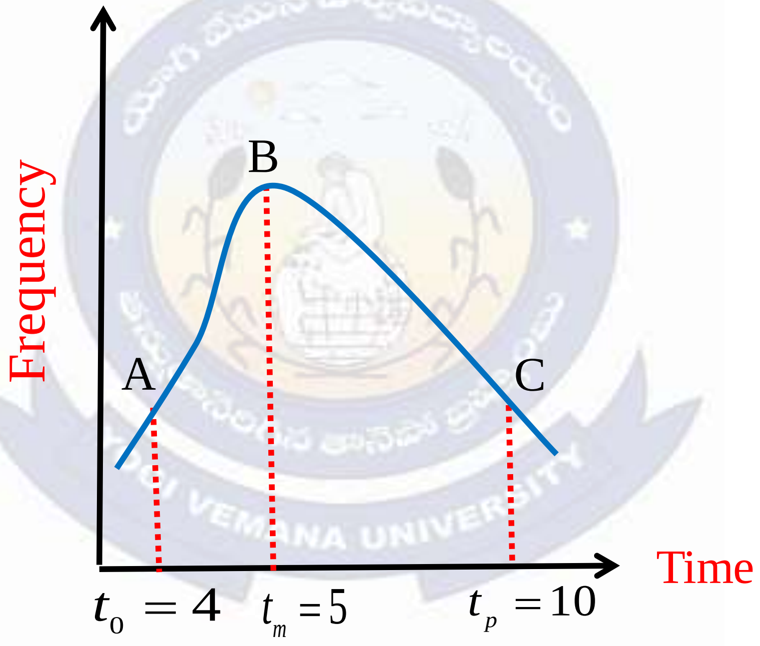


- It is a curve is plotted between the time of completion and the number of jobs completed in that time, a frequency distribution curve.
- In this curve point A corresponds to the optimistic time t_0, point C corresponds to pessimistic the t_p, and point B corresponds to most likely time t_m.
- It is clear from the curve that large number of activities are completed in most likely time

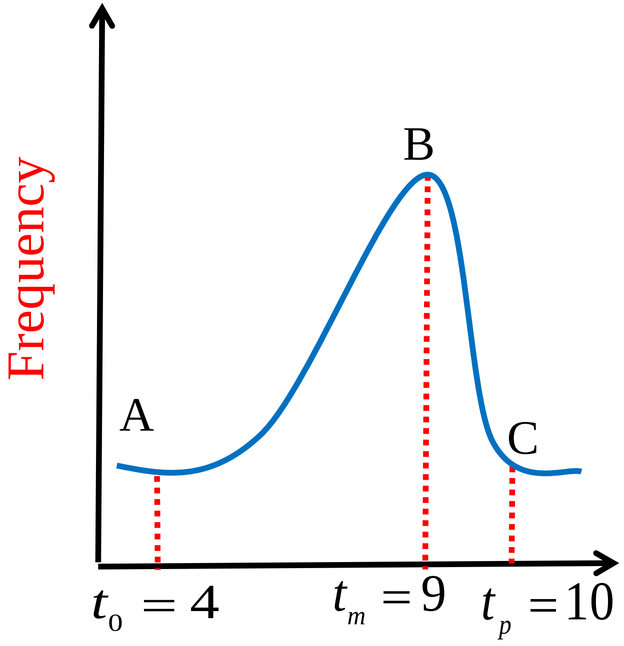
- Frequency distribution curves having a single hump or mode are called as Unimodal curves
- If the frequency distribution curve is symmetrical about modal axis it is known as a normal curve, otherwise it is said to be Skewed which may be left or right side.



Normal Curve



Left skewed Curve



Right Skewed Curve

Prof. B. Jayarami Reddy

MEAN, VARIANCE AND STANDARD DEVIATION OF THE DISTRIBUTION

1. Mean Time:

- It is defined as algebraic sum of time durations taken by various jobs divided by number of jobs.
- It is also called as Average Time or Mean of the distribution.

$$\bar{t} = \frac{t_1 + t_2 + t_3 + ... + t_n}{n} = \frac{\sum t}{n}$$

2. Deviation: It is the difference between the time under consideration and the mean time.

$$\delta = t - \bar{t}$$

Where

δ = deviation of any value x from the mean time value

t= value under consideration

Prof. B. Jayarami Reddy

3. Variance (σ^2): it is mean of the square of deviations.

$$\sigma^2 = \frac{\sum (t - \bar{t})^2}{n}$$

4. Standard deviation (σ): It is the square root of variance

$$\sigma = \sqrt{\frac{\sum (t - \bar{t})^2}{n}}$$

PROBABILITY DISTRIBUTION

- The three time estimates of PERT have some amount of uncertainties associated with it.
- Probability distribution can also be used for the analysis of time estimates.
- In frequency distribution we study the group behaviours, where as in probability distribution we have distribution of probability values for all possible outcomes.
- Hence probability distribution is preferred over the frequency distribution.
- The value of probability always ranges from 0 to 1.
- A probability distribution curve is a plot of a function *f(x)* called probability density function or simply probability function, with the height of curve standardized such that area under curve is equal to unity.

$$\int_{-x}^{x} f(x)dx = 1$$

Prof. B. Jayarami Reddy

The probability distribution curve can be represented by an equation.

