FLUID MECHANICS

Online Class-Analysis of Fluid Flow

by

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- Analysis of Pipe Flow
- Reynold’s Experiment
- Laminar Flow
- Transition Flow
- Turbulant Flow
- Reynold’ Number

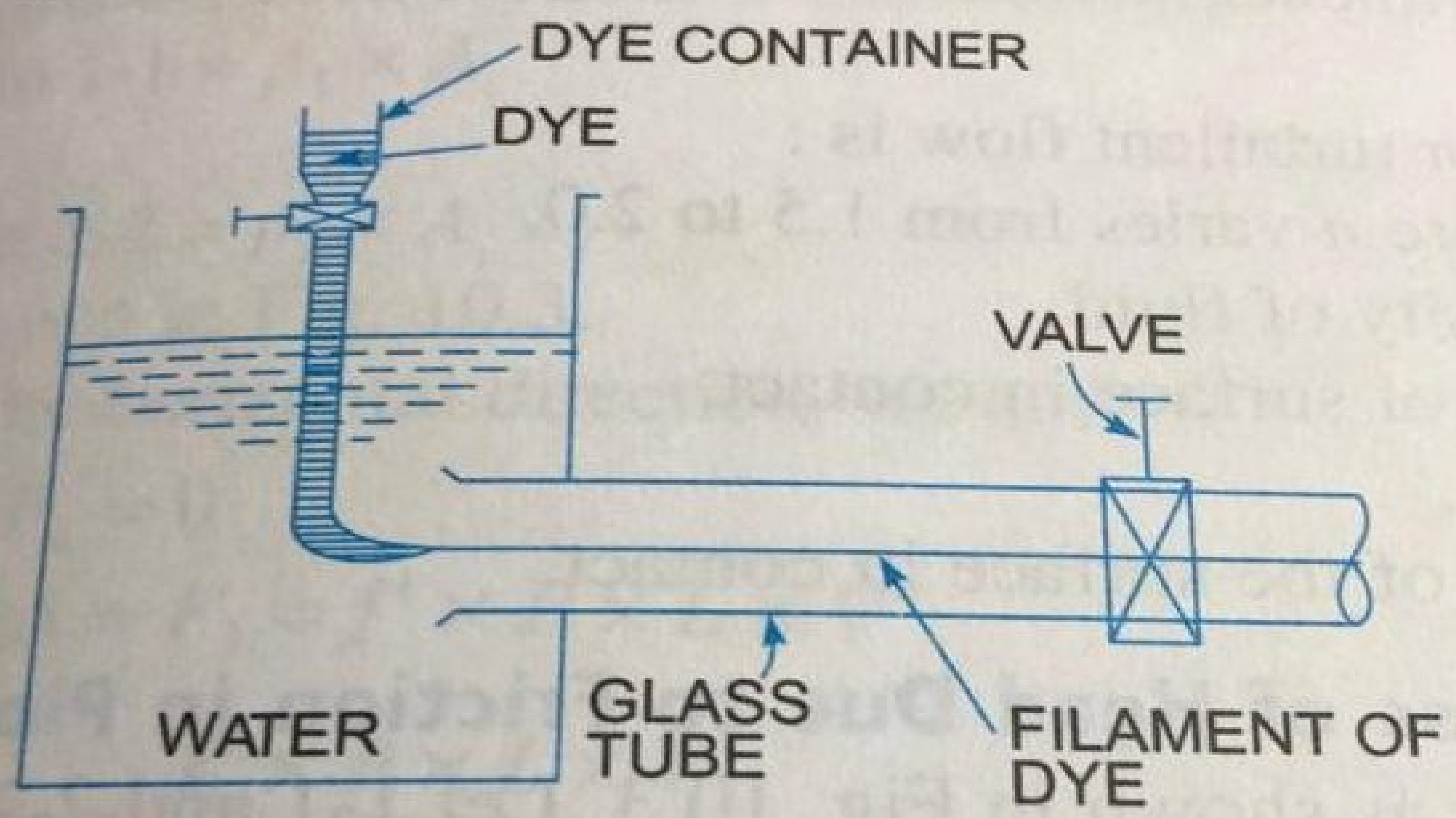


Fig. *Reynold apparatus.*

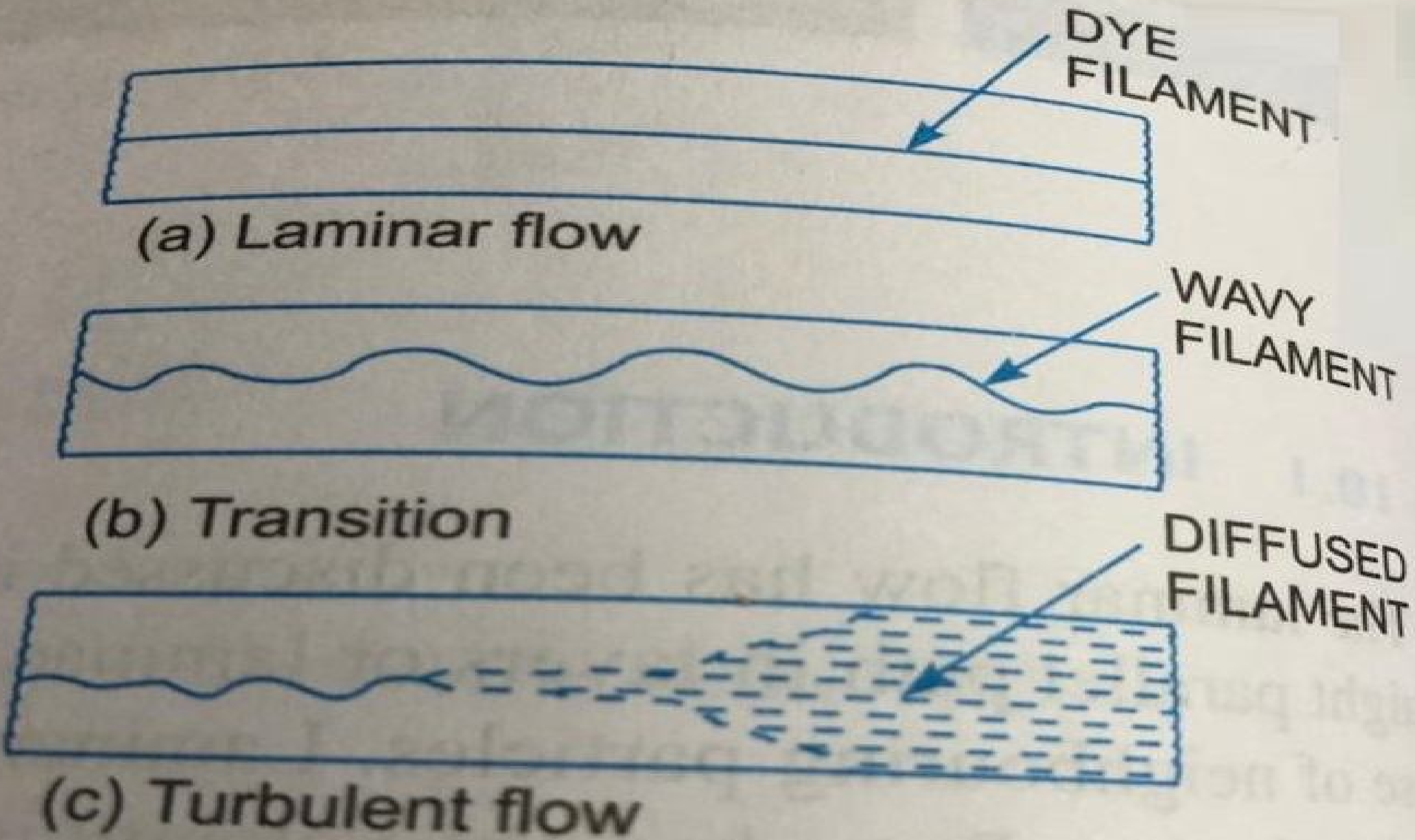


Fig. *Different stages of filament.*

- R_N= VD/ Kinematic Viscosity
- R_N= Inertia force/Viscous force
- <2000 LF, > 4000 TF
- Darcy Weisbach Equation
- $H_f = fLV^2/2gD$
- Hydraulic Mean Radius $R = A/P$, $S= H_f/L$
- Chezy's Formula $V = C (RS)^{1/2}$
- Manning's Equation $V= (1/n) \times R^{2/3} S^{1/2}$

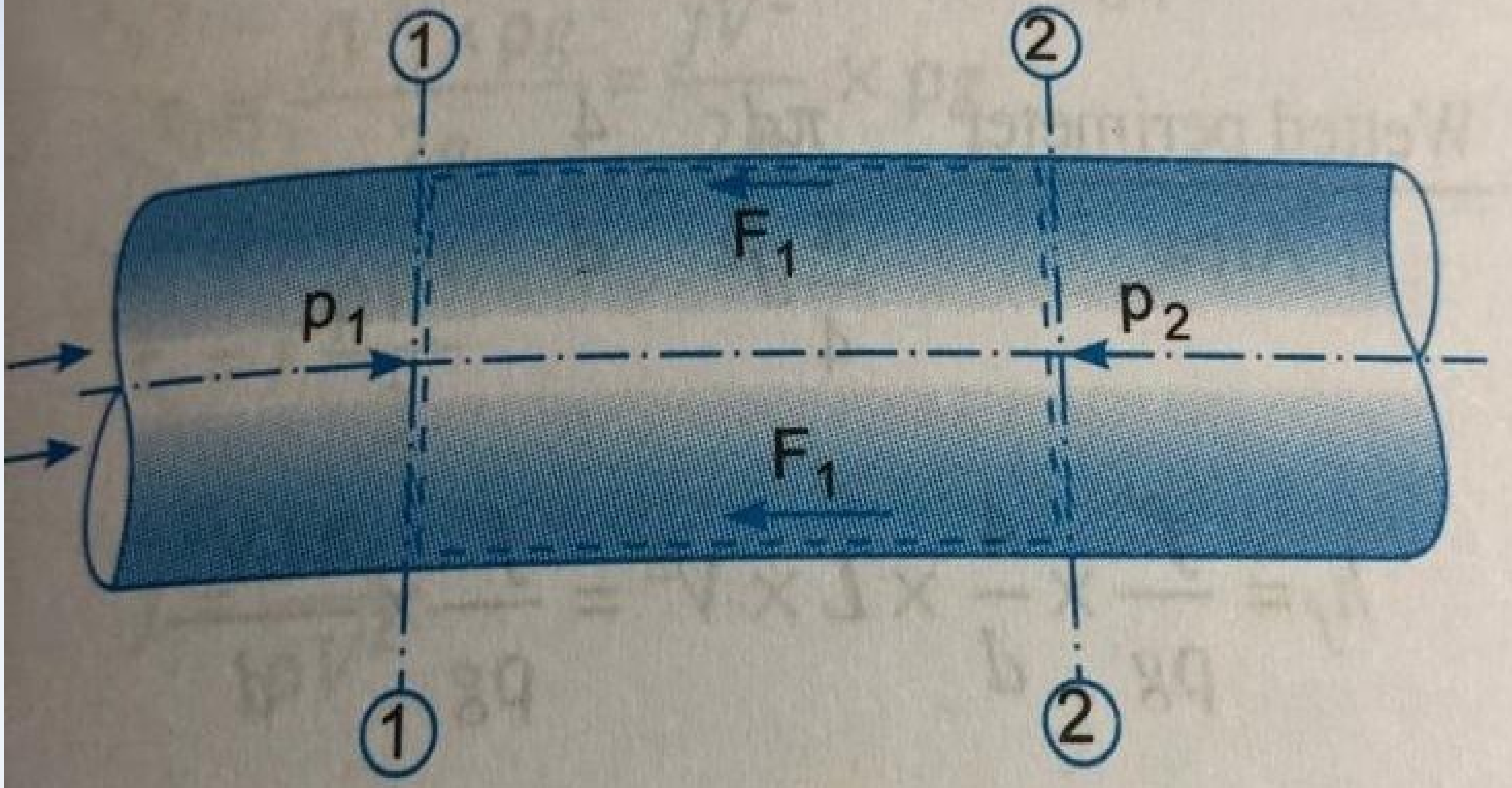


Fig. *Uniform horizontal pipe.*

- Energy Losses in Pipes
- Head loss due to sudden enlargement = $(V1-V2)^2/ 2g$
- Head Loss due to sudden contraction = $K V^2/2g$
- $K= ((1/Cc)-1))^2$
- Entrance Loss = $0.5V^2/2g$
- Exit Loss = $V^2/2g$
- Bend Loss = $KV^2/2g$
- Head Loss due to Gradual contraction or enlargement = $K (V1-V2)^2/2g$