$$y = f(x)$$

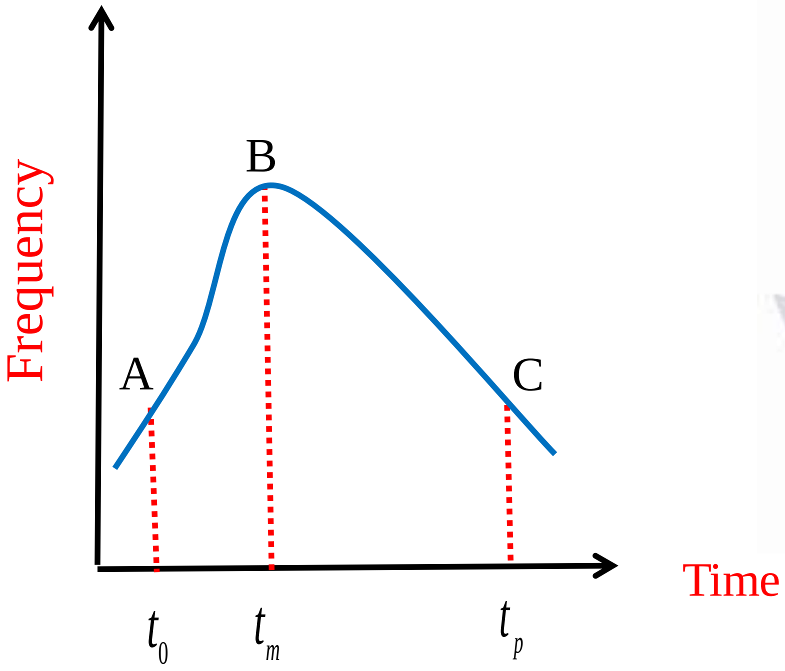
Probability value at any point x is given as the ratio of the shaded area to the total area of the curve.

If activity time duration $t = x$, then probability distribution curve can be drawn and the three time estimates can be obtained.

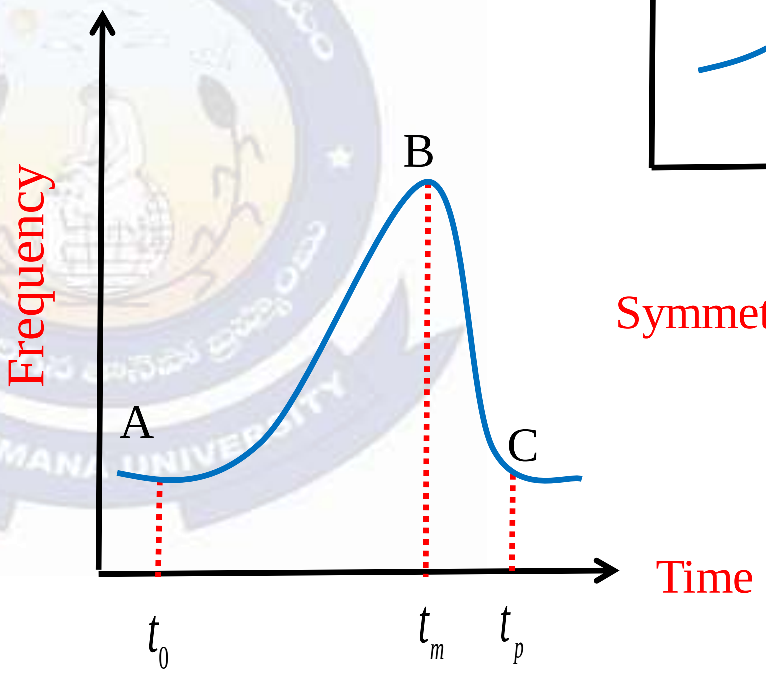
a. Symmetrical curve: Normal/Gaussian/bell shaped distribution

b. Unsymmetrical curve: (β)-distribution

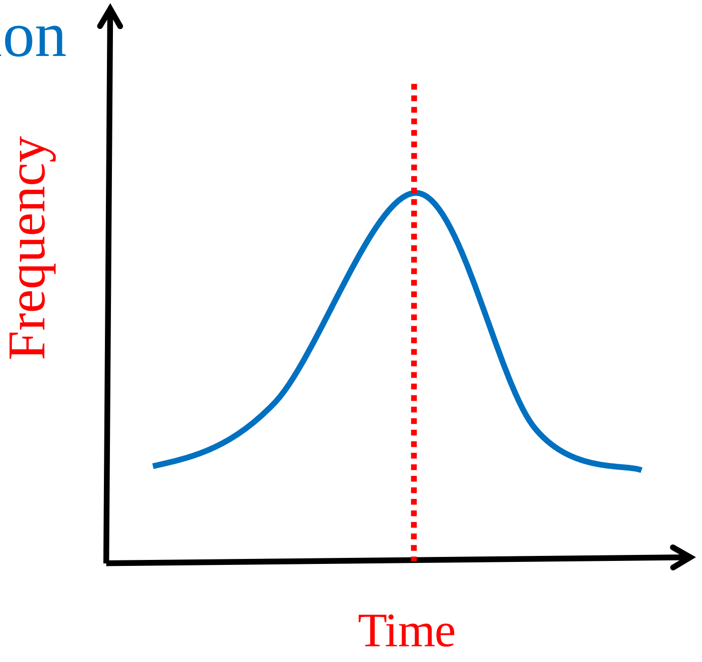
- PERT network usually follow β-distribution



Left skewed β – Distribution curve



Right skewed β – Distribution curve



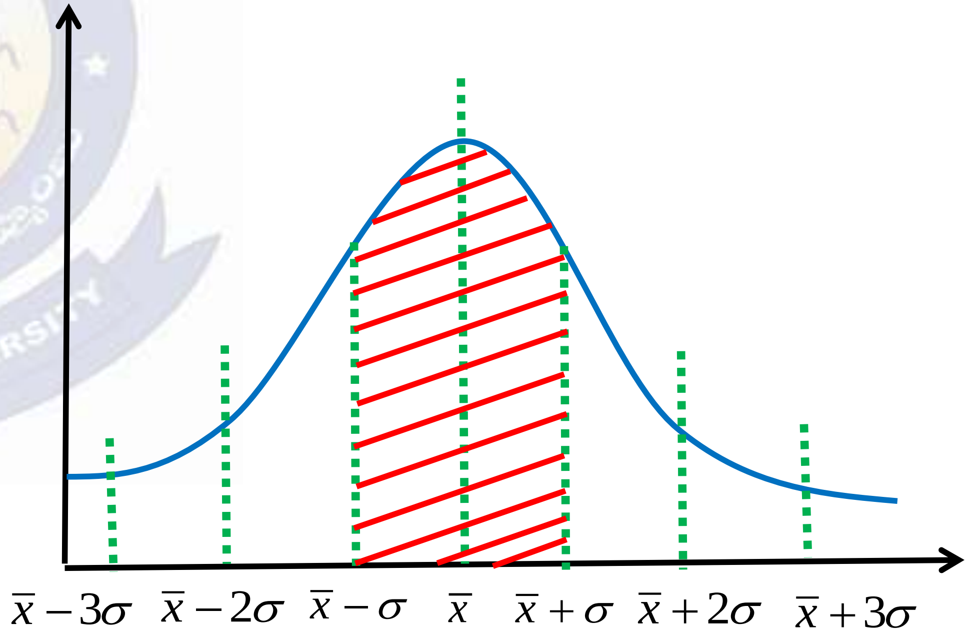
Symmetrical β – Distribution curve

Prof. B. Jayarami Reddy

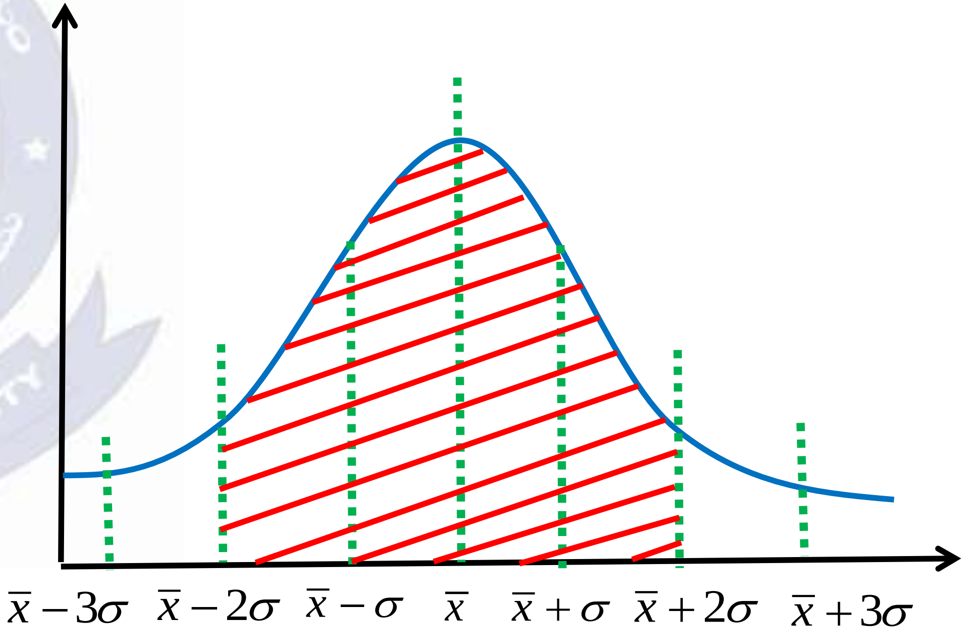
NORMAL PROBABILITY DISTRIBUTION

Normal Probability distribution curve is symmetrical about the mean of the distribution. Some of the observations made from the curve are

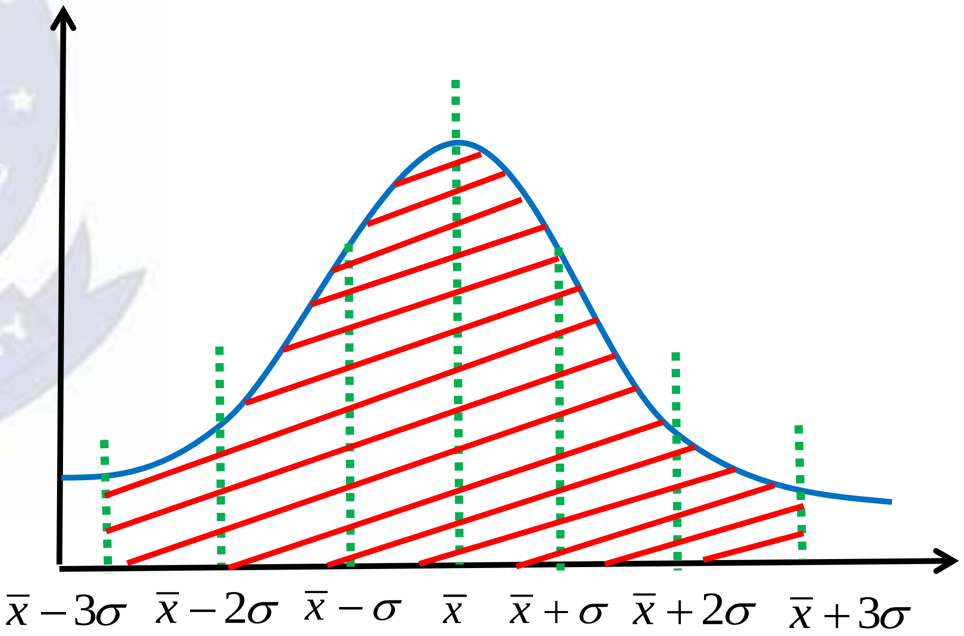
(i) Approximately 68% of all the values of the normal probability distribution curve lie within the range +σ from the average. It means that shaded area of the curve between $x = \bar{x} - \sigma$ to $x = \bar{x} + \sigma$ is 68% of total area.