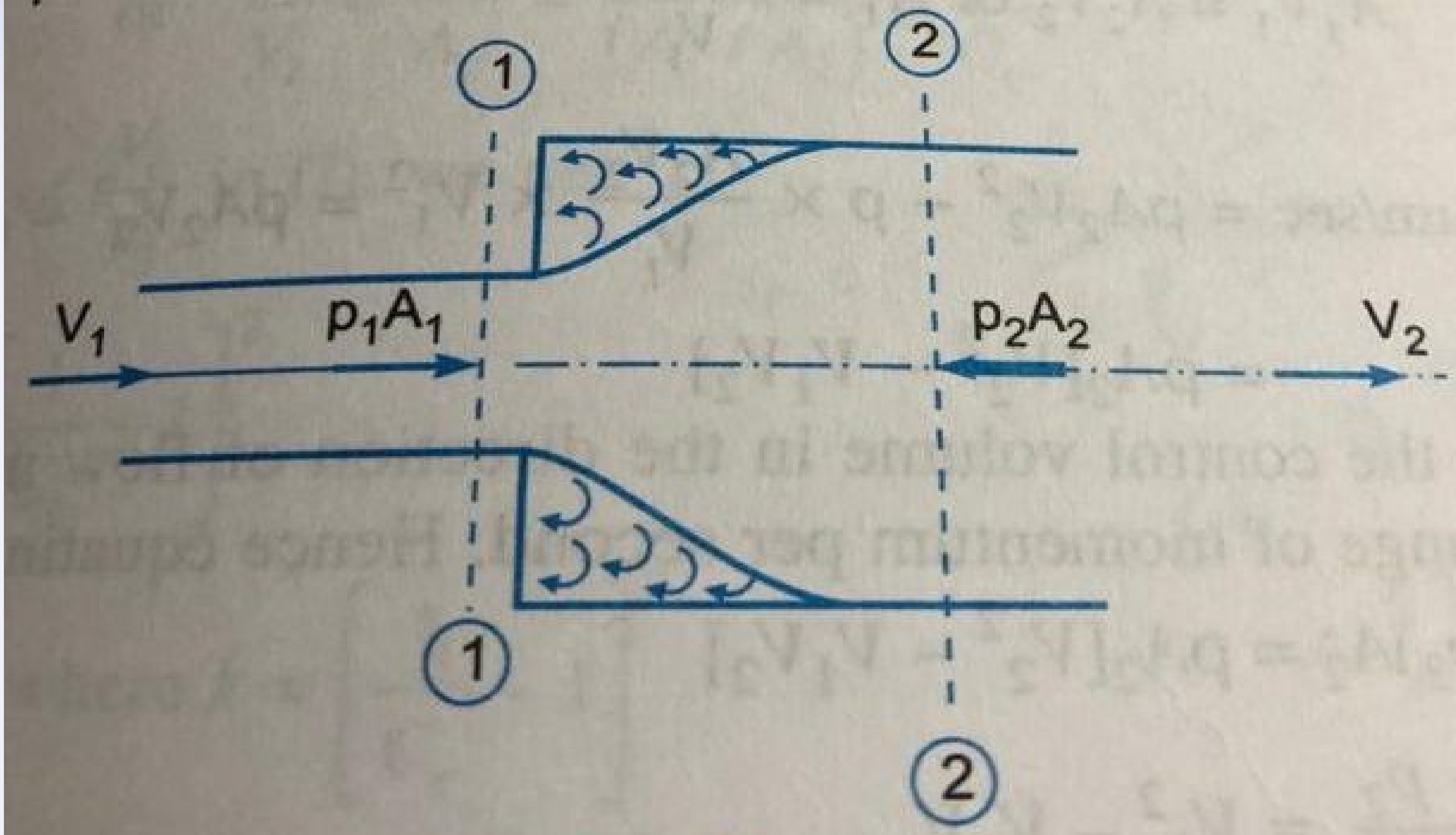


Fig. *Sudden enlargement.*

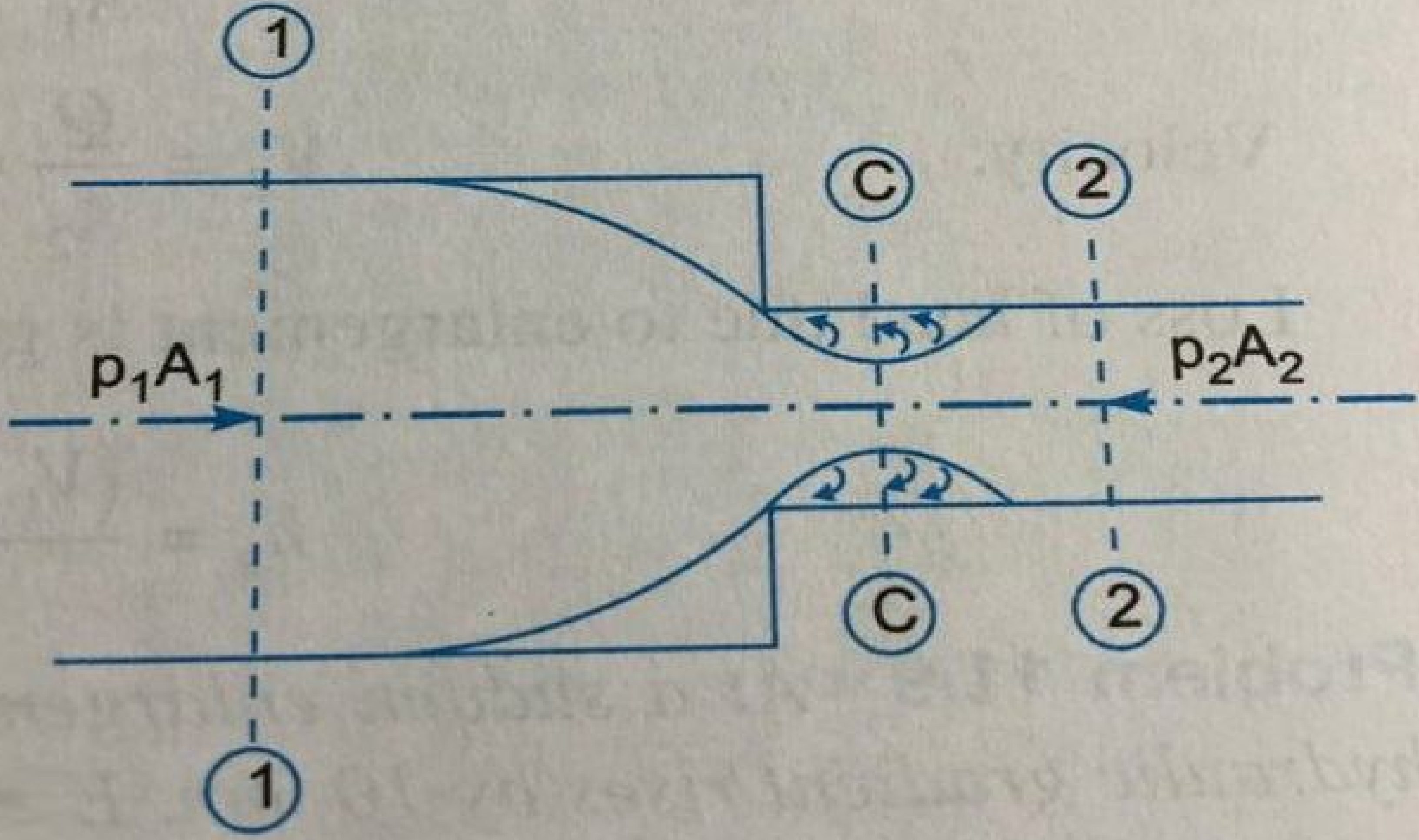


Fig. *Sudden contraction.*

- Coefficient of Friction $f = 64/R_N$ for Laminar flow (R_N less than 2000)
- Coefficient of Friction $f = 0.316/R_N^{1/4}$ for Turbulant flow (R_N more than 4000)
- Problem Find the head loss due to friction in pipe diameter 25 cm and length 60 m, through water is flowing at a velocity of 3.0 m/sec Using Darcy's formula and Chezy's formula. (C=55, Kinematic Viscosity = 0.01 Stokes. (R=A/P: S=Hf/L)
- R_N = VD/Kinematic Viscosity = 750000, f= 0.0107
- $H_f = fLV^2/2gD$ = 1.182m; Hf= 2.856,V = $C\ (RS)^{1/2}$

Problem: Calculate the rate of flow of water through a pipe of diameter 0.3 m (due to Friction) when the difference of pressure head between two ends of pipe 400 m apart is 5 m of water. (f=0.036)

- $H_f = fLV^2/2gD$, V= 1.429 m/sec,
- $Q=AV= 0.101\ m^3/sec$
- Flow through Long pipes
- $(H_A + Z_A) – (H_B + Z_B) = H = Hentry + H_f + Hexit$
- Water level difference between two tanks = sum of losses

- Pipes in Series
- H = Sum of losses
- Q = A1V1 = A2V2 = A3V3
- H= Hentry+ Hf1 + Hsc + Hf2+ Hse + Hf3 + Hexit
- Equivalent Pipe ($H_L = H_{L1}+H_{L2}+ H_{L3}$)
- L = L1 + L2 + L3
- Dupuitt’s Equation
- $L/D^5 = L1D1^5 + L2/D2^5 + L3/D3^5$