(ii) Approximately 97% of all the values of normal probability distribution curve lie within the range ±2σ from the average. It means that area of curve between $x = \bar{x} - 2\sigma$ to $x = \bar{x} + 2\sigma$ is 97% of total area.



(iii) Approximately 99.7% of all the values of normal probability distribution curve lie within ±3σ from the average. It means that area of curve between $x = \bar{x} - 3\sigma$ to $x = \bar{x} + 3\sigma$ is 99.7% of total area.



Prof. B. Jayarami Reddy

Now if we consider that optimistic time (t_0) and pessimistic time (t_p) are assumed such that they cover 99.7% of total area then we can write that

$$t_0 = \bar{x} - 3\sigma \text{(minimum time)}$$

$$t_p = \bar{x} + 3\sigma \text{(maximum time)}$$

$$t_p - t_0 = \left(\bar{x} + 3\sigma\right) - \left(\bar{x} - 3\sigma\right)$$

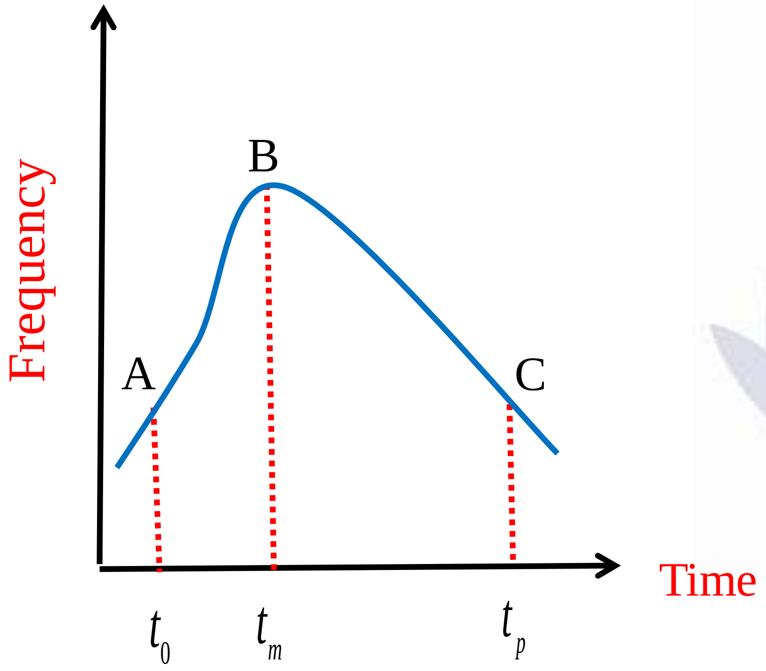
$$t_p - t_0 = 6\sigma$$

Standard deviation, $\sigma = \frac{t_p - t_0}{6}$ Similarly Variance, $\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2$

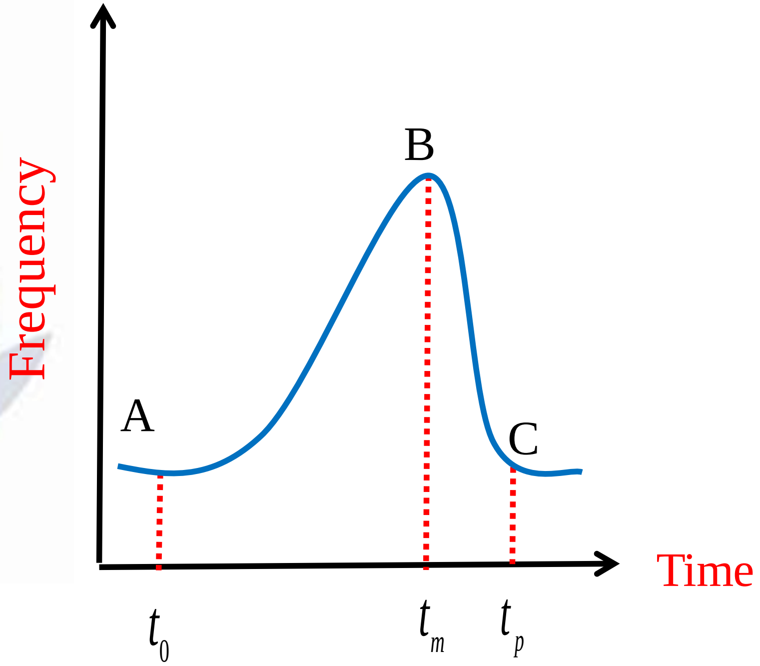
Prof. B. Jayarami Reddy

BETA PROBABILITY DISTRIBUTION

β - distribution curve is used in PERT network analysis because in this analysis individual activities generally have β - distribution which can be left or right skewed. If large number of activities are taken together, the skewedness will die out and for overall project time, the probability distribution function will normal distribution.



Left skewed β – Distribution curve



Right skewed β – Distribution curve

Standard deviation and variance for β- distribution curve can be approximately calculated as below.

Standard deviation, $\sigma = \frac{t_p - t_0}{6}$; Variance, $\sigma^2 = \left(\frac{t_p - t_0}{6} \right)^2$

Variance is a measure of uncertainty, greater the variance greater will be uncertainty.