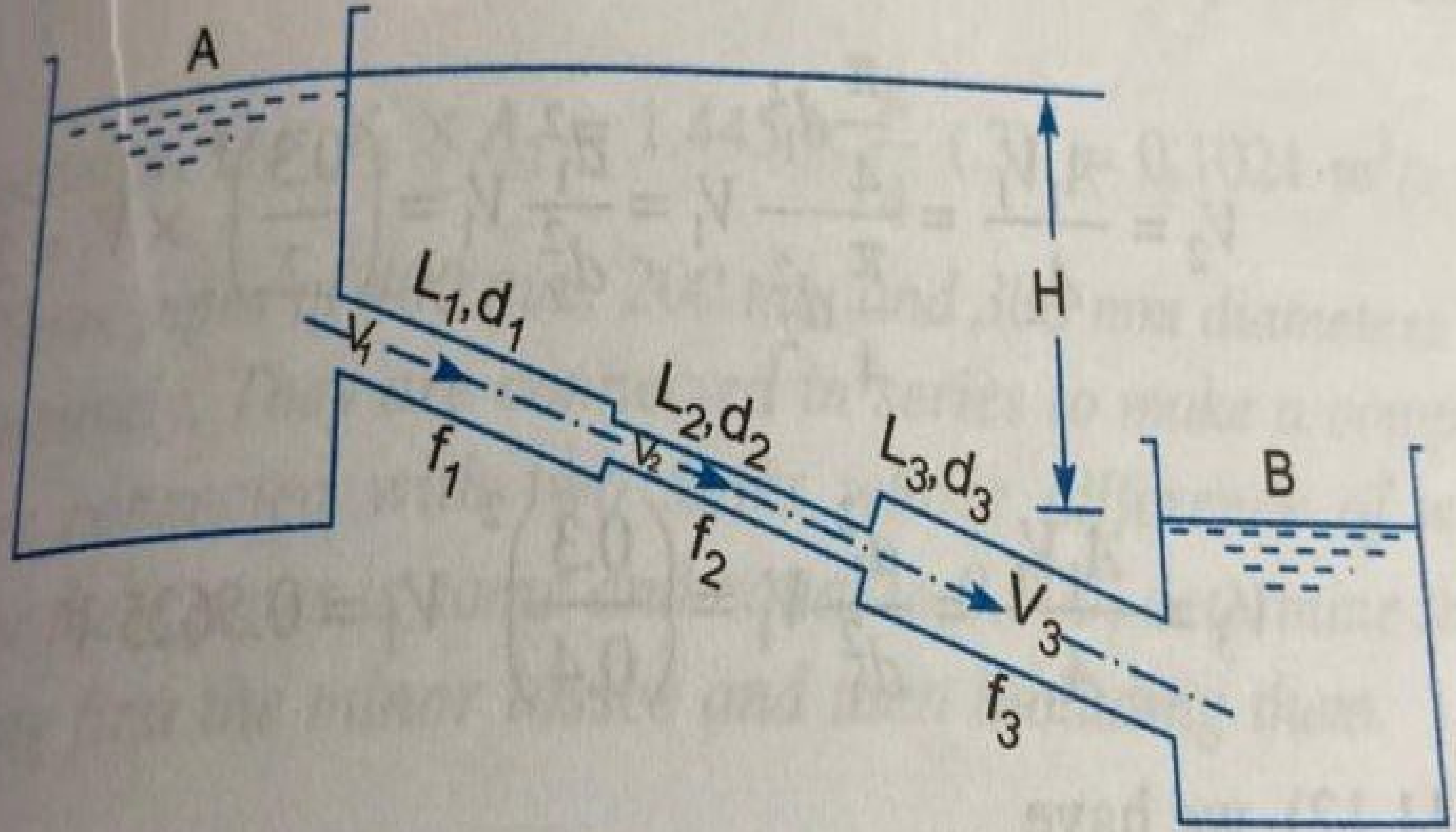


Fig.

- Pipes connected in Parallel
- $Q = Q1 + Q2$
- Losses in Pipe 1 = Losses in Pipe 2
- Siphon
- H = Sum of losses = Hentry + Hf + Hexit
- Application of Bernoulli's Equation to find summit height
- Hydraulic Gradient Line
- Total Energy Line

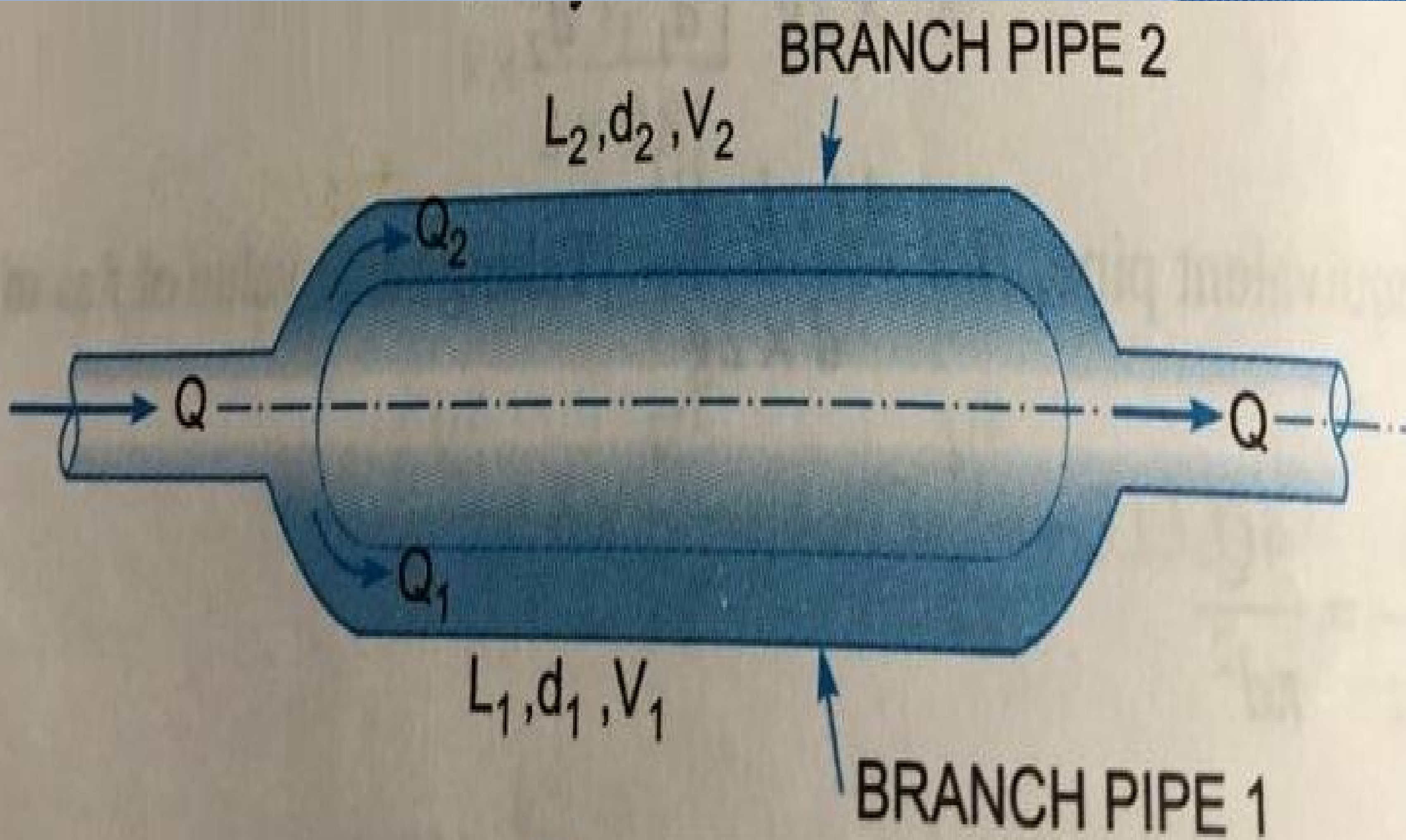


Fig.

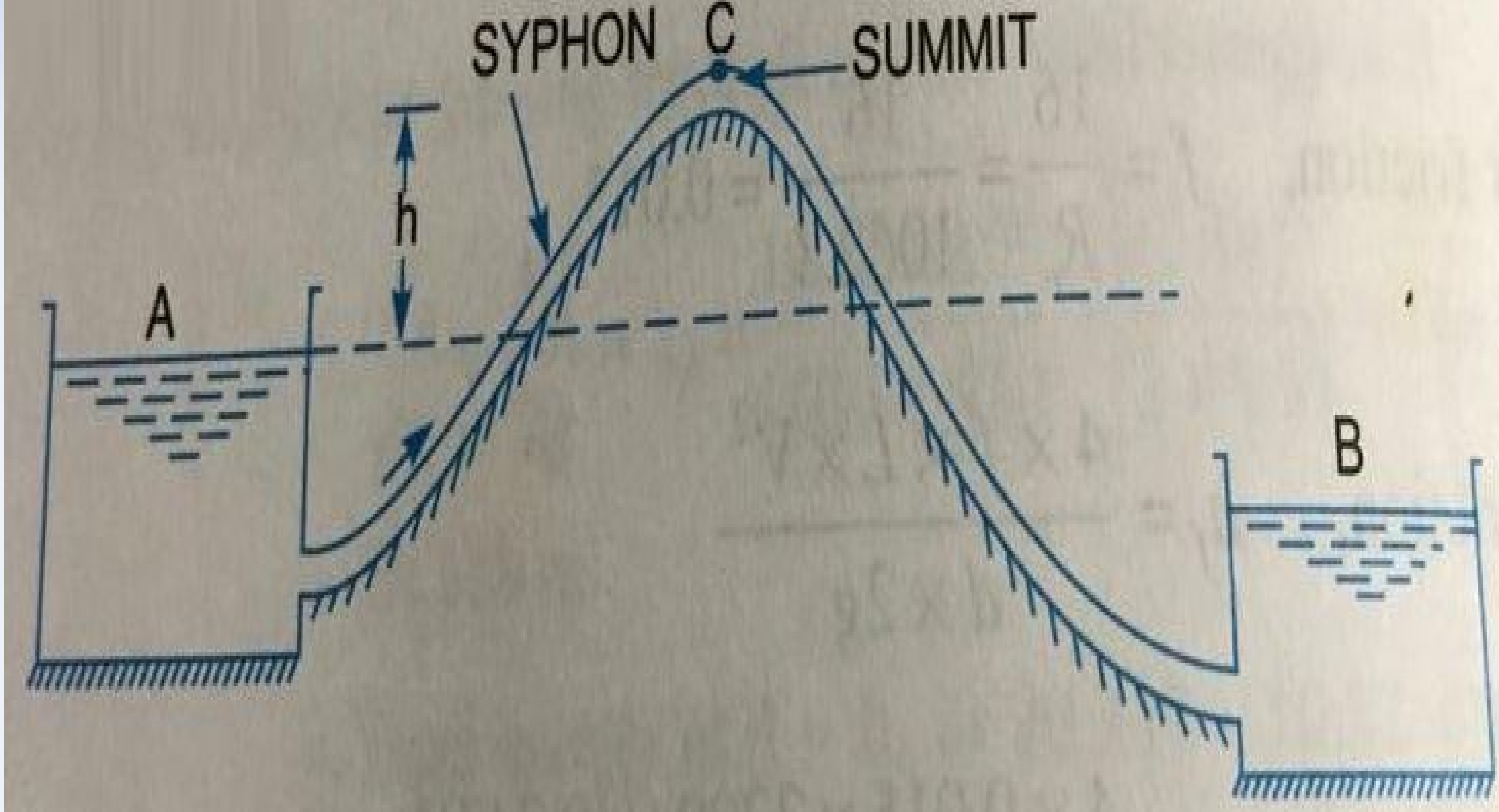


Fig.