Prof. B. Jayarami Reddy

EXPECTED TIME (t_e) OF AN ACTIVITY

- The average or mean time taken for the completion of an activity is called as Expected time.
- It should be noted that the expected time t_e represents the average value while the most likely represents the mode of the β – distribution.
- Expected time t_e represents a particular value on the distribution curve, that has both 50-50 chance of being exceeded and a 50-50 chance of being met.
- The vertical ordinate through t_e will divide the probability curve into two equal areas.
- Let us take examples of estimated times of completion of two jobs A & B, as under.

	t_0	t_m	t_p	t_e
Job A	4	6	11	$\frac{4+4\times6+11}{6}=6.5$
Job B	5	10	12	$\frac{5+4\times10+12}{6}=9.5$

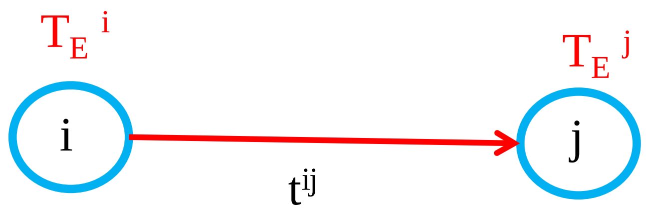
Prof. B. Jayarami Reddy

- For job A, the expected time falls to the right of the most likely time, hence the curve is skewed to left.
- For job B, the expected time falls to the left of the most likely time, hence the curve is skewed to right.

EXPECTED TIME FOR ACTIVITIES IN SERIES

- In a network, if a number of activities are in series, the expected time for the path along the activities is the sum of the expected time of the activities according to central limit theorem.

EVENT TIME



1. Earliest Expected Event occurrence time (T_E)

- It is the minimum time in which an event may occur.
- An event will occur when all the preceding activities are complete.
- It is also known as Earliest Expected time.
- It is determined by forward pass method.
- The earliest expected time (T_E) of a event is calculated by adding the expected time of the preceding event to the preceding activity.

$$T_E \text{ (Successor event)} = T_E \text{ (Predecessor event)} + t_e \text{ (activity)}$$

Where i and j refers to the predecessor and successor events respectively and i-j refers to the activity connecting the events i and j.

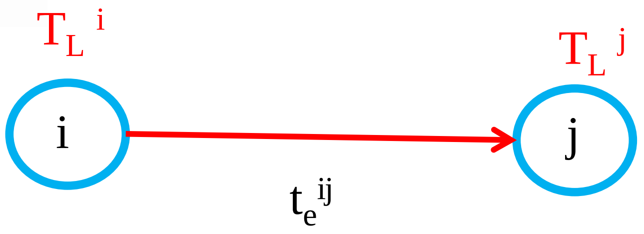
- If there are more than one predecessor event for a successor event j, this rule needs some modifications because an event j can not occur until all preceeding activities leading to that event are completed.

Note: Earliest expected time T_E for the first/initial event of the network diagram should be taken equal to 0, if nothing as mentioned in the question

Prof. B. Jayarami Reddy

2. Latest Allowable occurrence Time (T_L)

- It is the maximum time of an event by which it may be allowed to occur without affecting the completion time of the project.
- It is also called as Latest allowable time and denoted by T_L
- It is determined by Backward pass method.
- Almost all the projects are time bound which is known as contractual obligation time or scheduled completion time and is denoted by T_S.
- We know that for on time completion of the project $T_S = T_L$, of the last event in the network.
- The latest allowable time (T_L) of a event is calculated by subtracting the activity time (t_e^{ij}) from the latest allowable time of the successor event of that activity (T_L^j).
- If there are more than one successor for a event i, the latest allowable time for event i (T_L^j) will be minimum of ($T_L^j – t_e^{ij}$)



Prof. B. Jayarami Reddy

SLACK

Slack or Slack time of any event is defined as the difference between the latest allowable occurrence time and earliest expected recurrence time of the event.

It is denoted by S.

$$S=T_L-T_E$$

- Slack is associated with the event.
- It is the excess time available by which occurrence of an event can be delayed without affecting the project completion time.

Types of Slack

Depending on the values of T_L & T_E, Slack can be positive, zero and negative

1. Positive Slack (S > 0)

Positive Slack is obtained when T_L more than T_E(i.e.$T_L > T_E$) for an event.

$$+\text{Ve. } S = T_L > T_E$$

Prof. B. Jayarami Reddy

- Event is ahead of schedule by time period S.
- This event is also termed as Sub-Critical event.
- Such events can be delayed by time period equal to S, which will not result in any delay of the projects.
- Condition of excess resources deployed.

2. Zero Slack (S = 0)

Zero Slack is obtained when T_L is equal to T_E for an event.

$$S(0) = T_L - T_E$$

- Event is on schedule and can not afford to have any sort of delay.
- Any kind of delay in this event will result in delay of project and such events are termed as Critical events.
- Resources deployed are adequate or just enough.

3. Negative Slack (S < 0)

A negative Slack is obtained when T_L is less than T_E for an event.

$$S(-ve) = T_L < T_E$$

- Event is behind the schedule by the time period equal to Slack of that event.
- Any further delay in such events cause more delay in the project hence these events are called super critical events.
- Resource deployed are not adequate enough.

CRITICAL PATH

- After the project plan is completed and activity times are known, to estimate how long the project will continue we will have to determine the critical path.
- A critical path is the path in a project network, which commencing from the initial event, connects the events having zero or minimum Slack times, and terminates at the end event.
- Events having zero or minimum Slack times are called as critical events because any delay in their occurrence will cause delay in project.
- Critical path is the path connecting the critical events of the project.
- Activities along the critical path are called as critical activities.
- Critical path is the longest path in terms of time or most time consuming path from beginning to end of the network.