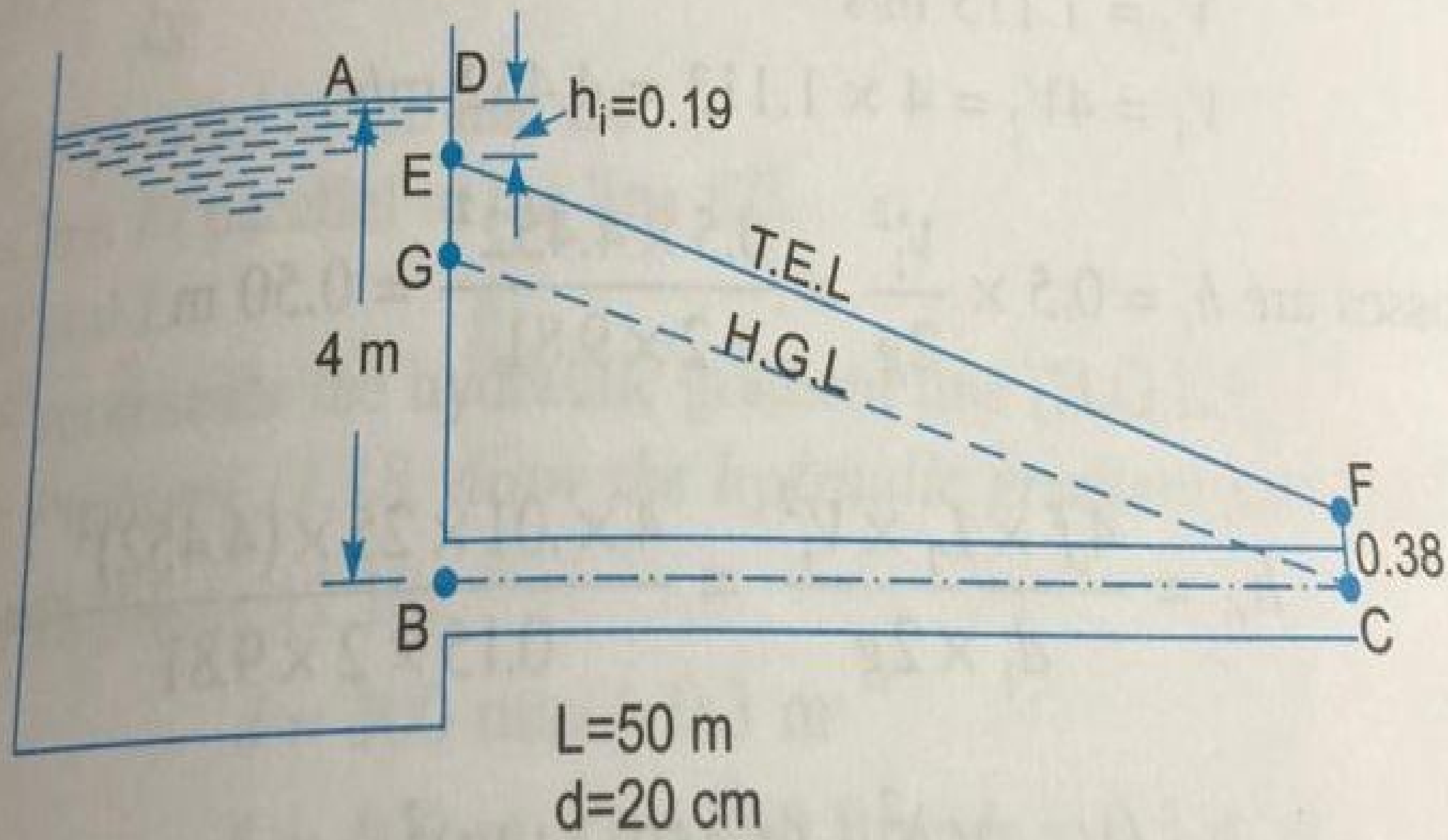


Fig.

- Branched Pipes
- When three or more reservoirs are connected by pipes having one or more junctions. (Q1=Q2+ Q3: Bernoulli's Equation)
- $p_1/w + V_1^2/2g + Z_1 = p_2/w + V_2^2/2g + Z_2$
- Problem: Three pipes of lengths 800 m, 600 m and 300 m and of diameters 40 cm, 30 cm, 20 cm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find the diameter of single pipe. (Dupuitts eqn
- D= 26.65 cm, $L/D^5 = L1/D1^5 + L2/D2^5 + L3/D3^5$
- Problem: A main pipe is divided into two parallel pipes which again form one pipe. The L,D of First pipe 1900 m and 1 m, L, D of Second pipe 1900 m and 0.8 m. Find the rate of flow in each parallel pipe, if Q = 2.5 m^3/sec.(Q=Q1+Q2, Hf1 = Hf2) Take f1 = f2 = 0.02
- $Hf = f\, L\, V^2/2gd$

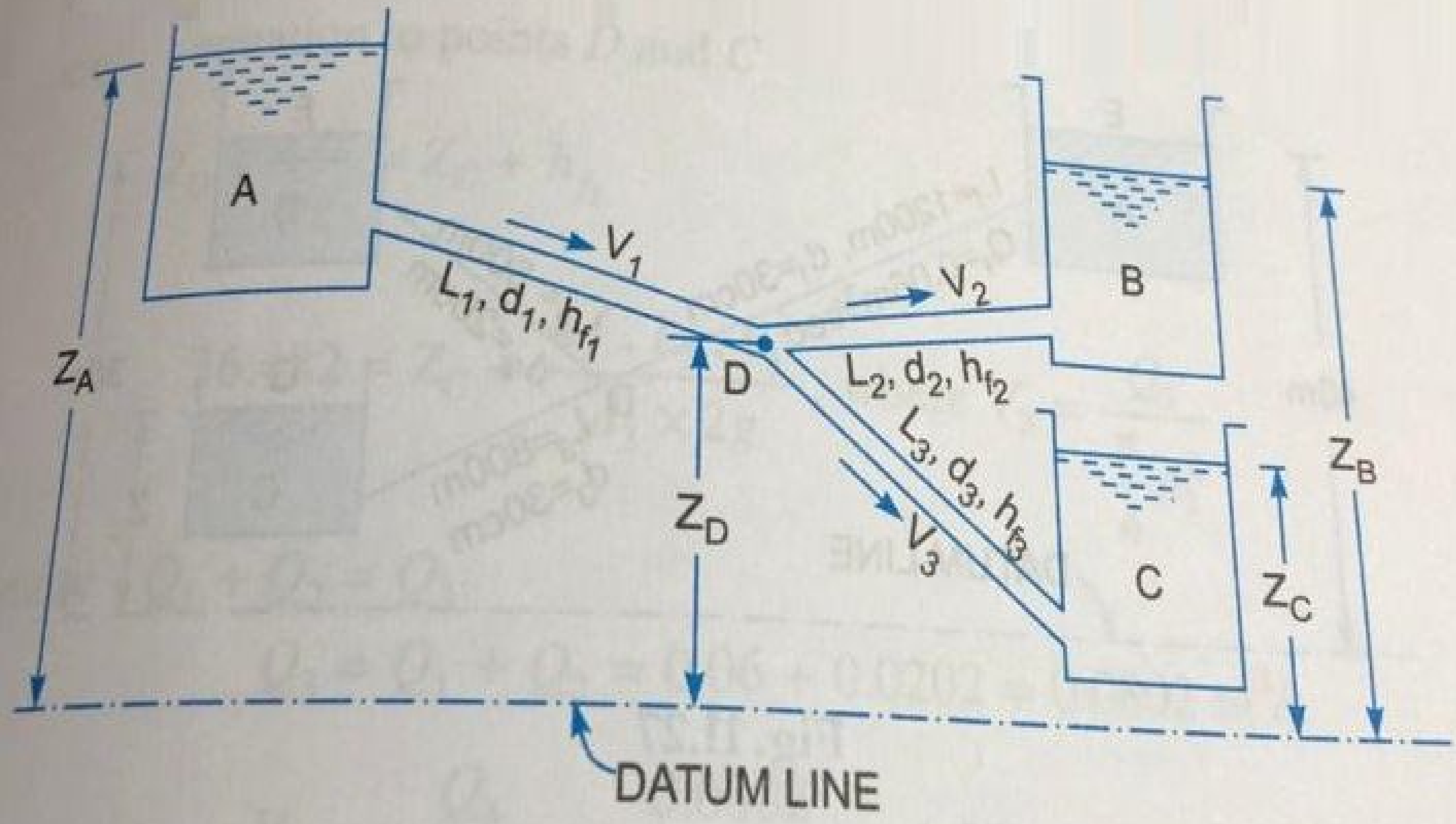


Fig.

- Problem: A crude oil of viscosity 0.9 poise and G = 0.8 is flowing through a circular pipe of diameter 8 cm and of length 15m. Calculate the difference of pressure at the two ends of the pipe, if the rate of flow of oil is 4.17 x 10^{-3} m^3/sec. (566.98 Kgf/Sq. m)
- V= Q/A = 0.82 m/sec P1-P2 = 32 x Dynamic Viscosity VL/ D^2

- The rate of flow of water through a horizontal pipe is 0.3 m^3/sec. The diameter of the pipe is suddenly enlarged from 25 cm to 50 cm. the pressure intensity in the smaller pipe is 1.4 kg(f)/cm^2. Determine the loss of head due to sudden enlargement and pressure intensity in the larged pipe.

- V1= 6.115 m/sec, V2 = 1.52 m/sec
- H_L= (V1-V2)^2/ 2g = 1.07 m, Z1 = Z2
- $p_1/w + V_1^2/2g + Z_1 = p_2/w + V_2^2/2g + Z_2 + H_L$
- $p_2 = 1.47 \times 10^4$ kgf/sq.m
- Problem: A 200 mm diameter of pipe reduces in diameter abruptly to 100 mm diameter. If the pipe carries water at $25 \times 10^{-3} m^3$/sec. calculate the pressure loss across the contraction. Take Cc = 0.6. (V1= 0.795, V2 = 3.18 m/sec)
- Head Loss due to sudden contraction = K $V_2^2/2g$ = 0.22 m
- K= $((1/Cc)-1))^2$
- $p_1/w + V_1^2/2g + Z_1 = p_2/w + V_2^2/2g + Z_2 + H_L$
- p1-p2 = 0.707 x 10^3 Kgf/sq.m

Objective Questions

GATE-1. **In flow through a pipe, the transition from laminar to turbulent flow does not depend on** **[GATE-1996]**

(a) Velocity of the fluid (b) Density of the fluid

(c) Diameter of the pipe (d) Length of the pipe

GATE-1. Ans. (d) It is totally depends on Reynolds number = $\frac{\rho V D}{\mu}$

GATE-2. **The velocity profile in fully developed laminar flow in a pipe of diameter D is given by u=u₀ (1-4r²/D²), where is the radial distance from the centre. If the viscosity of the fluid is μ, the pressure drop across a length L of the pipe is:** **[GATE-2006]**

(a) $\frac{\mu u_0 L}{D^2}$ (b) $\frac{4\mu u_0 L}{D^2}$ (c) $\frac{8\mu u_0 L}{D^2}$ (d) $\frac{16\mu u_0 L}{D^2}$

GATE-2. Ans. (d) By Hagen-Poiseuille law, for steady laminar flow in circular pipes

$$\tau = -\mu\frac{\partial u}{\partial r} \quad \Rightarrow \quad \tau = \frac{-\partial P}{\partial x}\cdot\frac{r}{2}$$

$$\mu\frac{\partial u}{\partial r} = \frac{\partial P}{\partial x}\cdot\frac{r}{2}$$

$$\mu u_0\left(\frac{-8r}{D^2}\right) = \frac{P}{L}\cdot\frac{r}{2} \qquad \text{---------}\left[\because u = u_0\left(1-\frac{4r^2}{D^2}\right)\right]$$

$$P = \frac{-16\mu L u_0}{D_2} \qquad \text{[(–) sign is due to drop]}$$

GATE-4. **A fully developed laminar viscous flow through a circular tube has the ratio of maximum velocity to average velocity as** [IES-1994, GATE-1994]

(a) 3.0 (b) 2.5 (c) 2.0 d) 1.5

GATE-4. Ans. (c) Ratio = $\dfrac{\text{Maximum velocity}}{\text{Average velocity}}$ for fully developed laminar viscous flow through a circular tube has value of 2.0

GATE-5. **For laminar flow through a long pipe, the pressure drop per unit length increases.** [GATE-1996]

(a) In linear proportion to the cross-sectional area
(b) In proportion to the diameter of the pipe
(c) In inverse proportion to the cross-sectional area
(d) In inverse proportion to the square of cross-sectional area

GATE-5. Ans. (d) $\dfrac{\Delta P}{L} = \dfrac{128\mu Q}{\pi D^4} \propto \dfrac{1}{D^4}$ $i.e. \propto \dfrac{1}{A^2}$

GATE-6. **In fully developed laminar flow in a circular pipe, the head loss due to friction is directly proportional to....... (Mean velocity/square of the mean velocity).** [GATE-1995]

(a) True (b) False (c) Insufficient data (d) None of the above

GATE-6. Ans. (a) $h_f = \dfrac{32\mu \bar{u} L}{\rho g D^2}$

GATE-3. Fluid is flowing with an average velocity of V through a pipe of diameter d. Over a length of L, the "head" loss is given by $h_f = \frac{fLV^2}{2g \times D}$.