- Critical path is shortest path in terms of time in which project can be completed as early as possible.
- Can be more than one or multiple critical path in a project.
- Most critical path in a network diagram having multiple critical path will be the critical path which has larger variance (or larger standard deviations) along the path.
- Variance of a path can be alleviated as per central limit theorem which states that variance of path is the sum of variance of activities along that path.

$$\text{Variance of a path or critical path} = \frac{\text{Sum of variance of activities along that path}}{\text{critical path}}$$

Standard deviation of a path or critical path =

$$\frac{\text{Square root of Sum of variance of activities along that path}}{\text{critical path}}$$

Prof. B. Jayarami Reddy

PROBABILITY OF COMPLETION OF PROJECT

For the determination of probability of completion of project with in the scheduled/assumed completion time, theory of probability is applied to the network analysis.

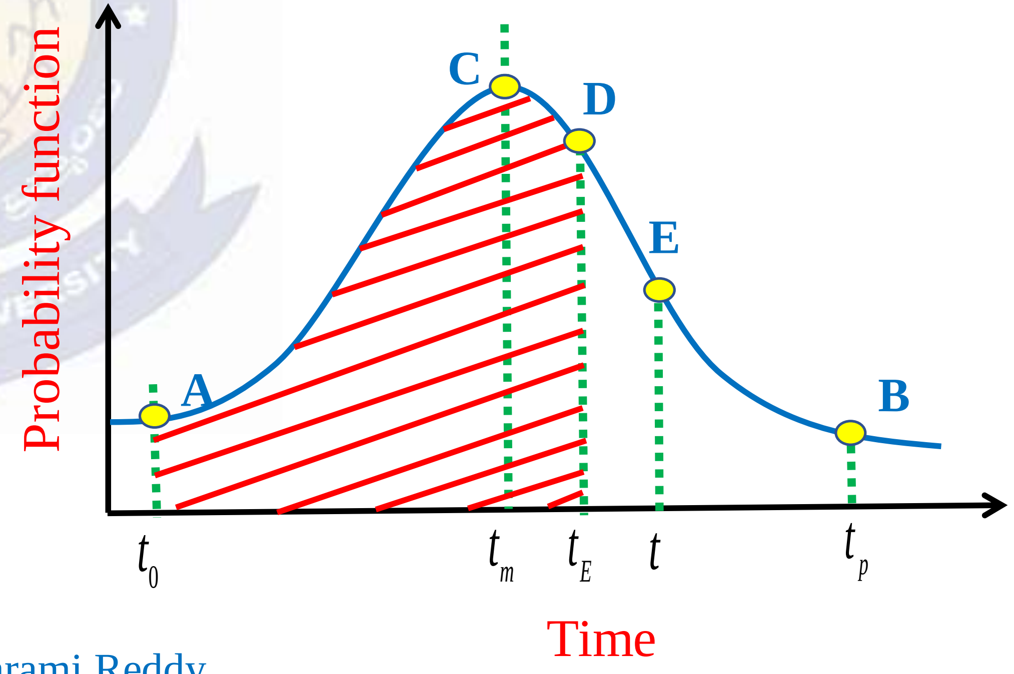
Expected time of each activity has 50 - 50 chance that activity will be completed within the expected time t_e i.e. if a probability distribution curve is drawn for this activity vertical line through t_e will divide the total area of probability distribution curve into two equal parts.

The probability of completion of the activity within some other time t would be given by ratio of the area under curve upto the vertical line at t and total area under the curve.

$$\text{Probability of completion of the activity within time t} = \frac{\text{Area under ACE}}{\text{Area under ACB}}$$

Prof. B. Jayarami Reddy

- The distance between T_E and T_S can be expressed in terms of standard deviation. For example if T_S is to the right of T_E at a distance of 1σ then area enclosed is 84.1% of total area.
- Similarly if T_S is to the left of T_E at a distance of 1σ, then area enclosed is 15.9% of total area.
- Therefore we can say that a distance of +1.0σ from the mean corresponds to 84.1% probability and -1.0σ from mean corresponds to 15.9% probability.



Prof. B. Jayarami Reddy

Procedure

Determine the standard deviation along the critical path. According to central limit theorem.

$$\sigma = \sqrt{\sum \sigma_{ij}^{2}}$$

$$\sigma = \sqrt{\text{sum of varibales along critical path}}$$

where, $\sigma = \left(\frac{t_p^{ij} - t_0^{ij}}{6} \right)^2$

Determine the time distance (T_S – T_E) and express it in terms of probability factor Z as below.

$$Z = \frac{T_S - T_E}{\sigma}$$

Where.

Z = Probability factor or normal deviate

T_S = Scheduled completion time

T_E = Expected completion time.

- Find % probability with respect to the normal deviate Z from the table.
- Probability factor Z can be positive, zero or negative.
- When Z is (+ve): T_S is at the right of T_E and chances of completing the project is more than 50%.
- When Z is (0): T_S is coinciding with T_E and chances of completing the project is 50%
- When Z is (-ve): T_S is at the left of T_E and chances of completing the project is less than 50%.