The friction factor, f, for laminar flow in terms of Reynolds number (Re) is........ [GATE-1994]

GATE-3. Ans. $\frac{64}{\text{Re}}$

GATE-4. Water flows through a 0.6 m diameter, 1000 m long pipe from a 30 m overhead tank to a village. Find the discharge (in liters) at the village (at ground level), assuming a Fanning friction factor f = 0.04 and ignoring minor losses due to bends etc. [GATE-2001]

GATE-4. Ans. (0.834 m³/s) $h_f = \frac{fLV^2}{2gD} = \frac{0.04 \times 1000 \times V^2}{2 \times 9.81 \times 0.6}$ $Therefore\, \Delta H = H - h_f = 30 - h_f$

$$V = \sqrt{2g\Delta H} \quad Or\, \Delta H = \frac{V^2}{2g} = 30 - h_f = 30 - \frac{0.04 \times 1000 \times V^2}{2 \times 9.81 \times 0.6} \Rightarrow V = 2.95 m/s$$

$$Q = VA = V \times \frac{\pi D^2}{4} = 2.95 \times \frac{\pi \times (0.6)^2}{4} = 0.834 m^3/s$$

GATE-5. Water at 25°C is flowing through a 1.0 km long G.I. pipe of 200 mm diameter at the rate of 0.07 m^3/s. If value of Darcy friction factor for this pipe is 0.02 and density of water is 1000 kg/m^3, the pumping power (in kW) required to maintain the flow is: [GATE-2009]

(a) 1.8 (b) 17.4 (c) 20.5 (d) 41.0

GATE-6. Ans. (b)

$$h_f = \frac{fLV^2}{2gD}$$

$$= \frac{8fLQ_1^2}{\pi^2 g D^5}$$

$$= \frac{8 \times 0.02 \times 1000 \times (0.07)^2}{\pi^2 \times 9.81 \times (0.2)^5} = 25.30$$

$$\text{Power} = \rho g Q\, h_f$$

$$= 1000 \times 9.81 \times 0.07 \times 25.30 \text{ W}$$

One Mark Questions

01. In Hagen-Poiseuille flow of viscous liquid, one of the following pairs of forces strike a balance

(a) Inertial and viscous forces

(b) Pressure and viscous forces

(c) Gravity and viscous forces

(d) Inertial and gravity forces

01. Ans: (b)

Sol: Hagen-poiseuille law deals with head loss in laminar flow where viscous forces will govern the resistance against flow.

According to Hagen-poiseuille law

$$h_f = \frac{P_1 - P_2}{\rho g} = \frac{32\mu VL}{\gamma D^2}$$

∴ Hagen-poiseuille law involves balancing of pressure and viscous forces.

02. The Reynolds number of a flow is the ratio of

(a) Gravity forces to viscous forces

(b) Gravity forces to pressure forces

(c) Inertial forces to viscous forces

(d) Viscous forces to pressure forces

02. Ans: (c)

Sol: $Re = \dfrac{F_i}{F_v}$

$$F_i \propto \rho V^2 L^2$$

$$F_v \propto \mu . V . L$$

$$\Rightarrow Re = \frac{\rho V^2 L^2}{\mu V L}$$

$$Re = \frac{\rho V L}{\mu}$$

03. The dimensions of a pressure gradient in a fluid flow are

(a) $ML^{-1}T^{2}$
(b) $ML^{-3}T^{-2}$
(c) $ML^{-2}T^{-2}$
(d) $M^{-1}L^{-3}T^{-2}$

03. Ans: (c)

Sol: Pressure gradient $= \frac{dP}{dx} = \left(\frac{N/m^2}{m}\right) = N/m^3$

$$= kg.\frac{m}{sec^2}/m^3 = kg/m^2.sec^2$$

Dimensional formula of $\frac{dP}{dx} = ML^{-2}T^{-2}$

04. For a steady incompressible laminar flow between two infinite parallel stationary plates, the shear stress variation is

(a) linear with zero value at the plates

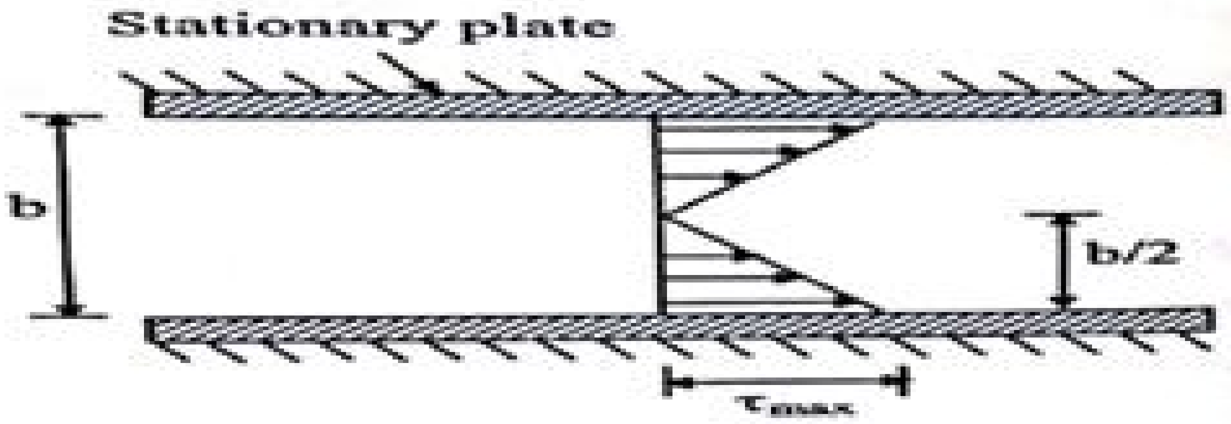
(b) linear with zero value at the center

(c) quadratic with zero value at the plates

(d) quadratic with zero value at the center

04. Ans: (b)

Sol:



Two Marks Questions

01. The shear stress in a fully developed laminar flow in a circular pipe is

(a) Constant over the cross section

(b) Varies parabolically across the section

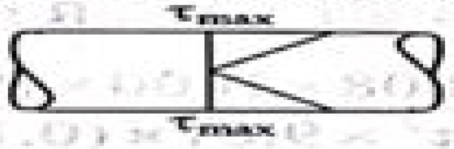
(c) Maximum at the pipe wall

(d) Maximum at the pipe center line

01. **Ans: (c)**

Sol: The resistance against Laminar flow is due to viscosity. Further, this resistance is maximum at pipe boundary.

∴ In laminar flow shear stresses varies linearly with zero at centre and maximum at pipe wall



02. Flow rate of a fluid (density = 1000 kg/m^3) in a small diameter tube is 800 mm^3/s. The length and the diameter of the tube are 2 m and 0.5 mm, respectively. The pressure drop in 2 m, length is equal to 2 MPa. The viscosity of the fluid is

(a) 0.025 $N\text{-}s/m^2$ (b) 0.012 $N\text{-}s/m^2$

(c) 0.00192 $N\text{-}s/m^2$ (d) 0.00102 $N\text{-}s/m^2$

02. Ans: (c)

Sol: Given data:

Density of fluid = 1000 $\frac{kg}{m^3}$

Flow rate through pipe

(Q) = 800 mm^3/sec

Q = 800×10^{-9} m^3/sec

Length of tube (L) = 2 m

Dia. Of tube

(D) = 0.5 mm = 0.5×10^{-3} m

Pressure drop (ΔP) = 2 MPa = 2×10^6 N/m^2

By Hazen – Poiseuille equation,

The pressure drop in pipe flow

$$(\Delta P) = \frac{32 \mu V L}{D^2} = \frac{128 \mu Q L}{\pi D^4}$$

$$2 \times 10^6 = \frac{128 \times \mu \times 800 \times 10^{-9} \times 2}{\pi (0.5 \times 10^{-3})^4}$$

$\therefore$ $\mu = 0.00192$ $N\text{-}sec/m^2$

04. Water is pumped at a steady uniform flow rate of 0.01 m^3/s through a horizontal smooth circular pipe of 100 mm diameter. Given that the Reynolds number is 800 and g is 9.81 m/s^2, the head loss (in meters, up to one decimal place) per km length due to friction would be ______

04. **Ans: 66.1**

Sol:

Length of pipe = 1 km = 1000 m

Given $R_e = 800 < 2000$

$$h_f = \frac{f\ell v^2}{2gd} = \frac{8f\ell Q^2}{\pi^2 \times gd^5}$$

For flow through circular pipes, if $R_e < 2000$, the flow is laminar

For laminar flow, $f = \frac{64}{R_e} = \frac{64}{800} = 0.08$

$$h_f = \frac{8 \times 0.08 \times 1000 \times (0.01)^2}{\pi^2 \times 9.81 \times (0.1)^5} = 66.1 \text{ m}$$

One Mark Questions

01. If bulk modulus of water is 2×10^4 kgf/cm^2 (19.62×10^8 N/m^2), then water hammer wave celerity through a rigid pipe line will be __ m/s.

01. Ans: 1400

Sol: For Water hammering rigid pipes

$$\text{Celerity of wave, } C = \sqrt{\frac{K}{\rho}}$$

$$= \sqrt{\frac{19.62 \times 10^8}{1000}}$$

$$= 1400 \text{ m/s}$$

K = Bulk modulus of elasticity of water, N/m^2

ρ = mass density of water, kg/m^3

02. The loss of energy at the exit of a submerged pipe is

(a) $\frac{V^2}{2g}$ (b) $\frac{0.5V^2}{2g}$

(c) $1.5\frac{V^2}{2g}$ (d) $2.0\frac{V^2}{2g}$

02. Ans: (a)

Sol:

It is a case of sudden expansion where $V_1 = V$; $V_2 = 0$

$$\Rightarrow h = \frac{(V_1 - V_2)^2}{2g}$$

$$\Rightarrow h = \frac{V^2}{2g}$$

03. "Eddy viscosity" means that it is

(a) A physical property of the fluid
(b) Same as the kinematic viscosity
(c) Always associated with laminar flow
(d) An apparent viscosity due to turbulent nature of flow

03. Ans: (d)

Sol: In turbulent flow number of eddies will be present due to inter mixing or hap hazard motion of fluid. These eddies will cause loss of head. Losses are generally due viscosity (dynamic or molecular viscosity) in laminar flow. Effect eddies is negligible in laminar flow. Has presence of eddies cause loss of head in turbulent flow, it is called eddy viscosity.

∴ Eddy viscosity is an apparent viscosity due to turbulent nature of flow.