1. A student attending to the online class late by 10 minutes, sometimes as much as 36 minutes and most of the time late by 20 minutes. Assumes that the student attending to the online class is an activity in PERT

a. The expected late time for attending online class

$$t_0 = 10\,\text{minutes} \quad t_m = 20\,\text{minutes} \quad t_p = 36\text{minutes}$$

$$\text{expected time, } t_e = \frac{t_0 + 4.t_m + t_p}{6}$$

$$= \frac{10 + 4 \times 20 + 36}{6} = 21\,\text{minutes}$$

b. Standard deviation is…….

$$\sigma^2 = \left(\frac{t_p - t_0}{6}\right)^2 \Rightarrow \sigma^2 = \left(\frac{36 - 10}{6}\right)^2 \Rightarrow \sigma = 2.08\,\text{minutes}$$

2. The optimistic, most likely and pessimistic estimation of time for an activity are 5 days, 12 days and 13 days respectively. The expected, completion time of the activity is….

$$t_0 = 5\,days, t_m = 12\,days, t_p = 13\,days$$

$$Expected\ time, t_e = \frac{t_0 + 4t_m + t_p}{6}$$

$$= \frac{5 + 4 \times 12 + 13}{6} = \frac{66}{6} = 11\,days$$

Prof. B. Jayarami Reddy

3. The time estimation from four contractors P, Q, R and S for executing, a project are given in table

Contractor	Optimistic Time, t_0	Most likely time, t_n	Pessimistic Time, t_p
P	6	10	13
Q	8	13	16
R	10	14	16
S	9	15	18

Which contractor is more certain about completing the project in time?

Contractor	Variance $\sigma = \left(\frac{t_p - t_0}{6}\right)^2$
P	1.36
Q	1.78
R	1.0
S	2.25

The contractor R is more certain about completing the project since the variance is minimum.

4. A project is expected to complete along the critical path is 18 months with a standard deviation of 3 months.

a. The probability of completing the project within 18 months is….

The probability for different values of probability factors are given as

Probability factor	Probability Percentage
-1	15.9
0	50
1	84.1
2	97.7
3	99.9

Standard deviation, $\sigma = 3$ months

Scheduled time, $T_s = 18$ months

Expected time, $T_E = 18\, months$

$$probability\ \ factor, Z = \frac{T_e - T_S}{\sigma} = \frac{18-18}{3} = 0$$

$$From\ \ table\ \ for\ \ z = 0,\ probability = 50\%$$

Prof. B. Jayarami Reddy

b. Probability of completing the project within 15 months is….

$$T_s = 18\,months \qquad T_e = 15\,months \qquad \sigma = 3\,months$$

$$\text{Probability factor}, z = \frac{T_e - T_S}{\sigma} = \frac{12-15}{3} = -1$$

$$\text{From table, for } z = -1, \text{ probability} = 15.9\%$$

c. Probability of completing the project within 24 months is….

$$t_t = 24\,months \qquad T_s = 18\,months \qquad \sigma = 3\,months$$

$$z = \frac{T_E - T_s}{\sigma} = \frac{24-18}{3} = 2$$

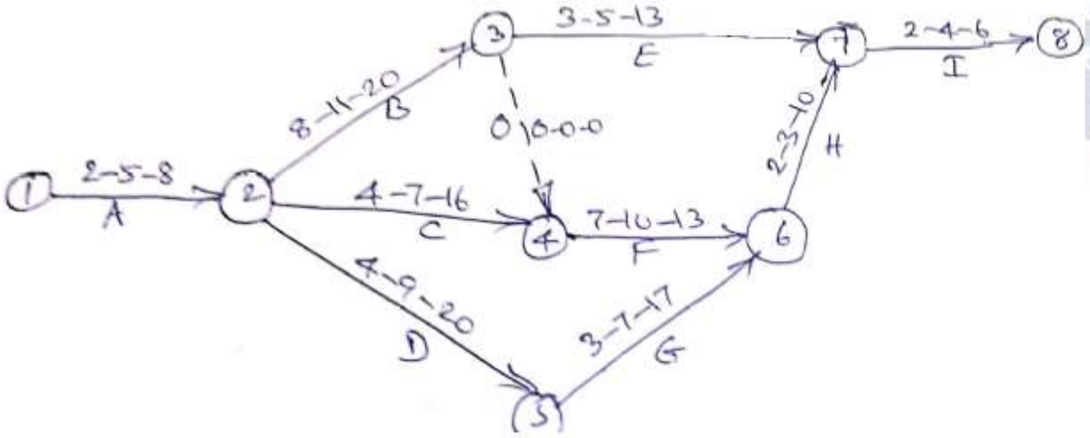
$$\text{From table, for } z = 2, \text{ probability} = 97.7\%$$

5. A construction work consists of activities with PERT durations in weeks as given below.

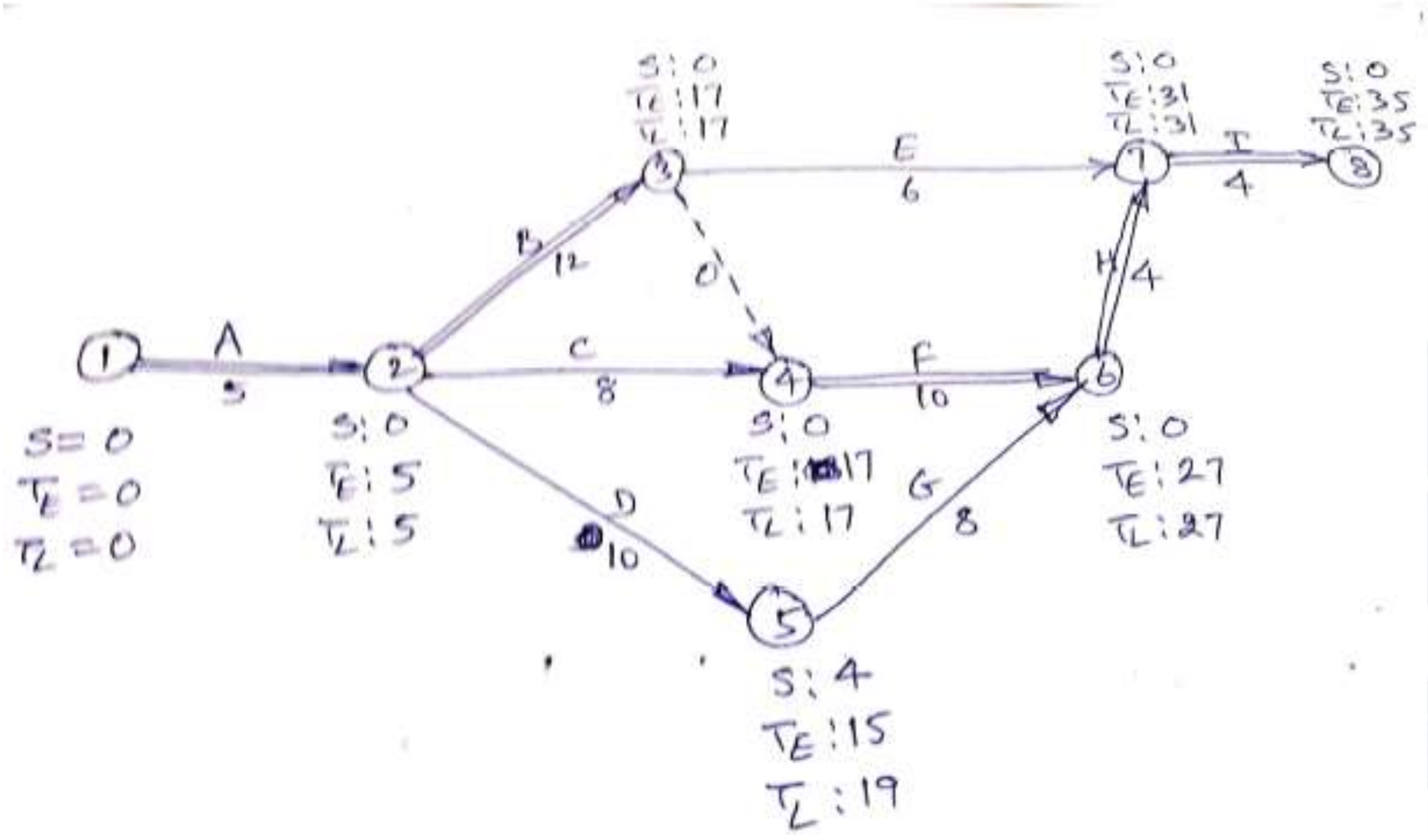
Activity	A	B	C	D	E	F	G	H	I
Predecessors	-	A	A	A	B,C	C	D	F,G	E,H
t_o, weeks	2	8	4	4	3	7	3	2	2
t_m, weeks	5	11	7	9	5	10	7	3	4
t_p, weeks	8	20	16	20	13	13	17	10	6

Z	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
Probability, %	81.59	84.13	86.43	88.49	90.32	91.92	93.32	94.52	95.54

a. The net work corresponding to the given activities is….



b. The critical path for the network is….



Activity	t_0	t_m	t_p	t_e	σ
A	2	5	8	5	1
B	8	11	20	12	2
C	4	7	16	8	2
D	4	9	20	10	2.67
O	0	0	0	0	0
E	3	5	13	6	1.67
F	7	10	13	10	1
G	3	7	17	8	2.33
H	2	3	10	4	1.33
I	2	4	6	4	0.67

Critical path: path joining the events of zero lack

:1–2–3–4–6–7–8

$: A - B - 0 - F - H - I$

Prof. B. Jayarami Reddy

c. Expected duration of the project is…..

Duration of critical path $= 5 + 12 + 0 + 10 + 4 + 4$

$= 35\, weeks$

$For\ \ path\ \ A - B - E - I : 5 + 12 + 6 + 4 = 27\, weeks$

$A - B - 0 - F - H - I : 5 + 12 + 0 + 10 + 4 + 4 = 35\, weeks$

$A - C - F - H - I : 5 + 8 + 10 + 4 + 4 = 31\, weeks$

$A - D - G - H - I : 5 + 10 + 8 + 4 + 4 = 31\ weeks$

Hence, critical path is: $A - B - 0 - F - H - I$

Expected duration of the project $= 35\, weeks$

d. Standard deviation for the network is…

$$\sigma = \sqrt{\sigma_A^2 + \sigma_B^2 + \sigma_0^2 + \sigma_F^2 + \sigma_H^2 + \sigma_I^2}$$

$$= \sqrt{1^2 + 2^2 + 0^2 + 1^2 + 1.33^2 + 0.67^2} = 2.867$$

e. The duration required for 95% probability of completing the project is….

$$\text{For 95\% probability}, Z = 1.6 + \frac{1.7 - 1.6}{95.54 - 94.52}(95 - 94.58) = 1.647$$

$$z = \frac{T_L - T_S}{\sigma} \Rightarrow 1.647 = \frac{T_L - 25}{2.867} \Rightarrow T_L = 39.72\, weeks$$

f. The duration required for 85% probability of completing the project is….

$$\text{for 85\% probability}, z = 1.0 + \frac{1.1 - 1.0}{86.43 - 84.13}(85 - 84.13) = 1.038$$

$$z = \frac{T_L - T_S}{\sigma} \Rightarrow 1.038 = \frac{T_L - 35}{2.867} \Rightarrow T_L = 37.98 \;\; weeks$$

g. The duration required for 50% probability of completion of the project is…..

$$\text{for } 50\% \text{ probability}, \sigma = 0$$

$$T_L = T_S = 35 \textit{ weeks}$$

h. The probability of completing the project in 32.133 weeks is……

$$Z = \frac{T_L - T_S}{\sigma} = \frac{32.133 - 35}{2.867} = -1$$

$$\text{Probability of completing the project} = 100 - 84.13 = 15.87\%$$

Prof. B. Jayarami Reddy