04. Water is pumped through a pipe line to a height of 10 m at the rate of 0.1 m^3/sec. Frictional and other minor losses are 5 m. Then the power of pump in kw required is ________.

04. Ans: 15

Sol: Given data

$H = 10$ m

$Q = 0.1$ m^3/sec

$h_f = 5$ m

$P = ?$

$$P = \rho g Q(H + h_f) \text{ kW}$$
$$= \gamma.Q(H + h_f)$$
$$= 9.81 \times 0.1 \times (10 + 5) = 9.81 \times 0.1 \times 15$$
$$= 14.715 \text{ kW} \simeq 15 \text{ kW}$$

05. The friction factor for a turbulent flow in smooth pipes varies

(a) Inversely as Reynolds number

(b) Directly as Reynolds number

(c) As square of Reynolds number

(d) Inversely as ¼ th power of Reynolds number

06. The stresses that arises due to fluctuations in the velocity components in a turbulent flow are

(a) Euler stresses (b) Limit stresses

(c) Reynolds stresses (d) Principal stresses

06. Ans: (c)

Sol: Reynold's stress = $-(\rho u'v')$

Where u' & v' are fluctuation components of velocities in 'x' and 'y' directions respectively in turbulent flow.

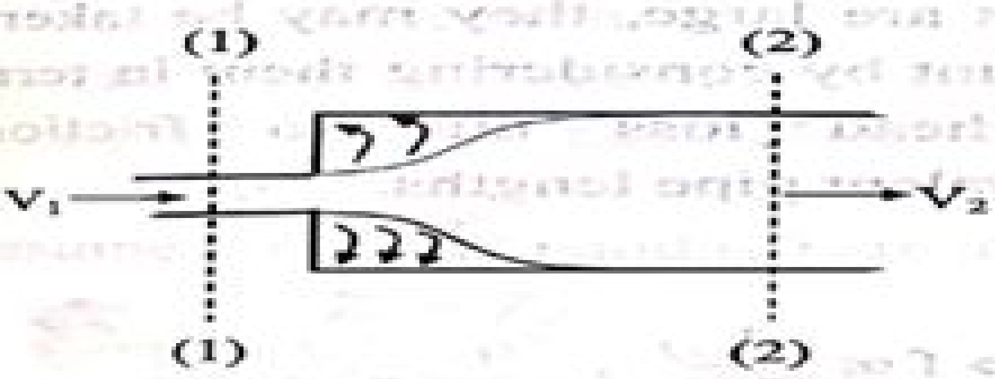
07. The head loss due to sudden expansion is expressed by

(a) $\frac{V_1^2 - V_2^2}{2g}$ (b) $\left(\frac{V_1 - V_2}{2g}\right)^2$

(c) $\frac{(V_1 - V_2)^2}{g}$ (d) $\frac{(V_1 - V_2)^2}{2g}$

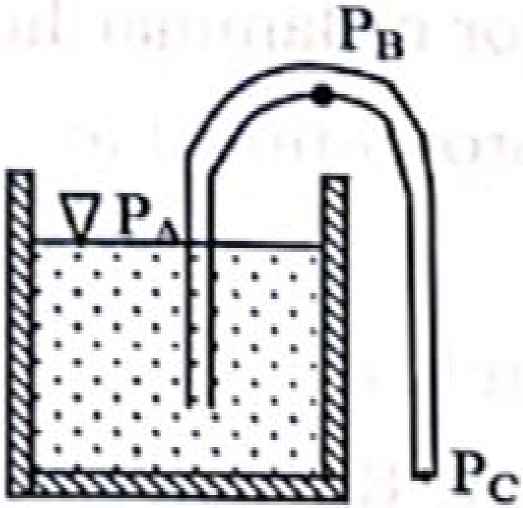
07. Ans: (d)

Sol: Since there is sudden change in the cross-sectional area of the flow passage, the liquid emerging from the smaller pipe is not able to follow the abrupt change of the boundary. Consequently at this section the flow separates from the boundary, forming regions of separation in which turbulent eddies are formed which result in the loss of energy which is ultimately dissipated as heat.



$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

08. In the siphon shown in figure below assuming ideal flow, pressure P_B



(a) $= P_A$

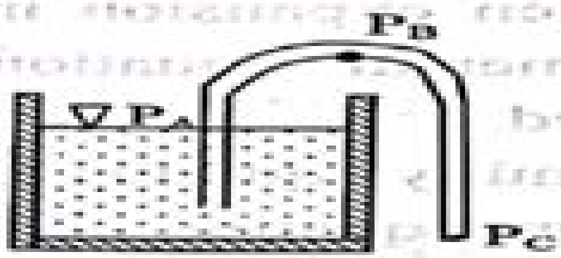
(b) $< P_A$

(c) $> P_A$

(d) $= P_C$

08. **Ans: (b)**

Sol:



Applying Bernoulie's equation between section A and section B (neglecting losses)

$$\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{P_B}{\gamma}+\frac{V_B^2}{2g}+Z_B$$

$V_A = V_B$ as pipe is of constant is dia

$Z_B > Z_A$

$$\frac{P_A}{\gamma}+Z_A=\frac{P_B}{\gamma}+Z_B$$

$\Rightarrow P_B < P_A$

Shortcut:

As we move up pressure will decrease

Point B is above the HGL and it is a summit so the pressure at this point is the least.

09. Flow in a pipe can be expected to be turbulent when the Reynolds number based on mean velocity and pipe diameter is

(a) = 0 (b) < 2000

(c) > 3000 (d) > 100

09. Ans: (c)

Sol: In general for flow through circular pipes

If $R_e < 2000$ Laminar flow

$2000 < R_e < 4000$ Transitional flow

$R_e > 4000$ Turbulent flow

Of the given, options 'c' is the most appropriate answer

12. The friction factor of laminar liquid flow in a circular pipe is proportional to

(a) Reynolds number
(b) Inversely to the Reynolds number
(c) Square of the Reynolds number
(d) Square root of the Reynolds number

12. Ans: (b)

Sol: $f = \dfrac{64}{Re} \Rightarrow f \propto \dfrac{1}{Re}$

13. While deriving an expression for loss of head due to a sudden expansion in a pipe, in addition to the continuity and impulse-momentum equation, one of the following assumptions is made:

(a) Head loss due to friction is equal to the head loss in eddying motion

(b) The mean pressure in eddying fluid is equal to the downstream pressure

(c) The mean pressure in eddying fluids is equal to the upstream pressure

(d) Head lost in eddies is neglected

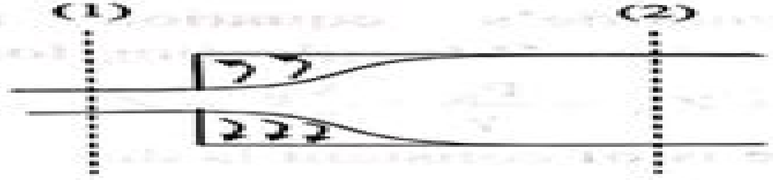
13. Ans: (a)

Sol: In the derivation of expression for loss of head due to sudden expansion in a pipe all the three fundamental equations of fluid mechanics are used.

Such as: (i) Continuity equation

(ii) Energy equation

(iii) Momentum equation



The assumptions made are:

- Between sections (1) and (2) the small shear force exerted on the walls between two sections can be neglected.
- Velocity over the flow cross sections (1) and (2) is assumed to be uniform.
- At section (1), the radial acceleration of fluid particles in the eddies along the surface is small. Hence generally hydrostatic pressure variation occurs across the section.
- The head loss due to friction is primarily equal to the head loss in a eddying motion.

Assume section (1) and (2) are at same level.

$$\therefore Z_1 = Z_2$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P^2}{\gamma} + \frac{V_2^2}{2g} + h_f$$

The loss term 'h_f' is taken as h_ℓ (eddy loss) due to sudden expansion

$$\therefore h_f = h_\ell$$

14. If a single pipe of length L and diameter D is to be replaced by three pipes of same material, same length and equal diameter d(d < D), to convey the same d and D are related by

(a) $d = \frac{D}{3^{2/5}}$ (b) $d = \frac{D}{2^{5/3}}$

(c) $d = \frac{D}{3^{2/3}}$ (d) $d = \frac{D}{2^{3/2}}$

14. Ans: (a)

Sol: For pipes in parallel

$$d = \frac{D}{n^{2/5}}$$

where n = no. of pipes in parallel

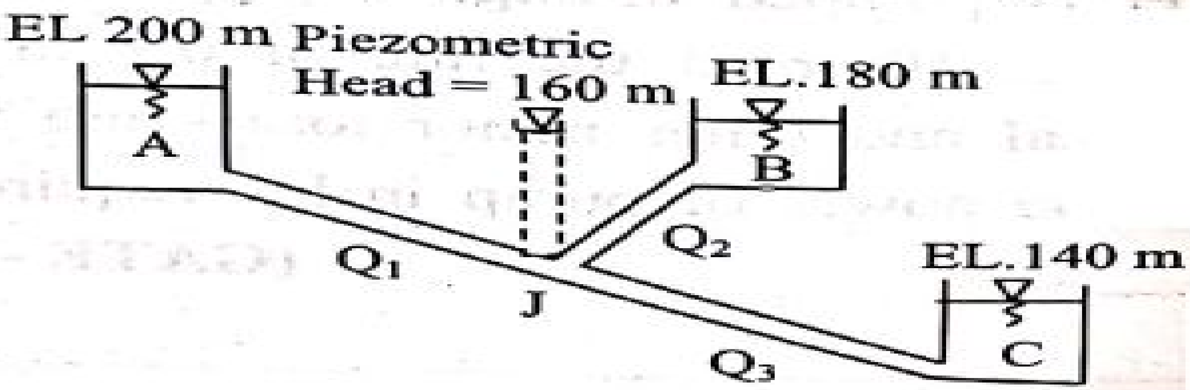
d = dia of each parallel pipe

D = dia of single pipe which is converted to parallel connection of number of small pipes of dia 'd'

n = 3

$$\therefore d = \frac{D}{3^{2/5}}$$

15. Three reservoirs A, B and C are interconnected by pipes as shown in the figure. Water surface elevations in the reservoirs and the Piezometric head at the junction J are indicated in the Figure.



Discharge Q_1, Q_2 and Q_3 are related as

(a) $Q_1 + Q_2 = Q_3$ (b) $Q_1 = Q_2 + Q_3$

(c) $Q_2 = Q_1 + Q_3$ (d) $Q_1 + Q_2 + Q_3 = 0$

15. Ans: (a)

Sol: At Junction 'J'

inflow = outflow,

H_A (200 m)>H_j (160 m), flow Q_1 from A → J

H_B (180 m) > H_j (160 m), flow Q_2 from B → J

H_j (160 m) > H_C (140 m), flow Q_3 from J → C

$\therefore \quad Q_1 + Q_2 = Q_3$

16. For steady incompressible flow through a closed-conduit of uniform cross-section, the direction of flow will always be :

(a) from higher to lower elevation

(b) from higher to lower pressure

(c) from higher to lower velocity

(d) from higher to lower piezometric head

16. **Ans: (d)**

Sol:

- Fluid flow direction always from higher total energy head to lower total energy head.
- Total energy head is sum of piezometric head (elevation head + pressure head) and velocity head.
- For uniform c/s of conduit, velocity is same.
- Hence piezometric head difference makes fluid to flow.

Two Marks Questions

01. An old pipeline which has relative roughness $\frac{K}{D} = 0.005$ operates at a Reynolds number which is sufficiently high for the flow to be beyond the viscous influence and the corresponding f = 0.03. The power increase required to maintain same rate of flow if 'f' increases to 0.0375 would be about

(a) 25%
(b) 50%
(c) 75%
(d) 100%

01. **Ans: (a)**

Sol: increase in friction factor

$$= \frac{0.0375 - 0.03}{0.03} = 0.25 = 25\%.$$

We know, $P = \gamma.Q.h_f$

As 'Q' is constant $P \propto h_f$

$h_f \propto$ friction factor

$\therefore$ Increase in power required to maintain same flow rate = 25%

02. A 15 cm diameter pipe is joined to a 30 cm diameter pipe by a reducing flange. For water flowing at a ratc of 0.115 m^3/s, the head loss when water flows from the smaller to the larger diameter pipe is ____________

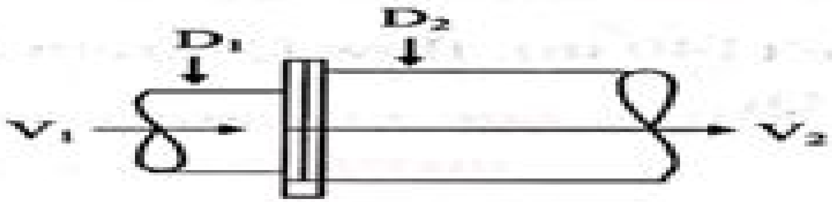
02. Ans: 1.214

Sol: Given data:

$D_2 = 30$ cm $= 0.3$ m

$D_1 = 15$ cm $= 0.15$ m

$Q = 0.115$ m^3/sec



Using continuity equation

$$Q = A_1V_1 = A_2V_2$$

$$V_2 = \frac{Q}{A_2} = \frac{0.115}{\frac{\pi}{4}(0.3)^2} = 1.627 \text{ m/s}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.115}{\frac{\pi}{4}(0.15)^2} = 6.508 \text{ m/s}$$

Head loss due to sudden enlargement

$$H_f = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{(6.508 - 1.627)^2}{2 \times 9.81} = 1.214 \text{ m}$$

05. Fill up the blank:

Due to ageing of a pipeline, its carrying capacity has decreased by 25%. The corresponding increase in the Darcy Weisbach friction factor f is ________%.

05. Ans: 50%

Sol:

Given data:

Decrease in discharge = 25%

Loss of head,

$h_f \propto Q^2$, $h_f = KQ^2$ $\rightarrow$ (i)

$$f \propto \frac{1}{Q^2}$$

$$\frac{df}{f} = -2\frac{dQ}{Q}$$

$$\frac{df}{f} = -2(25\%)$$

$$\frac{df}{f} = -50\%$$

–ve indicates the decrease

06. A farmer uses a long horizontal pipeline to transfer water with a 1 H.P pump and the discharge is 'Q' litres per min. If he uses a 5 H.P pump in the same pipe line and assuming the friction factor is unchanged the discharge is approximately

(a) 5Q

(b) $\sqrt{5}\,Q$

(c) $\sqrt{5Q}$

(d) $\sqrt[3]{5}\,Q$

06. Ans: (d)

Sol: Given, constant friction factor. Friction factor refers to turbulent flow.

Power = Sp.weight × discharge× Head loss

$$P = \gamma \, . \, Q . \, h_f$$

Where,

h_f = Darcy's head loss

$$= \frac{f.L.V^2}{2gD} = \frac{8fLQ^2}{\pi^2 g D^5}$$

$$\therefore P \propto Q.h_f$$

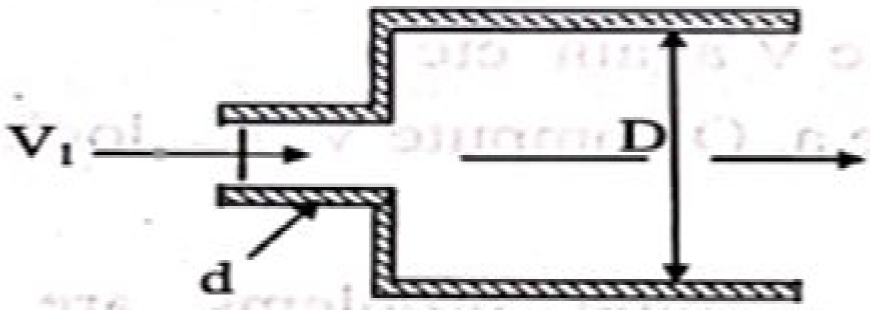
$$P \propto Q.Q^2 \Rightarrow P \propto Q^3$$

$$\frac{P_1}{P_2} = \frac{Q_1^{\,3}}{Q_2^{\,3}} \quad \text{given } \frac{P_1}{P_2} = \frac{1}{5}$$

$$\Rightarrow \frac{1}{5} = \frac{Q_1^3}{Q_2^3}$$

$$\therefore \quad Q_2 = Q.5^{1/3} = \sqrt[3]{5}.Q$$

07. The head loss coefficient in a sudden expansion shown in figure below is proportional to



(a) $\frac{d^2}{D^2}$

(b) $\left(1-\frac{d^2}{D^2}\right)^2$

(c) $\frac{D^2}{d^2}$

(d) $\frac{d^4}{D^4}$

07. Ans: (b)

Sol: Head loss, $h_L = \frac{(V_1 - V_2)^2}{2g}$

$$Q = A_1V_1 = A_2V_2$$

$$\frac{\pi}{4}d^2.V_1 = \frac{\pi}{4}D^2.V_2$$

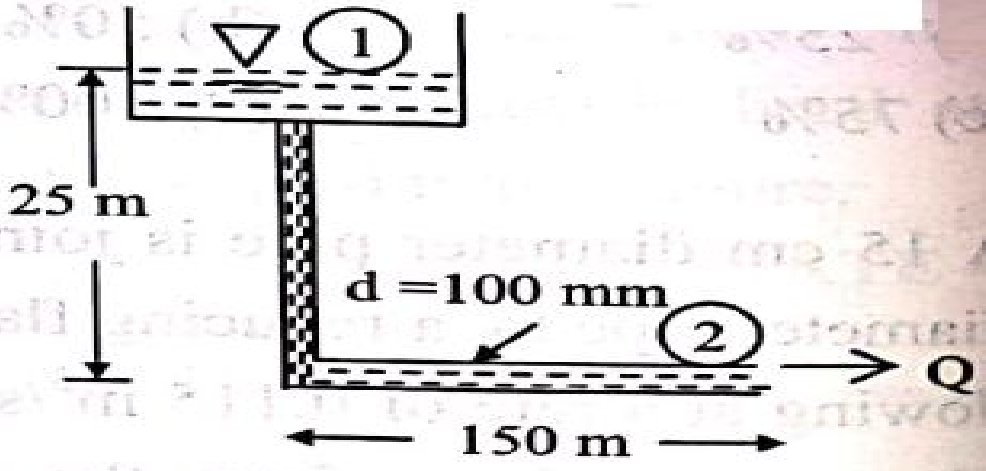
$$d^2.V_1 = D^2 V_2$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{d^2}{D^2}\right)^2$$

$$\therefore h_L \propto \left(1 - \frac{d^2}{D^2}\right)^2$$

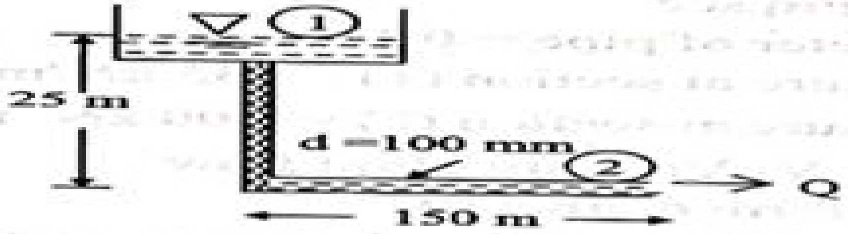
10. A fire protection system is supplied from a water tower with a bent pipe as shown in the figure. The pipe friction f is 0.03. Ignoring all minor losses, the maximum discharge, Q, in the pipe is



(a) 31.7 lit/sec
(b) 24.0 lit/sec
(c) 15.9 lit/sec
(d) 12.0 lit/sec

10. Ans: (b)

Sol:



Bernoulli's equation between (1) and (2)

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + h_L$$

$$0 + 25 + 0 = 0 + 0 + \frac{V_2^2}{2g} + h_L$$

$$25 = \frac{V_2^2}{2g} + h_L$$

$$25 = \frac{f.L.V_2^2}{2gd} + \frac{V_2^2}{2g}$$

$$25 = \frac{0.03 \times 175 \times V_2^2}{2 \times 9.81 \times 0.1} + \frac{V_2^2}{2 \times 9.81}$$

$\therefore V_2 = 3.028$ m/s

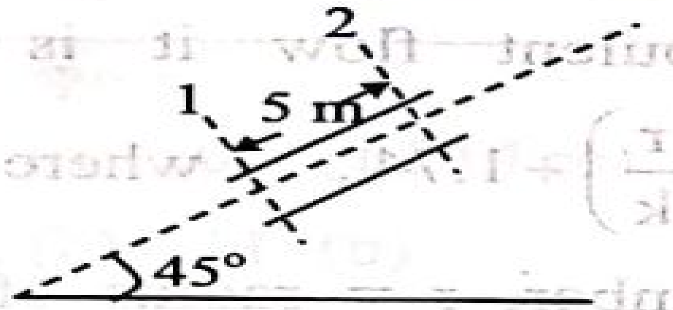
$$Q = A_2 V_2 = \frac{\pi}{4}(0.1)^2 \times 3.028$$

$= 0.024 \text{ m}^3/\text{sec} = 24$ lps

Note: velocity in the reservoir at section 1 is assumed to be zero. Both section 1 and 2 are subjected to atmospheric pressure.

Common Data for Questions 11 & 12

An upward flow of oil (mass density 800 kg/m^3, dynamic viscosity 0.8 kg/m-s) takes place under laminar conditions in an inclined pipe of 0.1 m diameter as shown in the figure. The pressures at sections 1 and 2 are measured as P_1 = 435 kN/m^2 and P_2 = 200 kN/m^2.



11. The discharge in the pipe is equal to

(a) 0.100 m^3/s (b) 0.127 m^3/s

(c) 0.144 m^3/s (d) 0.161 m^3/s

11. **Ans: (b)**

Sol: Given data:

ρ = Mass density of oil in pipe
= 800 kg/m^3

μ = Dynamic viscosity of oil
= 0.8 kg/m.sec

D = diameter of pipe = 0.1 m

P_1 = pressure at section (1) = 435 kN/m^2

P_2 = Pressure at section (2) = 200 kN / m^2

$V_1 = V_2 = V$=Velocity of oil flow (since uniform diameter)

By Bernoulli's equation between section (1) and section (2)

Assume section 1 as datum.

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + h_L$$

$$\frac{435 \times 10^3}{800 \times 9.81} + 0 + \frac{V^2}{2g} = \frac{200 \times 10^3}{800 \times 9.81} + 5.\sin 45° + \frac{V^2}{2g} + h_L$$

$55.428 = 25.484 + 3.536 + h_L$

h_L = 26.408 meters of oil

Where h_L = Loss of head due to viscosity of oil, $h_L = \dfrac{128\mu QL}{\pi \rho g D^4}$

$$\frac{128\mu.\ L.\ Q}{\pi\ \rho\ g\ D^4} = 26.408$$

$$\frac{128 \times 0.8 \times 5 \times Q}{\pi \times 800 \times 9.81 \times (0.1)^4} = 26.408$$

$\Rightarrow Q = 0.127$ m^3/sec

12. If the flow is reversed, keeping the same discharge, and the pressure at section 1 is maintained as 435 kN/m^2, the pressure at section 2 is equal to

(a) 488 kN/m^2 (b) 549 kN/m^2

(c) 586 kN/m^2 (d) 614 kN/m^2

12. **Ans: (d)**

Sol: For the flow reversed, by using Bernoulli's equation between (2) and (1)

$$\frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} = \frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} + h_L$$

$$\frac{P_2}{\rho g} + 5 \times \sin 45° = \frac{435 \times 10^3}{800 \times 9.81} + 0 + h_L$$

$$\frac{P_2}{800 \times 9.81} + 3.536 = 55.428 + 26.408$$

$$\therefore\ P_2 = 614.5 \times 10^3\ \text{N/m}^2 = 614.5\ \text{kPa}$$

14. Water flows through a 100 mm diameter pipe with a velocity of 0.015 m/sec. If the kinematic viscosity of water is 1.13×10^{-6} m^2/sec, the friction factor of the pipe material is

(a) 0.0015 (b) 0.032

(c) 0.037 (d) 0.048

14. Ans: (d)

Sol: **Given data:**

Type of fluid in flow through pipe = water

Diameter of pipe (D) = 100 mm = 0.1 m

Velocity of water flow

(V) = 0.015 m/sec

Kinematic viscosity of water (υ)

$= 1.13 \times 10^{-6}$ m²/sec

$$\text{Reynold's Number (Re)} = \frac{V.D}{\upsilon}$$

$$= \frac{0.015 \times 0.1}{1.13 \times 10^{-6}} = 1327.433$$

Reynold's number is less than critical Reynold's number value 2000. Hence fluid flow is laminar.

For laminar flow,

$$\text{Darcy's friction factor (f)} = \frac{64}{Re} = \frac{64}{1327.433}$$

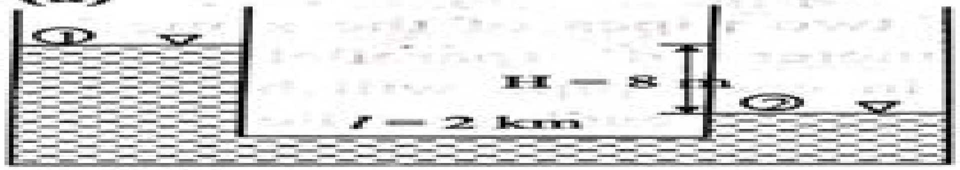
$$= 0.0482$$

16. A 2 km long pipe of 0.2 m diameter connects two reservoirs. The difference between water levels in the reservoirs is 8 m. The Darcy-Weisbach friction factor of the pipe is 0.04. Accounting for frictional, entry and exit losses, the velocity in the pipe (in m/s) is:

(a) 0.63 (b) 0.35 (c) 2.52 (d) 1.25

16. Ans: (a)

Sol:



Given data :

Length of the pipe, l = 2 km

= 2000 (m)

Diameter of the pipe, d = 0.2 (m)

Head of water, H = 8 m

Friction factor, f = 0.04

Minor losses :-

At entry = $\frac{0.5\,V^2}{2g}$

At exit = $\frac{V^2}{2g}$

Velocity of water in pipe (V) = ?

Total losses = Frictional loss + minor losses

$$= \frac{f.l.V^2}{2g.d} + \frac{0.5V^2}{2g} + \frac{V^2}{2g}$$

$$= \frac{0.04 \times 2000 \times V^2}{2 \times 9.81 \times 0.2} + \frac{0.5V^2}{2 \times 9.81} + \frac{V^2}{2 \times 9.81}$$

$$= 20.387\ V^2 + 0.0255\ V^2 + 0.051\ V^2$$

$$= 20.4635\ V^2$$

Applying Bernoulli's equation between (1) and (2):

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g} + \text{losses}$$

($P_1 = 0$, $P_2 = 0$, $Z_1 = 8$ m, $V_1 = V_2 = 0$)

$h_f = H$

$\therefore$ Total loss = Total head available

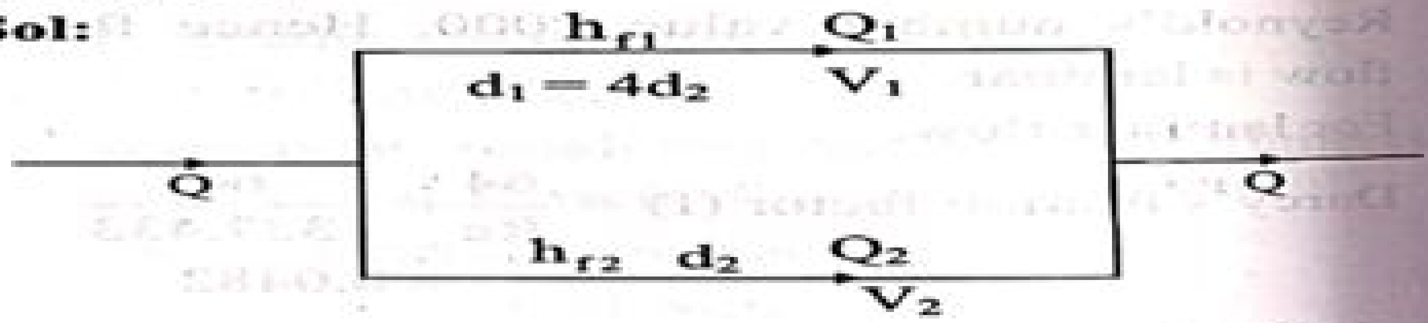
$20.4635V^2 = 8$

$\therefore$ V = 0.63 m/s

17. An incompressible fluid is flowing at a steady rate in a horizontal pipe. From a section, the pipe divides into two horizontal parallel pipes of diameters d_1 and d_2 (where $d_1 = 4d_2$) that run for a distance of L each and then again join back to a pipe of the original size. For both the parallel pipes, assume the head loss due to friction only and the Darcy-Weisbach friction factor to be the same. The velocity ratio between the bigger and the smaller branched pipes is ______

17. Ans: 2

Sol:



For parallel pipes, head loss is same

$$h_{f1} = h_{f2}$$

$$\frac{f_1.L_1.V_1^2}{2gd_1} = \frac{f_2.L_2.V_2^2}{2gd_2}$$

Given $L_1 = L_2 = L$

$$f_1 = f_2 = f$$

$$\therefore \frac{f.L.V_1^2}{2g.4d_2} = \frac{f.L.V_2^2}{2g.d_2}$$

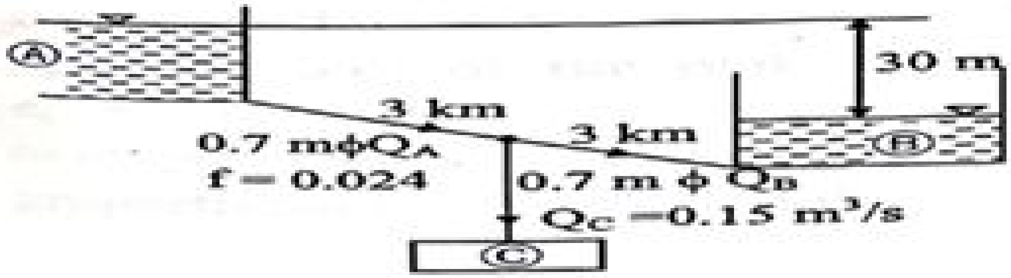
$$\frac{V_1^2}{V_2^2} = 4$$

$$\frac{V_1}{V_2} = 2$$

19. A pipe of 0.7 m diameter has a length of 6 km and connects two reservoirs A and B. The water level in reservoir A is at an elevation 30 m above the water level in reservoir B. Halfway along the pipe line, there is a branch through which water can be supplied to a third reservoir C. The friction factor of the pipe is 0.024. The quantity of water discharged into reservoir C is 0.15 m^3/s. Considering the acceleration due to gravity as 9.81 m/s^2 and neglecting minor losses, the discharge (in m^3/s) into the reservoir B is _____

19. Ans: 0.56 to 0.58

Sol:



By continuity equation:

$Q_A = Q_B + Q_C$

$Q_A = Q_B + 0.15$

For the flow from reservoir (A) to the reservoir (B)

$$\text{Loss of head} = \frac{f_A.\ell_B.Q_A^2}{12.1\times d_A^5} + \frac{f_B.\ell_B.Q_B^2}{12.1\times d_B^5}$$

$$30 = \frac{0.024\times 3000\times (Q_B + 0.15)^2}{12.1(0.7)^5} + \frac{0.024\times 3000\times Q_B^2}{12.1(0.7)^5}$$

Simplifying

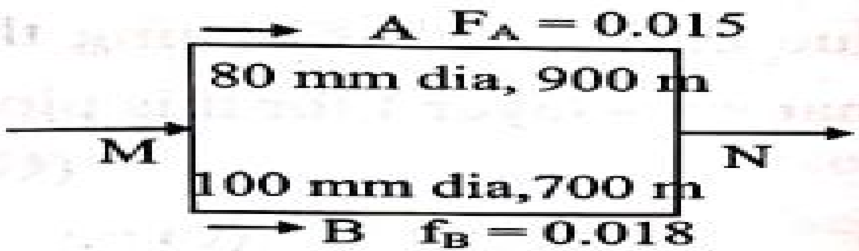
$Q_B^2 + 0.15Q_B - 0.413 = 0$

$\therefore\ Q_B = 0.572\ m^3/sec$

(considered positive value)

Five Marks Questions

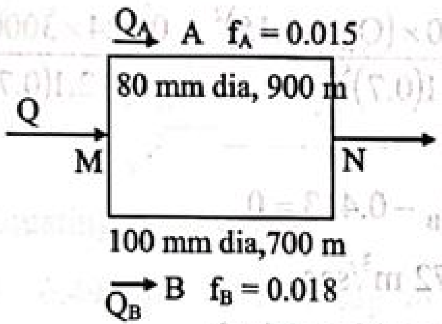
01. Two pipes A and B are connected in parallel between two points M and N as shown in the figure . Pipe A is of 80 mm diameter, 900 m long and its friction factor is 0.015. Pipe B is of 100 mm diameter, 700 m long and its friction is 0.018. A total discharge of 0.030 m^3/s is entering the parallel pipes throught the division at M. Calculate the discharge in the two pipes A and B.



01.

Sol: Total discharge, $Q = 0.030\ m^3/s$

Let Q_A and Q_B be the discharges in pipe A and B respectively



Length of pipe A, $L_A = 900$ m

Diameter of pipe A, $d_A = 80$ mm

Friction Factor of pipe A, $f_A = 0.015$

Friction factor of pipe B, $f_B = 0.018$

Diameter of pipe B, $d_B = 100$ mm

Length of pipe B, $L_B = 700$ m

$Q = Q_A + Q_B$

Head loss across pipe A = Head loss across pipe B (parallel pipes)

$$\therefore \frac{8f_A L_A Q_A^2}{\pi^2 g\, d_A^5} = \frac{8f_B L_B Q_B^2}{\pi^2 g\, d_B^5}$$

$$\Rightarrow \frac{0.015 \times 900 \times Q_A^2}{0.08^5} = \frac{0.018 \times 700 \times Q_B^2}{0.10^5}$$

$$\Rightarrow \quad Q_A = 0.55\, Q_B$$

$$Q_A + Q_B = 0.03$$

$$0.55\, Q_B + Q_B = 0.03$$

$$Q_B = \frac{0.03}{1.55} \simeq 0.02\ m^3/sec$$

$$\Rightarrow Q_A = 0.01\ m^3/sec$$


THANK YOU
VERY MUCH